Proceedings of Science and Mathematics
Faculty of Science, Universiti Teknologi Malaysia
Vol. 2, 2021, page 156-165

Investigation Of Wave On Guitar String Using Finite-Difference Method

[1]Mohd Asyraf Mohd Khadim and [2]Muhamad Najib Zakaria

[1,2]Department of Mathematical Sciences
Faculty of Science, Universiti Teknologi Malaysia,
81310 Johor Bahru, Johor, Malaysia.

e-mail: [1]mohdasyraf@graduate.utm.my 1, [2]mnajibz@utm.my

Abstract

We analyze a model of one-dimensional wave equation derived from a vibrating guitar string and investigate a plucked guitar string's transverse displacement and subsequent vibration motion. We used the finite difference methods to address the wave equation for a vibrating string and analyzed the waveforms for various values of the string variables. The findings reveal that the amplitude, pitch, or quality of the guitar wave of sound changes significantly depending on a string of tension, a string length, a linear string density, and soundboard material. However, is has some problems when strings it's being plucked. This occurs due strumming the string too hard that will produce "fret buzz" where it is an annoying sound coming from the guitar's strings. For solving the problem, the finite difference method will numerically solve the one-dimensional equation using Matlab software. By using central difference approximation will derive the wave equation to produce the final equation that is used for the value. As a result, we gain an understanding of how the one-dimensional wave equation pattern looks like in the strings of musical instruments.

.

Keywords buzz fret problem; guitar string; Matlab; Finite Difference.

1 Introduction

A wave is "a distortion in a medium or substance in which the individual component of the substance only cycle back and forth or up and down while the wave itself travels through the material." [1] Waves, such as electromagnetic waves, appear extensively in nature and in many different kinds of mechanical waves. Energy is transported via waves. The energy of a wave is seen in many ways. Earthquakes are caused by seismic waves, which carry a tremendous amount of energy and shake the ground. Electromagnetic waves transport information in several ways, allowing Internet connectivity, satellites, optical cables, and radios. In microwave ovens, the energy from microwaves is transformed into thermal energy. This research examines the mechanical waves generated by a guitar string mathematically. Wave energy is used in a variety of ways, [2] including "electromagnetic and communication satellites, and light waves in optical fibers." We might also mention that music is a kind of wave energy. The string is a substance composed of numerous twisted threads. A string is defined as a wire, nylon, and any other synthetic material that is thinner than its length and may be stretched between the two places in this research. A string that vibrate is only a representation of the various vibrating items seen in nature. The majority of vibrations produce wave motion. Waves, as previously said, convey the energy that may be utilized for human use. Wave theory may be used to simulate a variety of phenomena that are helpful in daily life. Musical instruments, weather forecasting, tsunami, and earthquake warning equipment are just a few items on the list. Because the variables can be readily adjusted, a vibrating string is a better and more accessible starting point for studying waves. A typical example of a multidimensional system is the vibration of a string, which is time and space are dependent. [3]"Partial Differential Equations are commonly used to describe multidimensional physical phenomena that rely on time and space. Electromagnetic, acoustics, mass, and heat transmission are some of the technical applications of the partial differential equation." For the sake of acoustics that have been listed on the PDE application, we will investigate a classical guitar string.

2 The Guitar

A guitar is a kind of stringed instrument. It is shaped like a violin with a long and dense neck, and there are segments called "fret" it is attached with several strings that can be played by plucking, generally using fingers or plectrum. Guitars are traditionally made of various types of wood, made of nylon, and there are also some modern guitars made of polycarbonate materials. Generally, guitars are divided into two types, namely acoustic guitars and electric guitars. The sound produced by different guitars will produce different sound waves and are available in the air space. A stringed musical instrument such as a guitar serves to transfer the bridge's vibrations into the surrounding air space. To produce this phenomenon, it requires a wide surface area to produce air that enters the front and back based on opinion [4]. The vibration factor of a guitar depends on the thickness of the strings used. The thicker the strings used, the vibration produced will be slower, while if the shorter strings are used, the vibrations produced will be stronger. This is due to the difference in string thickness on a guitar. The tension of the guitar strings will also change the frequency produced. If the guitar peg is tightened, the sound produced by the guitar will be louder. Next, the frequency also depends on the length of the strings on a guitar. When a string's duration is modified, it vibrates at a particular frequency. Short strings have more significant frequency and higher pitch [5].

