Proceedings of Science and Mathematics
Faculty of Science, Universiti Teknologi Malaysia
Vol. 2, 2021, page 148-155

Product Mix Optimization at Minimum Supply Cost of an Online Clothing Store using Linear Programming

[1]Ling Jyi Ying and [2]Nur Arina Bazilah Aziz

[1,2]Department of Mathematical Sciences
Faculty of Science, Universiti Teknologi Malaysia,
81310 Johor Bahru, Johor, Malaysia.

e-mail: [1]j.ying-1997@graduate.utm.my, [2]nurarina@utm.my

Abstract This study will investigate an optimal product mix by using Linear Programming (LP) to minimize the supply cost. An online clothing store is considered as the case study that contains two product lines supplied from four different suppliers. The demand of every product, minimum order quantity from suppliers, shipping fee and the total cost of each product have been collected from the article. As conclusion, this study has achieved the objective to construct an easy-to-use system that helps the user to make decision on their order quantity as well as to minimize the total supply cost.

Keywords Linear Programming; Online Clothing Store; Optimization; Product Mix; Simplex Method; Supply Cost.

1 Introduction

Recently, the pandemic COVID-19 had boosted the e-commerce rapidly. Amanda [1] found that the changes of lifestyle of the people had increased the e-commerce with the statistics from Google and Temasek which predicted that Malaysia's e-commerce economy could reach US$ 13 billion by 2025. This is because the pandemic affects the economic seriously, especially for the retailers. To adapt the situation, retailers started to promote their business online as people encourage to stay at home and this had changed their shopping behavior to online [2]. Thomas [3] revealed that ecommerce sales are predicted to have for 18.1% of retail sales worldwide. He commented that e-commerce is taking over slowly, and it broaden in various direction and becoming more structural part of the consumer experience globally. Although this trend provides tremendous opportunity for new entrepreneurs, but it was not easy to stand up and survive in this intense competition.

The goal can be achieved if they success in marketing their products and an optimal product mix. By optimizing the product mix, the production costs can be reduced by 5% to 10% [4]. The retailers are continually looking for ways to make the right decisions to achieve their objectives of either to maximize profit or minimize cost with limited resources. However, with no measurement system in place it will be difficult to achieve significant cost reduction because retailers will be relying on 'gut feel' and intuition rather than facts [5].

To determine these decision problems, various theoretical and quantitative techniques have been implemented. One of such techniques is the linear programming model, which apply mathematical method in finding for the ideal strategy in any decision situation under the constraint of limited resources and uncertainties. This study will investigate on an online clothing store that has been operating for two months using Linear Programming (LP) to minimize the product mix cost. Therefore, an optimal product mix will be obtained by using Solver and sensitivity analysis and develop a program with the Visual Basic of Applications (VBA) in Microsoft Excel.

The owner should aim to get the optimal solution of the product mix to minimize the cost which involves the purchasing cost and shipping cost [6]. The purchasing cost is the cost that purchase the cloths where the shipping costs vary due to the number of cloths. Thus, they need to estimate the quantity of every product and minimizes the total cost. However, some restrictions of limited resources and uncertainties that limit the owner to have the optimal solution. By modelling the LP model based on the constraints, the owner able to determine the number of each product to be purchase. The LP model provide an alternative strategy for optimization and the simulation supported by Microsoft Excel Solver.

Therefore, the objectives of the research are formulating a product mix linear programming model by using simplex method, to construct the sensitivity analysis in Microsoft Excel, to minimize the supply cost of the product mix and to develop the product mix planning system in Microsoft Excel.

2 Literature Review

This section presents an overview of operation research projects regarding product mix problem that have been conducted in various industries. Optimization approach is very important in production planning, and it was widely used to maximize the profit and minimize the costs. The concept of product mix problem and the contribution of optimization had been done will be discussed. Besides, this study also discusses the theory and application of LP in product mix problem

2.1 Product Mix Problem

Johnson and Montgomery [7] revealed that the product mix problem is modelled by mathematical technique. It is a prototype case of direct implementation of Linear Programming (LP) approach [8], and also defined as “deterministic product mix problems”. Traditionally, the problem in determining the quantity of each product to produce in a specific period, intention to maximize the total revenue of the organization, constraint by the resources availability is defined as the product mix problem, which is a common problem area in many industries [8]. It is also defined as the quantity determination of a provided set of variables subject to resource limitation to maximize profit or minimize cost, which able to explain as the “traditional production planning problem”. The similar case related with the product alternative decision is also called as “strategic production planning problem” in the study from Alonso-Ayuso et al [9]

