Proceedings of Science and Mathematics
Faculty of Science, Universiti Teknologi Malaysia
Vol. 2, 2021, page 118-127

Application of Markov Process in Outpatient Clinic Queuing System

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Abstract Queuing problem has become a critical issue in most of the outpatient clinics. In this research, we focus on the queuing problem in Pusat Kesihatan Universiti (PKU) in Universiti Teknologi Malaysia (UTM). This research aims to apply the Markov model and simulate the real-life queuing system in PKU UTM. The simulation model is constructed by using Simul8 to test the total performance and efficiency of the queuing system. In this study, there is no congestion in all the queues for the simulation model. Therefore, the management team of PKU UTM is recommended to maintain the current queuing system.

Keywords Queuing problem; Outpatient clinic; Markov model; Simulation model; Queuing system

1 Introduction

Currently, queuing problems arise and becoming challenging tasks to most outpatient clinics. Ineffective time management will cause patients' prolonged waiting time, which will lead to cancellation and finally contribute to patients' dissatisfaction level increased. In order to improve the patient's satisfaction level and reduce congestion at the outpatient clinic, an effective queuing system is essential in every outpatient clinic. There are some relevant and detailed research or studies starting from the year 2000. This indicates that the queuing problem has received much attention or concern from researchers.

Patient flow represents the movement of patients through medical facilities. Patient flow also refers to the healthcare system's capacity to support patients quickly and safely as they pass through the treatment. It encompasses the medical services, physical infrastructure, and internal processes required to bring the patients from the point of diagnosis to discharge while ensuring the performance and patients' satisfaction. When the queuing system in the outpatient clinic is working well, patients' flow will be very smooth without any delay [1]. On the other hand, if the queuing system is broken, patients will accumulate in the outpatient clinic and finally results in congestion or chronic delays. However, patient flow is one of the main elements in enhancing quality in the provision of health services [2]. An effective patient flows ensures that queuing is reduced, while a weak patient flow indicates that the patients need to experience significant queuing delays [3].

One of the main factors causing the patient flow problem in the outpatient clinic is poor patient flow management. Poorly controlled patient flow in the outpatient clinic may contribute to unfavourable health effects, including elevated re-admissions and mortality rates. Furthermore, unproductive scheduling of resources and activities will also influence the patient flow in the outpatient clinic. Inefficient scheduling leads to the shortage or inadequate of doctors, nurses and staff in the outpatient clinic. According to Hassan, Rahman & Lumpur [4], insufficient human resources to serve significant numbers of patients and lack of time management will increase patients' dissatisfaction level. Therefore, the long waiting time will cause the patients to give up their scheduled appointment and not show up when it is their turn [5]. Consequently, a contradictory situation arises where the waiting time is extended while the capacity is underused. Besides, inefficient systems that involve more effort than the required or excessive duplication of work cause queuing problems in the outpatient clinic. For example, the registration process takes longer than expected to record patients' details and personal information. Thus, an excellent queuing model is required to suggest decision-makers in solving the patient flow problem in the outpatient clinic.

2 Preliminary

Queuing theory is defined as the study of mathematics related to the function, queues, or the congestion of waiting lines. Queuing theory is widely used in the study of manufacturing and production systems. It is covered in the mathematical statistics of operational research on queue formation [6]. It has always been applicable and instrumental in validating complicated simulator models and designing models that support narrow alternatives in the initial phases of the method.

Queuing theory encompasses a wide variety of applications. Many researchers found that queuing theory is beneficial for healthcare. Afrane and Appah [3] reveal that the implementation of queuing theory will improve decision-making in terms of optimal efficiency. They study and investigate the performance of queuing theory and simulation to the queuing problem in the outpatient clinic at AngloGold Ashanti hospital in Ghana. In addition, Fomundam and Herrmann [7] sum up a variety of queuing theory results in a waiting period and utilisation analysis. They have explored the application of the queuing principle for the study of various forms of healthcare processes Obamiro [8] implemented the queuing theory to minimise a pregnant women's waiting time by determining the maximum number of staff available in an antenatal clinic of a Nigerian Hospital.

