Proceedings of Science and Mathematics
Faculty of Science, Universiti Teknologi Malaysia
Vol. 2, 2021, page 20-27

Analytical Solution of Brinkman Type Fluid with Heat Transfer through An Accelerated Plate by using Laplace Transform Method

[1]Aisyah Nadhirah Zulkefli and [2]Dr. Ahmad Qushairi Mohamad

[1,2]Department of Mathematical Sciences
Faculty of Science, Universiti Teknologi Malaysia,
81310 Johor Bahru, Johor, Malaysia.

e-mail: [1]a.nadhirah@graduate.utm.my , [2]ahmadqushairi@utm.my

Abstract The purpose of this present paper is to evaluate the analytical solution of Brinkman type fluid through an accelerated plate with heat transfer. The dimensional governing equations reduced to non-dimensional form by using appropriate dimensionless variables. The non-dimensional equations are solved to obtain velocity and temperature profiles by using Laplace transform method. This research is to determine how physical parameters, Prandtl number, Grashof number, Brinkman type fluid parameter and time embedded in the fluid flow models effect the behaviour of velocity and temperature profiles. The effect of these parameters is illustrated graphically by using MATHCAD software.

Keywords Brinkman type fluids, heat transfer, accelerated plate, analytical solution.

1 Introduction

The movement of energy in a medium caused by a difference of temperature is known as heat transfer. Heat transfer is a process of heat flowing through a boundary of a system from a higher temperature surrounding to a low temperature surrounding due to difference of temperature and environment. Heat describes as a transport or in a system, it is called boundary [1]. Convection is one of the methods for the heat to transfer.

Convection incorprates heat transfer and circulation in conduction to cause molecular in the air to travel from hotter to cooler regions.The motion of heated molecules is based on two forms of convections which are free convection and mechanical convection. Free convection generates a circulating current that distributes heat uniformly across the prepared material.

In this research paper, Brinkman type fluid is one of the examples of non-Newtonian fluid as suggested by H.C. Brinkman. The fluid flows over a highly porous media and it has its own special term of viscosity [2].

2 Literature Review

2.1 Free Convection Flow of Newtonian Fluid by using Laplace Transform

A Newtonian fluid is a fluid in which at any point, the viscous stresses resulting from its flow are linearly correlated to the local strain rate which the rate of change over time of its deformation.

There are few differences between Newtonian and non-Newtonian fluid stated in some of the researches. Non-Newtonian fluid has twice velocity as the Newtonian fluid velocity, and the volume flux amplitude of non-Newtonian fluid is higher than the amplitude of Newtonian fluid [3]. The harvested voltages and electrical power in Newtonian fluid are higher than in non-Newtonian fluid [4]. Laplace transform method can solve the second problem of stokes for Newtonian fluid simplier and direct [5].

2.2 Free Convection Flow of Brinkman Type Fluid by using Laplace Transform

Non-Newtonian fluid has greater energy dissipation and better heat transfer than Newtonian fluid [6]. The influence of thermal radiation on unsteady free convection is studied by considering MHD flow of incompressible Brinkman type fluid in a porous medium with Newtonian heating. They considered four important situations of flow due to impulsive motion of the plate., uniform acceleration of the plate, non-uniform acceleration of the plate and highly non-uniform acceleration of the plate. Laplace transform method is used to solve the combined effect of heat and mass diffusion on time fractional free convectional incompressible flow of Brinkman type fluid over an oscillating plate [7]. Laplace transform method together with Hankel transform can be developed to get analytical solution for the influence of a transverse magnetic field in a cylindrical tube for Brinkman type fluid [8].

2.3 Accelerated Plate

When the temperature of accelerated moving vertical plate has a temporarily ramped profile, unsteady hydromagnetic free convection mass and heat transfer flow with hall current of a viscous, incompressible, electrically conducting, heat absorbing and optically radiating fluid past a plate and Laplace transform method is used to solve the exact solutions of momentum, energy and concentration equations [9]. A study is discovered that an accelerated motion of the porous plate generated the flow of unsteady MHD flow of a viscous incompressible electrically conducting fluid under the transverse magnetic field and Hall current, Laplace transform technique is used to form analytical solution of governing equation [10]. Fourier Sine and Laplace transform method can be used to obtain the solutions for the velocity and shear stress corresponding to the unsteady flow induced by an infinite constantly accelerating and oscillating plate in an incompressible generalized Maxwell fluid [11].