2.2 Concept of Linear Programming

The simplex method is used in this study to solve the linear programming blending algorithm for minimizing supply costs. This method is an algorithm that moves successively from one maximum edge to another until it achieves the optimal solution. If the beginning of the objective function is feasible, it will act as the initial extreme point. This method considers all nearby extreme points at each iteration and moves to the one that provides the most increment in the objective function. The iteration keep going from one nearby maximum value to another and lastly terminate when the optimal solution is found [5]. The simplicity of the simplex method had encouraged its application in optimization.

2.3 Assumptions of Linear Programming

Before formulating the LP Model, the following assumptions were taken into consideration:
1. Proportionality: The contribution of each decision variable in the objective function and in the constraints is directly proportional to the value of the variables.
2. Additivity: The total contribution of all the variables in both the objective function and the constraints is the sum of the individual contributions of each variable.
3. Certainty: All the objectives and constraints coefficients are deterministic and do not change during the period being studied.
4. Divisibility: The solutions need not be in whole numbers and may take any fractional value.

2.4 Limitations of Linear Programming

Although LP has several benefits and applications, but it is not free from shortcomings. There are some limitations in solving the problem. Vinay [10] had discussed that LP is less realistic. In real life situations, most of the business and product mix problems are nonlinear in nature while LP can only solve the objective function and constraints in linear form. LP also deals with only single objective, whereas we may struggle multi objective problems in real life situations. Also, parameters determination in LP are expected to be constant, but in real life situations it depends in many uncertainties. Vinay [10] also pointed that LP presents trial and error solutions to problems, and it is difficult to find real optimal solutions to various business problems. Despite these limitations, linear programming is extensively used in taking business decisions. Most of the limitations of linear programming can be solved by developing nonlinear programming techniques [11]. Furthermore, considering the technicalities, managers who are familiar with its rigorous models avoid its usage on the grounds that they do not understand how it could be applied.

3 Methodology

From the previous chapter, it was shown that many researchers' work about the product mix problem. This chapter will discuss the methodology of formulating linear programming (LP) model, solve it by using simplex method and the application of sensitivity analysis. The information of the data is obtained based on research done by Molina [6]. However, the result of the research only shows the final quantity of the product that was difficult for the owner to apply the LP formulation by him/herself in future and interpret the results. Therefore, this study also designs the Product Mix Plan System for the owner by just inserting the parameters and shows the result with just one click.

An LP model consists of three basic components, namely: decision variables, objective function, constraints. Decision variables are the variables to be determined; objective function is the goal to be optimized (maximize or minimize); and the constraints are the restrictions or conditions to be satisfied. The table 3.1 to 3.3 shows the formulation of LP model

Table 3.1 Indices used in the mathematical model

Indices	Definition	
i	Supplier	*(i=1,..,4)*
j	Size of the cloths	*(j=1,..,5)*
k	Type of products	*(k=1,2)*

Table 3.2 Decision variable in mathematical model

Decision variable	Definition	
x_{ij}	Quantity of clothes allocated	$i = 1,2,3,4 \text{ } and \text{ } j = 1,2,3,4,5$

Table 3.3 Coefficients in mathematical model

Coefficients	Definition	Unit
C_{ij}	Cost	RM
D_{kj}	Demand	quantity
Q_i	Quantity of product to order according to shipping fee	quantity
M_i	Minimum order quantity	quantity

The objective function aims to minimize the total supply costs which include the shipping cost. However, the total of shipping fees is dependent on the quantity of products ordered, then the shipping cost need to be calculated after determining the optimal quantity of the product to be ordered. Therefore, it need not be reflected in the objective function. The objective function is shown as equation 3.1.

$$\text{Min } Z = \sum_{j=1}^{5} \sum_{i=1}^{4} C_{ij} x_{ij} \tag{3.1}$$

Each of the size of the products have its demand, let the demand of the product denoted by D_{kj} where k is the type of the products, k = 1, 2. The constraints can be illustrated as equation 3.1.

$$\sum x_j \geq D_{kj} \tag{3.2}$$

The sum of the is x_j the same type of product but from different suppliers.