2.1 A vibrating of wave Equation on Guitar String's

One guitar line is taken out of the center to obtain a wave equation, and u (x, t) represents a location along the string. According to [6] from Figure 3.1 we can see at points (x+dx), respectively, T_1 and T_2 are tangential. There will always be a constant state of T in the vertical part of the rope, as the wave will spread from point x to (x+dx).

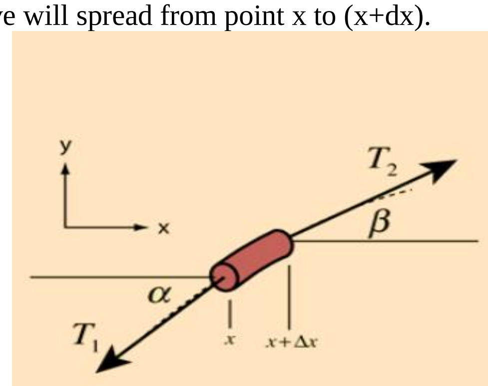

Figure 1.1: Waves in Ideal String

Based on the Figure 3.1 above let the length of a string be (Δx),its mass, and the linear density is (m), .When the angles of α ,and β, are tiny, a constant T, for which the net horizontal force is zero may estimate the horizontal components of tension on either side. Accordingly, the horizontal pressures on both sides of the string segment are given with the small-angle approach, this implies:

3 Mathematics Formula and Equations

The implies:

$$T_{1x} = T_1 \cos(\alpha) \approx T$$
$$T_{2x} = T_2 \cos(\beta) \approx T$$
$$T_{1x} \cos(\alpha) = T_{2x} \cos(\beta) \approx T$$

$$T_1 = \frac{T}{\cos(\alpha)} \text{ and } T_2 = \frac{T}{\cos(\beta)}, \tag{1}$$

where T_{1x} and T_{2x} are the guitar string's tangential tensions.

We differentiate α and β due respect to *t* from the previous equation,

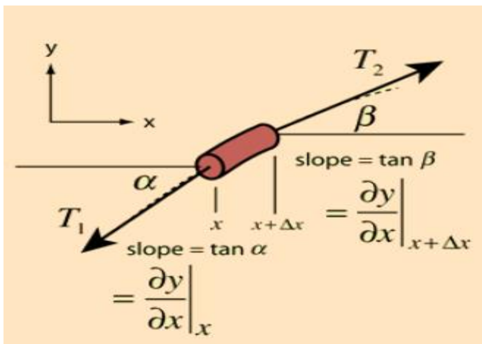

Figure 1.2: strings structure

Based on [6] that associate the equation (3.2) following the vertical motion of the string that match with Newton's second law. Where newton's second law stated that the rate of changing momentum of a body overtime is directly proportional to the force that applied, and it will occur in the same direction as the applied force.

$$F = \frac{d(mv)}{dt} = m\frac{dv}{dt} = ma$$

Where F is net force applied, *m* is the mass of body and *a* is the acceleration

$$\sum F_y = T_{2y} - T_{1y} = T_2 \sin\beta - T_1 \sin\alpha = \Delta ma \tag{2}$$

$$T_2 \sin \beta - T_1 \sin \alpha = \rho \Delta x \frac{d^2 y}{dt^2} \qquad (3)$$

where, ρ is the mass per unit length and m is an acceleration $= \rho \Delta x \frac{d^2 y}{dt^2}$

Next we substitute equation (1) into (3), the replacement process substitutes equation (1) for equation (3), Which equates as below

$$\frac{T_2 \sin \beta}{T_2 \cos \beta} - \frac{T_1 \sin \alpha}{T_1 \cos \alpha} = \frac{\rho \Delta x \frac{d^2 y}{dt^2}}{T} \qquad (4)$$

$$\mathrm{Ttan}(\beta) - \mathrm{Ttan}(\alpha) = \frac{\rho \Delta x \frac{d^2 y}{dt^2}}{T}$$

This acquire basic calculus.