Queuing system is a series of tools and sub-systems that help monitor traffic movement, track waiting times, and enhance customers' experience in different industries. Queuing system must follow the first-come-first-serve technique. A queuing problem is defined as the queue exists when there are more people than workers to serve them. If all of the servers are busy when new customers arrive, these will generally wait in line for the next available server. Problems emerge when lines get more protracted than expected, and this will lead future customers to leave. Putting a good queue management system in place will enhance customers' satisfaction levels and decrease the waiting times simultaneously.

A queuing model will be formulated in solving the real-life queuing system problem. Queuing models are utilised in many kinds of research to solve queuing problems in different aspects of the hospitals. For example, Vass and Szabo [9] implement an M/M/3, multiple server queue model in the emergency department to characterise the flow of patients at Mures Country, Romania. The research demonstrates how queuing models are used for decision-makers in identifying the optimal solutions. Furthermore, Basri and Mardiah [2] implemented M/M/1/1 queue model in the appointment system at Indonesia's public hospital. The queue system is a single channel multi-phase system of two single servers and is based on a first-come-first-serve basis. The main focus of the research is to improve the productivity and efficiency of capital and capacity.

Simulation modelling is the process by which a virtual prototype of a physical model is created and analysed to estimate its efficiency in actual life. The queuing and patient flow systems are always connected to the simulation models. The simulation model is an effective instrument for testing and examining emerging prototypes of structures, modifying existing systems, and recommending improvements to control systems [5]. Bahadori et al. [10] modelled and simulated the queuing system based on the pharmacy performance in a military hospital in Iran using ARENA software, version 12. They aim to improve the operation of studied ambulatory pharmacy through the development of useful queuing theory and simulation models. The results obtained by applying the simulation model in this research showed that the pharmacy's queue characteristics for morning and evening shifts were particularly unfavourable. In another study, Nor Aziati & Hamdan [5] used ARENA simulation software to model and simulated the queuing system according to the patient situation in the outpatient department of the Public Health Clinic. Based on the findings of the simulation model of patient flow in the outpatient clinic, the waiting time is proven to be achieved according to the Ministry of Health patient charter.

3 Queuing Model

A queuing model provides advantages to the outpatient clinics, and the outcome will contribute to the clinics that aim to improve the healthcare service system and shorten the waiting time of patients at the outpatient clinic. The main characteristics of the queuing model are the arrival and service patterns, queue discipline, system capacity, number of service channels and service phase.

3.1 Arrival Rate of Patients

In March 2019, there are 4456 patients visited PKU UTM for different purposes. In this study, to calculate the arrival rate of the patients, we are required to decide between the register time and the patients' scan time in the consultation room. A one-tailed Z-test is used to test whether the mean time difference between the register time and the scan time exceeds two minutes. The test statistic, z_{test} obtained is 0.3202 which is less than the critical value, $z_{0.05} = 1.6449$, hence there is statistical evidence to show that the H_0 is not rejected at $\alpha = 0.05$. We can conclude that the mean time difference between the register time and scan time of the patients was not exceeded two minutes.

The register time of the patients is chosen to calculate the interarrival time of the patients The daily arrival rate is calculated by using the formula

$$Mean\ Interarrival\ time = \frac{Total\ interarrival\ time\ (hour)}{Total\ number\ of\ patients} \tag{1}$$

$$Arrival\ rate, \lambda\ = \frac{1}{mean\ interarrival\ time} \tag{2}$$

Therefore, the arrival rate, λ of the patients in PKU UTM is 16.1778 patients per hour.