Then, every supplier has a list of shipping fee according to the quantity of the product purchased. The shop owner has its maximum price, P to purchase shipping fee on every supplier. There is maximum quantity of the cloths, Q_i, to order from every supplier that corresponding to the cost limit, P. In addition, some of the suppliers offer free shipping with a minimum of a certain quantity of orders. The constraint is denoted by equation 3.3 and 3.4.

$$\sum x_i \leq Q_i \tag{3.3}$$

If $Q_i >$ quantity that offers free shipping,

$$\sum x_i \geq Q_i \tag{3.4}$$

Every supplier had the minimum order of the quantity, M_i , the constraints is denoted by equation 3.5.

$$\sum x_i \geq M_i \tag{3.5}$$

where $x_{(i)}$ is the sum of the product under same supplier.
Lastly, for the non-negativity constraints,
$x_{ij} \geq 0, i = 1,2,3,4 , j = 1,2,3,4,5$

3.1 Design and Implementation of Product Mix Planning System

To solve the problem, the retailers need to set up the objective function and constraints by themselves but sometimes it is inconvenient for them to learn and apply it. Therefore, an easy-to-use system for the problem is needed. A system is developed in MS-Excel with the Solver to solve the product mix problem. The product mix plan system includes four parts: cost of cloths, constraints, solver, and sensitivity analysis. The dashboard of the system is as shown as figure 3.1 as below:

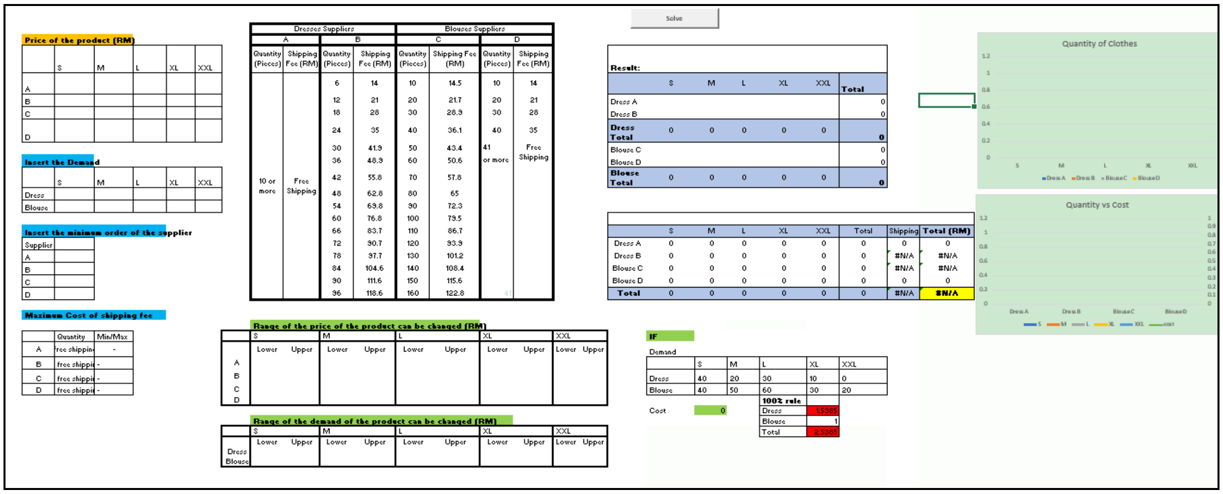


Figure 3.1 Dashboard of the system

4 Result

The LP model modelled in previous chapter was solved using the simplex method. To hold the simplex procedure, the Microsoft Excel software was used. The optimal solution for this model is as table 4.1:

Table 4.1 Result

Products	S	M	L	XL	XXL	Total
Dress A	0	20	4	10	0	34
Dress B	40	0	26	0	0	66
Dress Total	40	20	30	10	0	100
Blouse C	0	0	50	30	20	100
Blouse D	40	50	10	0	0	100
Blouse Total	40	50	60	30	20	200

From the table, the optimal solution is:

$x_{21} = 40, x_{12} = 20, x_{13} = 4, x_{23} = 26, x_{14} = 10, x_{41} = 40, x_{42} = 50, x_{33} = 50, x_{43} = 10, x_{34} = 30, x_{35} = 20$