$$\tan \alpha = | \frac{dy}{dx} |_x$$

and

$$\tan \beta = | \frac{dy}{dx} |_{x+\Delta x} \qquad (5)$$

Next, equation (5) will be substituted into (4). In the limit $\Delta x \rightarrow 0$ this will become

$$\frac{1}{\Delta x}\left[\left| \frac{dy}{dx} \right|_{x+\Delta x} - \left| \frac{dy}{dx} \right|_x \right] = \frac{\rho}{T} \frac{d^2 y}{dt^2} \qquad (6)$$

At equation (6) you extract the equation for a wave, with,

$$\frac{\partial^2 y}{\partial x^2} = \frac{\rho}{T} \frac{\partial^2 y}{\partial t^2} \qquad (7)$$

In order to clarify, $\frac{T}{\rho} = C^2$ where T is the tension and p is the density , the equation (7) would be,

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 y}{\partial t^2}. \qquad (8)$$

4 Numbering Equations

In the previous part, we have seen the formula or one-dimensional wave equation and the derivation of the formula. We would like to see how to relate the one-dimensional wave equation to the finite difference method for this part. For this study, we will see the central approximation of finite difference methods.

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 y}{\partial t^2}$$

From the equation of one dimensional wave equation above we can transform to central approximation finite difference method.

$$\frac{u(x+dx,t)-2u(x,t)+u(x-dx\,,t)}{(dx)^2}=\frac{1}{c^2}\frac{u(x,t+dt)-2u(x,t)+u(x,t-dt)}{(dt)^2}$$

In order to solve the problem of one-dimensional waves equation, we are going to add the i & j notation into the equation such a $u(x,t)=u_{i,j}$ and distribute dx=m , dt=n to generates the new equation such ;

(9)

$$\frac{u_{i+1,j}-2u_{i,j}+u_{i-1,j}}{m^2}=\frac{1}{c^2}\frac{u_{i,j+1}-2u_{i,j}+u_{i,j-1}}{n^2}$$

The equation can be reorganized into:

(10)

$$\frac{c^2n^2}{m^2}u_{i+1,j}-2u_{i,j}+u_{i-1,j}=u_{i,j+1}-2u_{i,j}+u_{i,j-1}$$

So we conclude the $\lambda=\frac{nc}{m}$ and replace λ with equations (11):

(11)

$$u_{i,j+1}=\lambda^2u_{i-1,j}+2(1-\lambda^2)u_{i,j}+\lambda^2u_{i+1,j}-u_{i,j-1}$$

Since we have the strings that fixed at end point for both side such at x = 0 and x = L, we will have the boundary condition u (0, t) = 0 and u (L, t) = 0. These conditions are satisfied by the function $f(x,t)=A\sin(kx-\omega t)$ when the value of L is set to L = 1 and the amplitude, A is constant value. In order to determine the value of the wave function at the first time step, we need the value of $u_{i,-1}$ to start the finite difference scheme which is the value of i and j notation must start with (i = 0 and j = 0).

(12)

$$\frac{\delta u(x,0)}{\delta t}=h(x)$$

Based on the initial condition $u_t(x,0)=h(x)$ we know that the string is plucked from rest and the velocity for its initial is h(x) = 0

(13)

$$\frac{u(x,t+\partial t)-u(x,t-\partial t)}{2\,\partial t}=h(x)$$

For i and j notation we will get

(14)

$$u_{i,-1}=u_{i,1}-2\,\partial t h(x)$$

Replace $u_{i,-1}$ to the equation 13 to get :

$$u_{i,j+1} = \frac{1}{2}\lambda^2 u_{i-1,J} + (1-\lambda^2)u_{i,j} + \frac{\lambda^2}{2}u_{i+1,j} - \partial th(x) \quad (15)$$

We will get a new equation by substituting h(x)=0

$$u_{i,j+1} = \frac{1}{2}\lambda^2 u_{i-1,J} + (1-\lambda^2)u_{i,j} + \frac{\lambda^2}{2}u_{i+1,j} \quad (16)$$

Therefore, the final equation that we will use in Matlab to calculate the graph is from equation (16)

5 Matlab calculation

The outcome of the calculation of the wave equation using the FDM technique will be seen for this segment. We also succeeded in having a calculation for the wave pattern. We have the validation outcome of FDM and the estimation of the frequency are also seen. In this chapter will also be shown a comparison of FDM calculations and graph sketches, as well as a comparison of different lambda values.