3 Mathematical Formulation

By taking unidirectional and one-dimensional flow, and using the Boussinesq approximation, the free convection flow is governed by the momentum and energy equations:

$$\frac{\partial u'}{\partial y'} + \beta^* u' = \mu \frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T'_\infty) \tag{1}$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} \tag{2}$$

$$u' = 0 \qquad T' = T'_\infty \qquad \text{for } y' \geq 0 \text{ and } t' \leq 0 \tag{3}$$

$$u' = At' \qquad T' = T'_w \qquad \text{at } y' = 0 \text{ fot } t' < 0$$

$$u' \to 0 \qquad T' \to T'_\infty \qquad y' \to \infty \text{ for } t' > 0$$

Some suitable dimensionless variables are introduced in order to convert equation (1), (2), (3) into dimensionless form.

$$u = \frac{u'}{(\mu A)^{\frac{1}{3}}} \quad y = \frac{y' A^{\frac{1}{3}}}{\mu^{\frac{2}{3}}} \quad t = \frac{t' A^{\frac{2}{3}}}{\mu^{\frac{1}{3}}} \quad T = \frac{T' - T'_\infty}{T'_w - T'_\infty} \tag{4}$$

By substituting equation (4) into (1) and (2) together with initial and boundary conditions (3), then dimensionless equation for both momentum and energy equations are obtained as,

(i) **Dimensionless form for momentum equation** (1)

$$\frac{\partial u}{\partial t} + \beta_1 u = \frac{\partial^2 u}{\partial y^2} + Grt \tag{5}$$

where;

$$\beta_1 = \frac{\beta^* \mu^{\frac{1}{3}}}{A^{\frac{2}{3}}} \text{ and } Gr = \frac{g}{A}\beta(T'_w - T'_\infty)$$

(ii) **Dimensionless form for energy equation** (2)

$$\Pr\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial y^2} \tag{6}$$

where:

$$\Pr = \frac{\mu \rho c_p}{k}$$

The initial and boundary conditions in dimensionless forms are,

$$u = 0 \quad T = 0 \quad \text{for } y \geq 0 \text{ and } t \leq 0 \tag{7}$$

$$u = t \quad T = 1 \quad \text{at } y = 0 \text{ for } t > 0$$

$$u \to 0 \quad T \to 0 \quad \text{as } y \to \infty \text{ for } t > 0$$

4 Laplace and Inverse Laplace transform

In order to solve momentum and energy equation to obtain velocity and temperature profiles, Laplace transform method is used.

(i) **Energy equation**:

Transformation equations and initial conditions (7) will be used to transform equation (6) into ordinary differential equation together by implementing homogeneous equation technique, eventually we can get,

$$\bar{T}(y,s) = \frac{1}{s} e^{-y\sqrt{s\Pr}}\,. \tag{8}$$

(ii) Momentum equation

By implementing Laplace transform identity and non-homogeneous equation technique, together with the initial conditions (7),

$$\bar{U}(y,s) = \left[\frac{1}{s^2} - \frac{Gr}{s^s(1-\mathrm{Pr}) + s\beta_1}\right] e^{-y\sqrt{s+\beta_1}} + \left[\frac{Gr}{s^2(1-\mathrm{Pr}) + s\beta_1}\right] e^{-y\sqrt{sPr}}. \tag{9}$$

Next, implying the inverse Laplace transform to obtain the analytical solutions for the velocity and temperature profiles,