3.2 Service Rate of Patients

The service time of a patient is obtained by calculating the time difference between the calling time of the first and second patient. The calling time for the second patient indicates the completion of the first patient's service. The service rate, μ of the queuing system can be calculated by using the formula

$$\begin{aligned} Service\ rate, \mu &= \frac{Total\ number\ of\ patients}{Total\ service\ time\ (hour)} \\ &= \frac{1}{Mean\ service\ time} \times 60\ minutes \end{aligned} \tag{3}$$

However, by conducting a single factor or one-way Analysis of Variance (ANOVA) to determine whether there is any significant difference between the mean service time of all consultation rooms. The results showed that the F value is 0.0678 which is less than the $F\ crit\ (f_{0.05,5,100})$ value, 2.3053, this indicates insufficient evidence to reject H_0 at 5% significance level. This concludes that the mean service time for all the consultation rooms is the same. The calculation of the service rate, μ for lab sample collecting room, X-ray room and pharmacy is the same by using equation (3). The service rate, μ for each station is shown in Table 1.

Table 1: Service Rate for each Station in PKU UTM

Station	Consultation Room	Lab Sample Collecting Room	X-ray Room	Pharmacy
Service rate, μ (patients per hour)	3.4362	4.2390	4.9440	17.7649

3.3 Limiting Probabilities

In March 2019, there are 4456 patients visited PKU UTM for different purposes. There are more than half of the patients which is around 77% went to the consultation room for their first station. Therefore, our main focus is the patients who went to the consultation room as their first station.

In the PKU UTM system, there are six consultation rooms with unlimited queuing capacity. Therefore, it is a multiple server model with Kendall-Lee notation, $(m/m/6):(GD/\infty/\infty)$. From the notation, m denotes as the Markovian arrivals or departures of the patients, 6 shows that there six servers in the queuing system, GD is defined as General Discipline and ∞ shows the infinite number of patients allowed in the system and calling source. The system utilization, ρ is 4.7080 calculated by using the formula

$$\rho = \frac{\lambda}{\mu} \tag{5}$$

Next, the limiting probabilities of the queuing system for 0 and n patients in the system are computed by using formula

$$P_0 = \left[\sum_{n=0}^{c} \frac{\rho^n}{n!} + \frac{\rho^c}{c!}\left(\frac{\frac{\rho}{c}}{1-\frac{\rho}{c}}\right)\right]^{-1} \tag{6}$$

$$P_n = \begin{cases} \frac{1}{n!}\rho^n P_0, & n \le c \\ \frac{1}{c!\,c^{n-c}}\rho^n P_0, & n > c \end{cases} \tag{7}$$

Hence, the limiting probability of the queuing system for 0 patient is 0.0069 while the limiting probabilities of n patients in the system are listed in Table 2.

Table 2: Limiting Probabilities of n Patients in the System

n	P_n	n	P_n	n	P_n	n	P_n	n	P_n
1	0.0325	7	0.0819	13	0.0191	19	0.0045	25	0.0010
2	0.0765	8	0.0643	14	0.0150	20	0.0035	26	0.0008
3	0.1200	9	0.0504	15	0.0118	21	0.0027	27	0.0006
4	0.1412	10	0.0396	16	0.0092	22	0.0022	28	0.0005
5	0.1330	11	0.0310	17	0.0072	23	0.0017	29	0.0004
6	0.1044	12	0.0244	18	0.0057	24	0.0013	30	0.0003