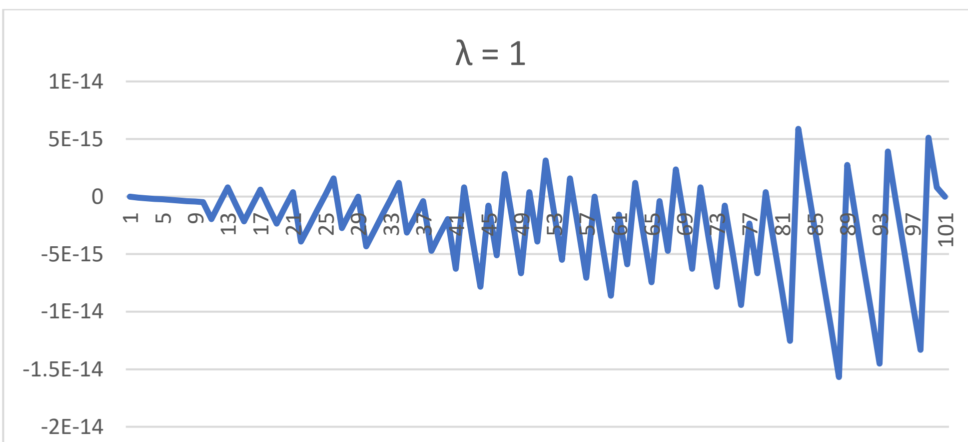

Figure 1.3: The form of the wave at lambda = 1

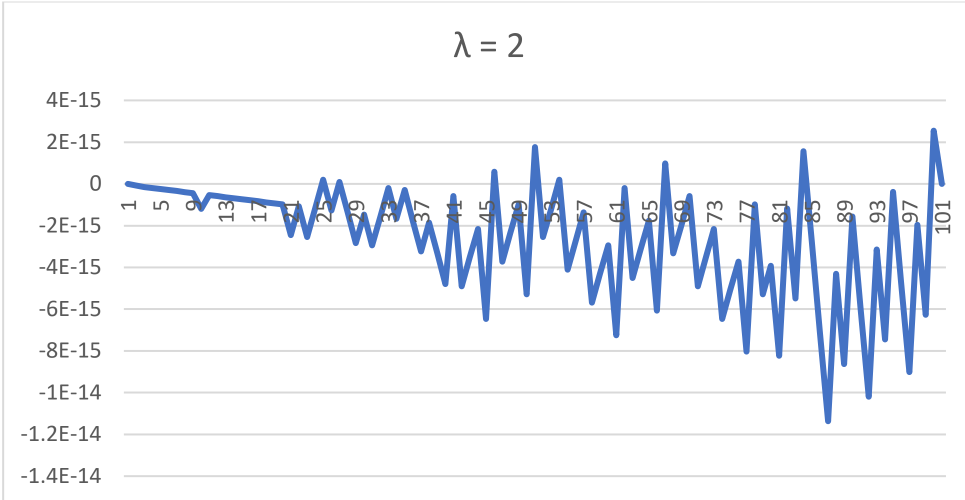

Figure1.4: The form of the wave at lambda = 2

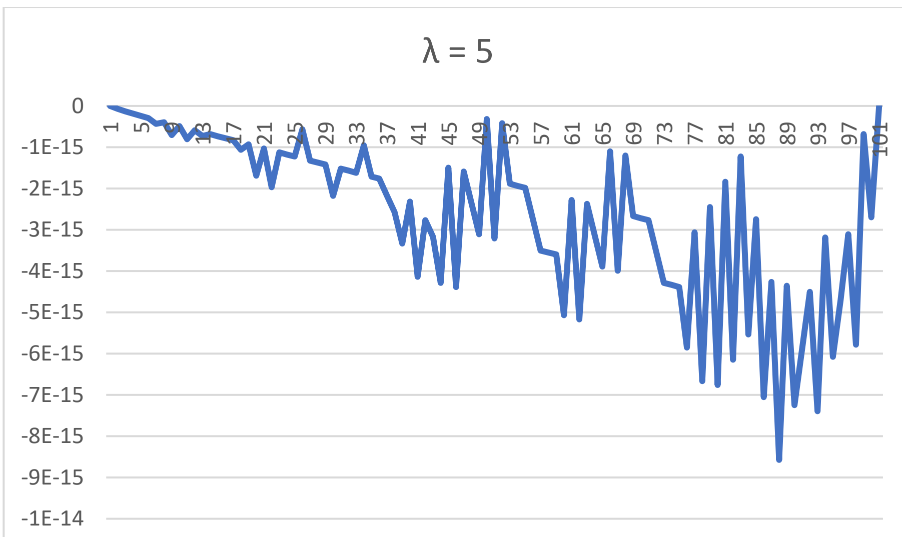

Figure1.5: The form of the wave at lambda = 5

We can see that each graph that has a different lambda value will produce a different graph pattern. For example, Figure 1.3 shows a slow wave and circulate at the zero value only when the lambda value increases. For example, as in Figure 1.4, the graph pattern starts to change, initially, it undulates slowly and subsequently undulates below the level of an empty value and indicates a negative value. Since the value of lambda, $\lambda = Cn/m$, the effects of the phase sizes, n, and m, are seen differently. Higher values of lambda produce Non-uniform amplitude waveforms. We can see from Figures 1.5 when the value of lambda is 5, and it shows that larger step-width values will cause a loss of consistency. This shows that the larger value for wavelength, the smaller of energy is going to produce.