$x_{11} = x_{22} = x_{14} = x_{15} = x_{25} = x_{31} = x_{32} = x_{44} = x_{45} = 0$

$z = \text{RM}3921.8$

Considering the shipping fee charged by the suppliers, we have free shipping offered by supplier A provided that the minimum number of dresses (at least 10) was satisfied. Based on the optimal solution obtained, the total number of dresses to be purchased is 34, hence free shipping fee may be availed. For supplier B, since based on the optimal solution obtained, the total number of dresses to be purchased is 66, hence the shipping fee to be charged is RM83.70. For supplier C, since based on the optimal solution obtained, the total number of blouses to be purchased is 100, hence the shipping fee to be charged is RM79.50. For supplier D, since based on the optimal solution obtained, the total number of blouses to be purchased is 100, hence free shipping may be availed.

Therefore, adding the shipping fees (RM83.70 + RM79.50 = RM163.20) charged to the optimal supply cost (RM3921.80), the most economical total supply cost is RM4850.00. The most economical product mix is summarized in table 4.2.

Table 4.2 Optimal Product Mix

	S	M	L	XL	XXL	Total	Shipping	**Total (RM)**
Dress A	0	406	88	237	0	731	0	731
Dress B	680	0	527.8	0	0	1207.8	83.7	1291.5
Blouse C	0	0	465	306	220	991	79.5	1070.5
Blouse D	372	510	110	0	0	992	0	992
Total	372	510	575	306	220	3921.8	163.2	**4085**

4.1 Discussion

Although the optimal solution is determined, there will some uncertainties in the real world. The variable of the constraint could change from time to time. However, it is time costly if we want to recalculate the parameter of the decision variable. By observing sensitivity report, we could determine the effect of the changes to the cost without changing the optimal solution. The table 4.3 and 4.4 shows the range that the variable could change without changing the optimal solution.

Table 4.3: Range of the price of the product can be changed (RM)

x	S		M		L		XL		XXL	
	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
A	18.70	infinity	0.00	20.40	22.00	22.00	0.00	23.70	0.00	1.70
B	0.00	17.00	18.60	infinity	20.30	20.30	22.00	infinity	0.00	infinity
C	7.60	infinity	8.50	infinity	9.30	9.30	0.00	10.20	0.00	11.00
D	0.00	9.40	0.00	10.20	11.00	11.00	11.90	infinity	12.70	infinity

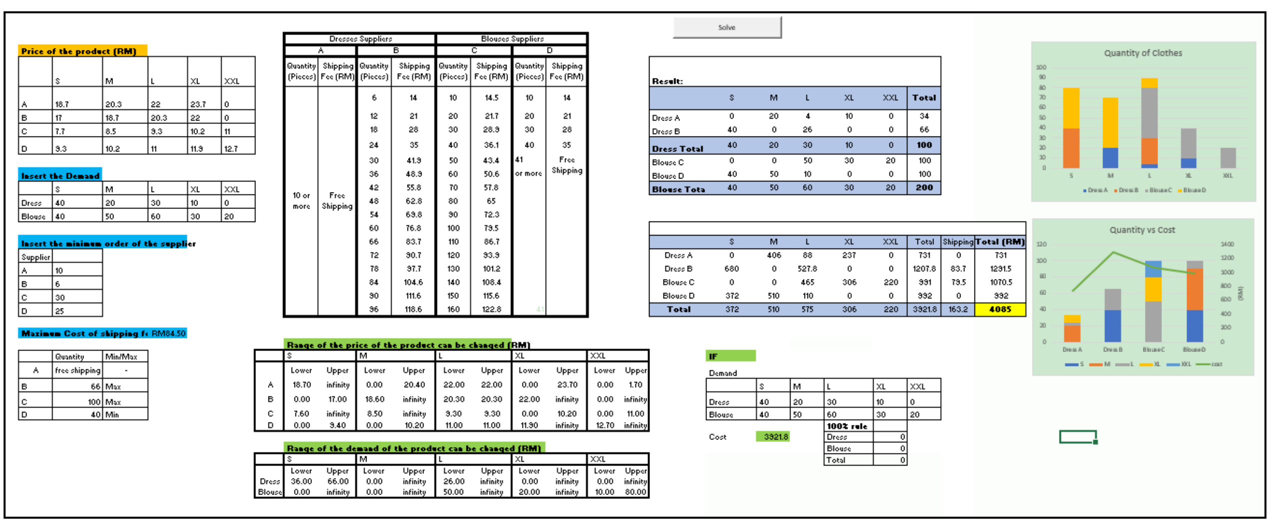
Table 4.3 indicate that the range of price could change without changing the optimal solution. For example, if the price of the medium size of dress from supplier A is recorded wrongly which the price is actually RM20.40 instead of RM 20.30, since the price is within the optimality, therefore it would not change the optimal solution and the new cost would be: RM4085 + (RM0.10*20) = RM4087.00.