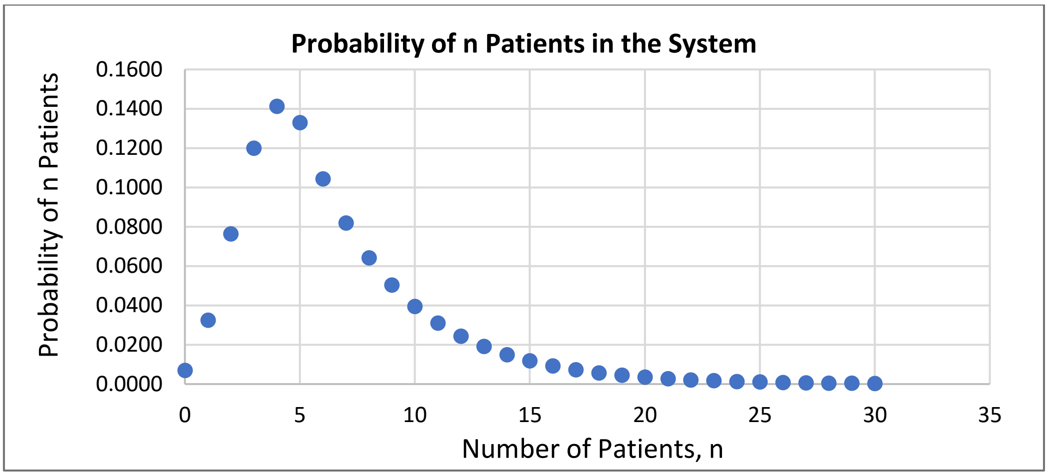

Figure 1: Probability of n Patients in the System

According to Table 2 and Figure 1, it shows that the highest probability is 0.1412 where $n = 4$. This indicates that the most likely number of queries in the system is $n = 4$.

3.4 Hypothesis Testing

Hypothesis testing is vital to check whether the data is followed the specified distribution. However, we study Chi-square Goodness-of-fit-test in this research. The Chi-square goodness-of-fit test also known as χ^2 test is a valid statistical hypothesis test when the statistics of the Chi-square test are distributed in the null hypothesis. The objective of this Chi-square test is to assess how likely the observed frequencies are to assume that the null hypothesis is true. In this study, Chi-square goodness-of-fit test is chosen to test the probability distribution of the arrival process and service time. Both interarrival time and service time are assumed to follow the exponential distribution. Test Statistics is given by

$$\chi_0^2 = \sum_{j=1}^{k} \frac{(O_j - E_j)^2}{E_j} \tag{8}$$

The probability density function of the exponential distribution is

$$f(x) = \begin{cases} \lambda e^{-\lambda t}, & x \geq 0 \\ 0, & otherwise \end{cases} \tag{9}$$

Hence, the probability function of the interarrival time is

$$P(x < A < y) = \int_x^y \mu \times e^{-\mu a}\, da \tag{10}$$

and the expected frequency, E for each interarrival time interval is computed by

$$E = \sum O \times P(x < A < y) \tag{11}$$

Since the test statistic, $\sum \frac{(O_i - E_i)^2}{E_i}$ is approximated to χ^2 when all the expected frequencies are greater or equal to 5; therefore, when the expected frequency is less than 5, the group must merge with other groups until the condition is satisfied. The hypothesis and results of hypothesis testing for arrival process and service time are listed in Table 3 and Table 4. The hypothesis testing is conducted at 5% significance level

Table 3: Chi-Square Goodness-of-Fit Test for Arrival Process

Hypothesis	Conclusion
H_0: PKU data on interarrival time follows exponential distribution H_1: PKU data on interarrival time does not follow exponential distribution	Test statistic, $\chi^2 = 10.3787$ less than critical value, $\chi^2_{0.05,9} = 16.9190$. This indicates there is insufficient evidence to reject H_0. We can conclude that PKU data on interarrival time follows the exponential distribution.

Since the result of single-factor ANOVA showed that the mean service time for all the consultation rooms is the same. Consultation room 2 is chosen for the chi-square goodness-of-fit test to determine the probability distribution of the service time.