6 The result of frequency

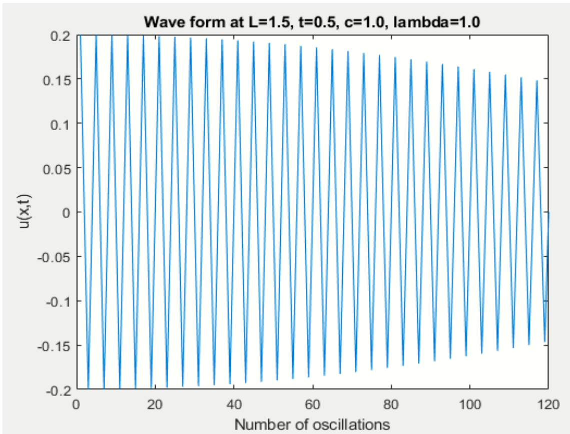

Figure 1.7: A waveform graph of u (x, t) at λ = 1, c = 1, t = 0.5 and L = 1.5

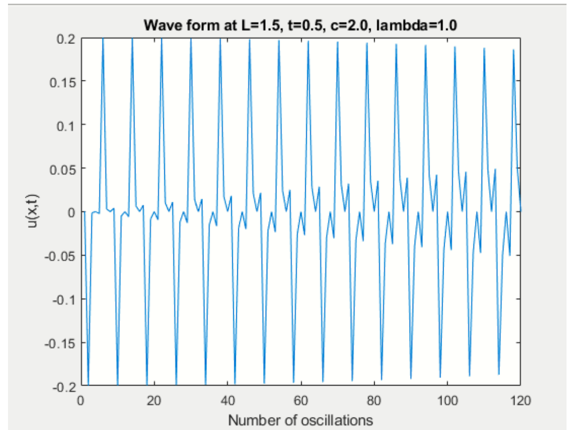

Figure 1.8: A waveform graph of u (x, t) at λ = 1, c = 2, t = 0.5 and L = 1.5

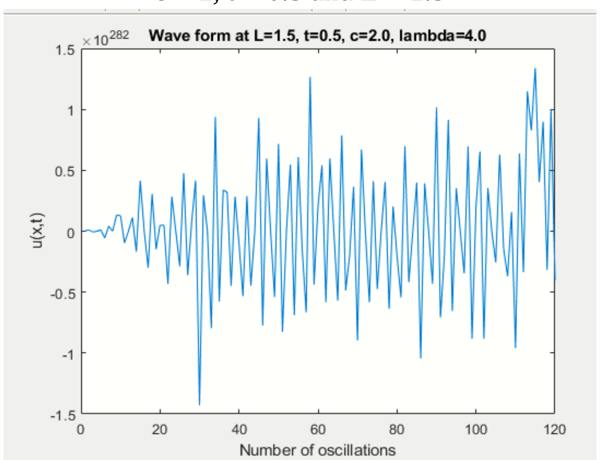

Figure 1.9: A waveform graph of u (x, t) at λ = 4, c = 2, t = 0.5 and L = 1.5

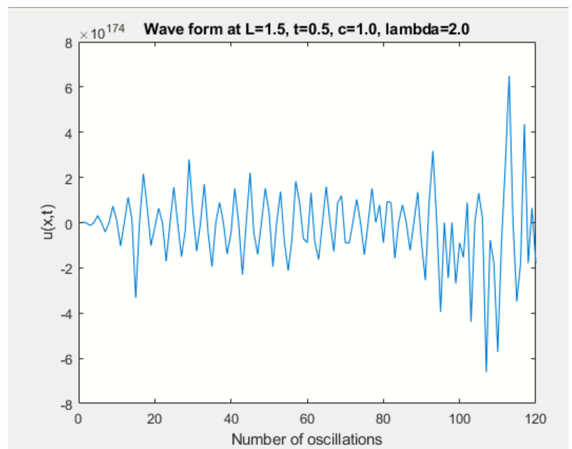

Figure 1.10: A waveform graph of u (x, t) at λ = 2, c = 1, t = 0.5 and L = 1.5

As we can see in Figures 1.7, where the value of L is 1.5, the waveform shows it oscillates uniformly from 0 oscillations approaching 120th oscillations compared to the Figures 1.8, where the value of *L* is 1.5. Still, the value of c is increasing from 1 to 2. It indicates a waveform in Figures 1.7 is a more stable and uniform frequency. The parameters, in this case, are approximately standard.

However, Figures 1.8 shows the wave oscillated with a sharp peak starting from the beginning until the end of the graph. This may have happened owing to truncation errors in the derivation of the finite difference system, or it's happened when the value of *c* is higher. For example, the values are more than 1. The main point to be considered in the graph of Figures 1.8 is that the wave occurred just above the rest position, which might not be correct. Therefore, the approximate solution is inaccurate.

We can see in Figure 1.9 portrays the impact of high values of λ. When λ is 4, and the value of c is 2, the frequency is not uniform, and the non-uniform amplitudes are observed. This implies that the calculated results of $u(x, t)$ aren't close to the actual values. From the graph of Figures 1.10, we can see the value of *c* is 1, and the value of lambda is 2, where the value of c is smaller than the value of lambda. The frequency is not uniform and has a peak value at approximately 110th oscillations.

From the four graphs plotted above, we can conclude that when the value of *c* is higher than the value of lambda, the graph form is not accurate, and when the value of lambda is higher than the value of c, the frequency is not uniform. When the value of lambda is high, the frequency produces is low, which means the sound produces also will be slow. We know by using lower lambda, the sound would be loud. One way to avoid the buzz fret problem, a musician must avoid strumming too hard because when strumming the string too hard, it could produce a loud sound.