Table 4.4: Range of the RHS constraint of demand can be changed (RM)

	S		M		L		XL		XXL	
	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Dress	30.00	60.00	0.00	infinity	20.00	infinity	0.00	infinity	0.00	infinity
Blouse	0.00	infinity	0.00	infinity	50.00	infinity	20.00	infinity	10.00	80.00

Table 4.4 shows the effect on the cost if the constraint of the demand changes. For example, if the demand of the small size of dress is decreased from 40 to 30, which is decreased by 10 and it is within the range and the new cost is estimated as RM3921.8 – (10*18.7) = RM3734.8 while the shipping fee remains constant RM3734.8 + RM163.2 = RM3898.

The result of the optimal product mix in the system is displayed as figure 4.2:



5 Conclusion

Product Mix Problem has been a topic of business management for many years. It is used to determine the optimal product mix with subjected relative constraints such as minimum order quantity, demand. This study focused on product mix determination by considering an online clothing store as a case study with the aim of applying linear programming in determining the most economical product mix. Upon applying the simplex method using Microsoft Excel software, this study also aims to develop an easy-to-use system for the owner to solve the product mix problem by his/herself. The system is called Product Mix Planning System. It was found that the most economical product mix to be purchased is 20 medium size dresses and 4 large size dresses and 10 extra-large size dress from supplier A; 40 small size dresses, 26 large size dresses, from supplier B; 50 large size blouses, 30 extra-large size blouses, and 20 double extra-large blouses from supplier C; and 40 small size blouses, 50 medium size blouses and 10 large size blouses from supplier D. Hence, adding the shipping fees (RM83.70) charged to the optimal supply cost (RM79.50), the most economical total supply cost is RM4850.

6 Future Work

This study is recommended to further study on determine the optimal shipping cost. This is because the maximum shipping cost in this study is decided by the owner his/herself. LP unable to determine the minimum total shipping cost from all suppliers this is because the shipping cost is varied due to the total number ordered from every supplier. It is recommended to use other method such as evolutionary algorithm to determine the optimal shipping cost. Besides, the uncertainties of the LP model are not considered in this study, the demand of the products is expected to predict by stochastic approach so that it is closer to real world problem.

References

1. Lim, A., *What's happening in Malaysian ecommerce in 2020?* 2021.
2. *E-Commerce Fashion Industry In Malaysia 2020 – 2021*. 2021; https://blog.boxme.asia/malaysia-fashion-industry-trends-2020/]. Available from: https://blog.boxme.asia/malaysia-fashion-industry-trends-2020/.
3. Law, T.J., *19 Powerful ecommerce statistics that will guide your strategy in 2021*, in *Oberlo*. 2021: https://www.oberlo.com/blog/ecommerce-statistics.
4. Aldeseit, B., *Linear Programming-Based Optimization of Synthetic Fertilizers Formulation.* Journal of Agricultural Science, 2014. **6**.
5. Merenda, J., *A Linear Programming Algorithm to Minimize Glass Batch Costs*. 1996, Colorado School of Mines: Ann Arbor. p. 98.
6. Molina, M., *Product Mix Optimization at Minimum Supply Cost of an Online Clothing Store using Linear Programming.* 2018.
7. Johnson, L.A. and D.C. Montgomery, *Operations research in production planning, scheduling, and inventory control*. 1974: New York (N.Y.) : Wiley.
8. Ravindran, A.R., ed. *Operations research applications*. 2008, CRC Press.
9. Alonso-Ayuso, A., et al., *On the product selection and plant dimensioning problem under uncertainty.* Omega, 2005. **33**(4): p. 307-318.
10. Vinay, C., *Limitations of Linear Programming.* 2015.
11. Pradeep, S., *7 Limitations of linear programming – Explained!* 2015.