Table 3: Chi-Square Goodness-of-Fit Test for Service Time for each Station

Station	Hypothesis	Conclusion
Consultation Room 2	H_0: PKU data on service time follows exponential distribution H_1: PKU data on service time does not follow exponential distribution	Test statistic, $\chi^2 = 14.7724$ less than $\chi^2_{0.05,10} = 18.3070$. There is sufficient evidence to reject H_0 at $\alpha = 0{,}05$. We can conclude that the PKU data on service time of consultation room 2 follows the exponential distribution. Hence, the service time for all consultation rooms is followed exponential distribution.
Lab Sample Collecting Room	H_0: PKU data on service time follows exponential distribution H_1: PKU data on service time does not follow exponential distribution	Test statistic, $\chi^2 = 12.9786$ less than $\chi^2_{0.05,8} = 15.5073$. There is no enough evidence to reject H_0. The PKU data on service time of the lab sample collecting room follows the exponential distribution.
X-ray Room	H_0: PKU data on service time follows exponential distribution H_1: PKU data on service time does not follow exponential distribution	Test statistic, $\chi^2 = 3.5056$ less than $\chi^2_{0.05,2} = 5.9915$. There is insufficient evidence to reject H_0 at 5% significance level. We can conclude that the PKU data on service time of the X-ray room follows the exponential distribution.
Pharmacy	H_0: PKU data on service time follows exponential distribution H_1: PKU data on service time does not follow exponential distribution	Test statistic, $\chi^2 = 16.3022$ less than $\chi^2_{0.05,9} = 16.9190$. There is no enough evidence to reject H_0. We can conclude that the PKU data on service time of pharmacy follows the exponential distribution.

The hypothesis testing results showed that the interarrival time of the patients and service time for all stations are followed the exponential distribution.

4 Simulation Model

Simulation modelling is the method by which a computer prototype of a physical object is generated and analysed to predict its real-life results. In this study, we chose Simul8 as our simulation software to conduct the simulation for PKU UTM. Simul8 is a discrete event simulation software that enables the users to construct a simulation model considering the real-life constraints, failure rates and other aspects that influence the overall output performance and quality. The purpose to build the simulation model is to provide a more efficient queuing model for the management team in the outpatient clinic.

The duration of the simulation is 5 days a week, starting from 8 a.m. and the duration is 9 hours per day. The Simulation model starts with a starting point which is considered as the arrival of patients in the outpatient clinic. There is a queue after every starting point and work center. After that, all the patients are directed to the registration counter which is Work Centre 1. The registration time or service time of the patients on the registration counter is assumed deterministic with a mean of 2 minutes per patient.

After the registration counter, all the patients went to the consultation rooms as their first station. In this queuing system, there are six consultation rooms which are known as Consultation Room 1, 2, 3, 4, 5 and 6. The queue follows the first come first serve (FCFS) queue discipline. Next, all the patients went to their next station for different purposes according to their requirements such as lab sample collecting room, X-ray room, pharmacy or exit. Since the simulation model is constructed according to the actual scenario of the outpatient clinic, hence the routing out percentage to each station is calculated. After that, all the patients are eventually going to the same endpoint in the outpatient clinic.

Figure 2 below shows the simulation model 1. By referring to the simulation model below, it shows that there is no congestion in each station.