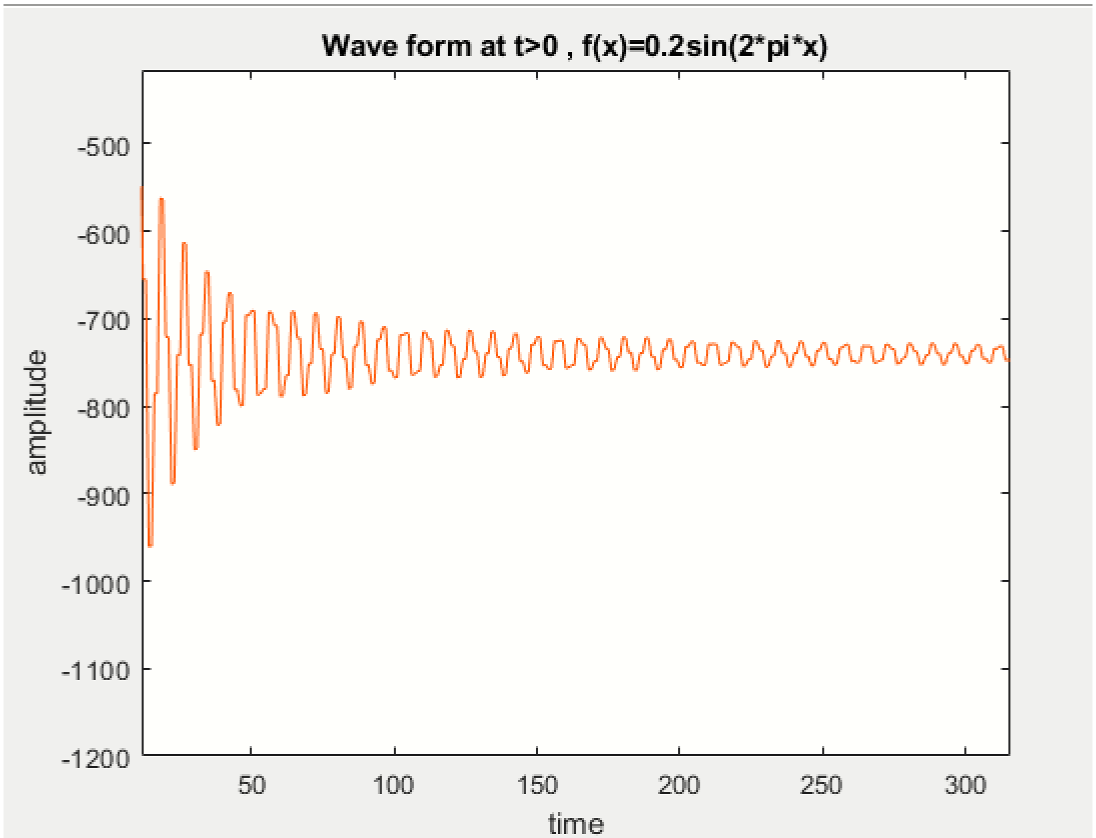

Figure 1.11 : A waveform graph of $u(x, t)$ at $t > 0$, $f(x)=0.2\sin(2\pi x)$

Based on Figure 1.11, the graph produced is the result of the calculation from the method separation of variables. The amplitude of the graph decreases constantly, and the waveform is more uniform compared to Figure 1.19. Based on the graph shown above shows that the waveform does not fluctuate randomly. The graph shows that the amplitude approaches zero as time increases. The waveform oscillates uniformly. The method used to plot the graphs of Figure 1.11 is the separation of variables. This method is used to compare the results obtained from the central approximation method. The conclusion we can see is that these two methods show almost the same results. However, using the separation method of variables, the resulting graph obtained is more constant and accurate.

7 Conclusion

In conclusion this study aims to investigate the application of a one-dimensional wave equation on the finite difference method by using the central approximation approach. We are using this method because it is easy to calculate and easy to apply. For this project, we mainly derive the one-dimensional wave equation and use Matlab software to calculate the answer and run the coding. We are looking to observe the pattern of the graph produce by using Matlab, and we would like to see the frequency of the guitar string when the musician plucks it. For this project, we want to see how the musician faces the problem when they are using the guitar, which is the buzz fret sound that came out when they plucked the strings. We would like to see and improve the quality of music from this experiment, which is to reduce the buzz fret problem. From the result that we have seen based on the graph produces, we know when the wavelength of the string is short, the frequency that produces is high; therefore, the sound produces by the string is loud. We know that several causes cause the buzz fret sound, one of them are when strumming the string too hard. From this observation, we know that when we reduce the frequency by plucked the strings slowly, it can help the musician play the strings instrument way better.

8 References

[1] T. a. R. R. Lutz, "Multidimensional String Models. Proceedings of the Online Symposium for Electronic Engineers (SEE' 00).," 2000.

[2] G. Alan, "Numerical Methods for Physics.," *1st Edn., Prentice Hall, Englewood Cliffs, ISBN-10:,* 2000.

[3] G. L. B. a. P. Y. Evans, "Numerical Methods for Partial Differential Equations.," *1st Edn., Springer, Berlin, ISBN-10: 354076125X. pp: 290,* 2000.

[4] Grimes, "String Theory," *The Physics of String-Bending and Other Electric Guitar Techniques,* 2014.

[5] M.H, adel and khader, "Advance different equaton," *Numerical solutions of fractional wave equations using an efficient class of FDM based on the Hermite formula,* 2016.

[6] Nave, "the fundamental vibrational mode of a stretched string," 2016. [Online]. Available: http://hyperphysics.phy-astr.gsu.edu/hbase/Waves/string.html.