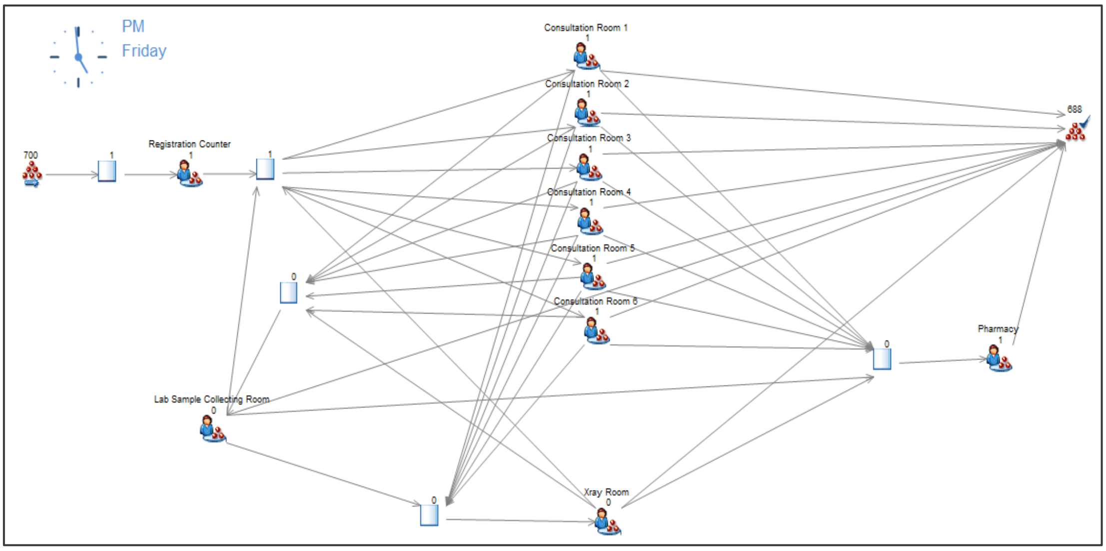

Figure 2: Simulation Model 1

The setting for each starting point, queues and work centers are listed in Table 4, Table 5 and Table 6. All the settings for consultation rooms are the same except the routing out percent discipline.

Table 4: Setting of Simulation Model 1

Activity	Distribution	Setting
Starting Point	Average	Mean interarrival time = 3.7080
Consultation Room 1	Exponential distribution	Mean service time = 17.4611
Consultation Room 2		
Consultation Room 3		
Consultation Room 4		
Consultation Room 5		
Consultation Room 6		
Lab Sample Collecting Room	Exponential distribution	Mean service time = 14.1542
X-ray Room	Exponential distribution	Mean service time = 12.1360
Pharmacy	Exponential distribution	Mean service time = 3.3774
All Queues	Default	
End Point	Default	

Table 5: Routing Out Percent Discipline of Consultation Rooms

Consultation Room	1	2	3	4	5	6
Lab Sample Collecting Room	12.8857 %	10.1338 %	9.8507 %	11.9382 %	12.3223 %	6.5886 %
X-ray Room	5.0817 %	4.3977 %	3.1343 %	2.9494 %	1.8957 %	2.7818 %
Pharmacy	68.7840 %	70.3633 %	71.9403 %	71.7697 %	73.4597 %	77.8917 %
End Point	13.2486 %	15.1052 %	15.0746 %	13.3427 %	12.3223 %	12.7379 %

Table 6: Routing Out Percent Discipline of Lab Sample Collecting Room and X-ray Room

	Consultation Rooms	Lab Sample Collecting Room	X-ray Room	Pharmacy	End Point
Lab Sample Collecting Room	68.0328 %	-	3.2787 %	17.7596 %	10.9290 %
X-ray Room	41.4063 %	7.0313 %	-	40.6250 %	10.9375 %

Next, we reduced the number of consultation rooms from 6 to 5 while the number of counters for other stations remained the same. In this case, we removed consultation room 6 and reconstructed a simulation model called simulation model 2. All the settings for consultation rooms are the same except the routing out percent discipline. The simulation model is shown in Figure 3. Figure 3 shows that there is congestion in the queue for consultation rooms and pharmacy. There are still 29 patients waiting in the queue for consultation rooms while there are 6 patients wait for their turns in the pharmacy at the end of the simulation.

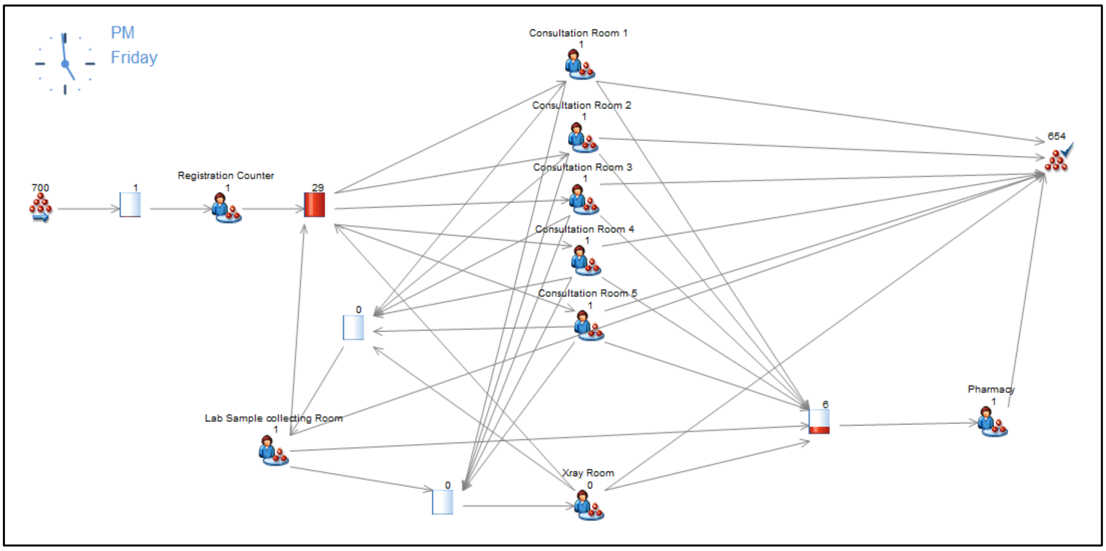

Figure 3: Simulation Model 2

The queue results for simulation model 1 and 2 listed in Table 7. It shows that the queue size and queuing time of the patients at the registration counter remained the same. However, the average queuing time and queue size in the consultation rooms increased drastically due to the reduction of consultation room 6. For the lab sample collecting room, the average queuing time and queue size of the patients rose. Moreover, both queuing time and queue size of the patients in the and X-ray room pharmacy decreased.

Table 7: Queues Results for Simulation Model 1 and 2

	Simulation Model 1				
Queues	Registration Counter	Consultation Rooms	Lab Sample Collecting Room	X-ray Room	Pharmacy
Average Queuing Time	1.04	6.62	3.68	2.59	13.06
Maximum Queuing Time	7.76	31.6	42.45	38.59	63.96
Average Queue Size	0.26	1.85	0.08	0.02	2.79
Maximum Queue Size	4	12	2	1	15
	Simulation Model 2				
Queues	Registration Counter	Consultation Rooms	Lab Sample Collecting Room	X-ray Room	Pharmacy
Average Queuing Time	1.04	33.93	7.24	0.65	5.82
Maximum Queuing Time	7.76	140.14	85.43	8.33	32.59
Average Queue Size	0.26	9.78	0.23	0.01	1.18
Maximum Queue Size	4	47	4	2	10

5 Conclusion and Recommendation

This study focuses on the queuing problem in the outpatient clinic which has become a critical issue in our country especially the outpatient clinic in the hospital. Hence, there happened congestion and prolonged waiting time of patients and this has caused the inefficiency of healthcare services in the outpatient clinic. Therefore, it is vital to improve the queuing system and enhance the resource planning of the outpatient clinic.

In this study, we focus on the queuing system in the outpatient clinic in UTM. For the limiting probabilities, we mainly focus on the first station which is the consultation rooms. The multiple server queuing model, $(m/m/6):(GD/\infty/\infty)$ is obtained according to the behaviour of the queuing system in PKU UTM. From the simulation results, there is congestion in the queue for consultation rooms and pharmacy for simulation model 2. This indicates that six consultation rooms are sufficient compared to five consultation rooms. The average waiting time for a patient and queue size in all the stations for simulation model 1 is reasonable which is not more than 15 minutes compared to simulation model 2.

The management team of PKU UTM is recommended to maintain the number of resources in the current queuing system. The replacement of absence doctors is required to maintain the number of resources in the queuing system. According to the Malaysian Ministry of Health's patient charter's suggestion, the waiting time for a patient to see the first provider and the moment a pharmacist receives the patient's prescription until the dispensing of medicine should less than 30 minutes. The management team is suggested to include the estimated waiting time in the patient's registration slip. Moreover, PKU UTM can provides the drive thru services for the patients who need to see a doctor or medicine dispensing. This will reduce the chance of a patient in contact with other patients in the clinic and decrease the waiting time of a patients Lastly, PKU UTM is suggested to open a special counter as an express lane for the patients who have done their medical check-up. Therefore, the patients will not require to repeat the queuing process for the consultation room.

6 References

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