(i) Velocity profiles

$$u(y,t) = u_1(y,t) - u_2(y,t) + u_3(y,t) + u_4(y,t) - u_5(y,t) \tag{10}$$

where;

$$u_1(y,t) = \left(\frac{t}{2} + \frac{y}{4\sqrt{\beta_1}}\right)\exp\left(y\sqrt{\beta_1}\right)\mathrm{erfc}\left(\frac{y}{2\sqrt{t}} + \sqrt{\beta_1 t}\right) + \left(\frac{t}{2} - \frac{y}{4\sqrt{\beta_1}}\right)\exp\left(-y\sqrt{\beta_1}\right)\mathrm{erfc}\left(\frac{y}{2\sqrt{t}} - \sqrt{\beta_1 t}\right)$$

$$u_2(y,t) = \frac{\beta_3}{2}\exp\left(y\sqrt{\beta_1}\right)\mathrm{erfc}\left(\frac{y}{2\sqrt{t}} + \sqrt{\beta_1 t}\right) + \frac{\beta_3}{2}\exp\left(-y\sqrt{\beta_1}\right)\mathrm{erfc}\left(\frac{y}{2\sqrt{t}} - \sqrt{\beta_1 t}\right)$$

$$u_3(y,t) = \frac{\beta_3}{2}\exp\left(-\beta_2 t + y\sqrt{\beta_1 - \beta_2}\right)\mathrm{erfc}\left(\frac{y}{2\sqrt{t}} + \sqrt{(\beta_1 - \beta_2)t}\right) + \frac{\beta_3}{2}\exp\left(-\beta_2 t - y\sqrt{\beta_1 - \beta_2}\right)\mathrm{erfc}\left(\frac{y}{2\sqrt{t}} - \sqrt{(\beta_1 - \beta_2)t}\right)$$

$$u_4(y,t) = \beta_3 \mathrm{erfc}\left(\frac{y}{2}\sqrt{\frac{\mathrm{Pr}}{t}}\right)$$

$$u_5(y,t) = \frac{\beta_3}{2}\exp\left(-\beta_2 t + yi\left(\sqrt{\beta_2 \mathrm{Pr}}\right)\right)\mathrm{erfc}\left(\frac{y}{2}\sqrt{\frac{\mathrm{Pr}}{t}} + i\sqrt{\beta_2 t}\right) + \frac{\beta_3}{2}\exp\left(-\beta_2 t - yi\left(\sqrt{\beta_2 \mathrm{Pr}}\right)\right)\mathrm{erfc}\left(\frac{y}{2}\sqrt{\frac{\mathrm{Pr}}{t}} - i\sqrt{\beta_2 t}\right)$$

(ii) Temperature profiles

$$T(y,t) = \mathrm{erfc}\left[\frac{\mathrm{y}}{2}\sqrt{\frac{\mathrm{Pr}}{t}}\right]. \tag{11}$$

5 Result and Discussion

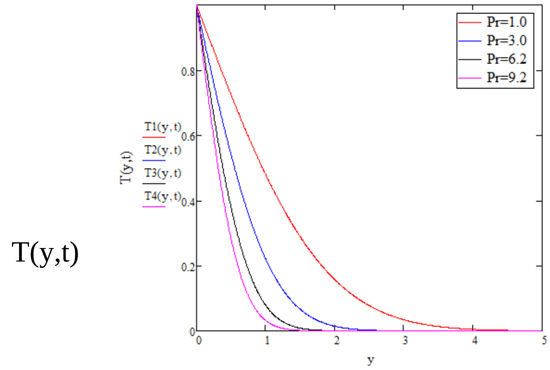

Figure 5.1 Temperature profiles for different values of Pr with *t* = 1.0.

Figure 5.1 shows the effect of Pr on the temperature profile corresponds to constant *t*. It is observed that the temperature decreases as the value of Pr increases. The graph above shows that the curve of the temperature profile declines more rapidly as the value of Pr increases.

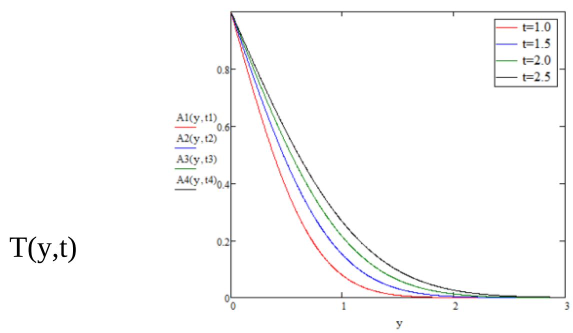

Figure 5.2 Temperature profiles for different values of *t* when Pr = 0.7.

Figure 5.2 displays that the effect of time, *t* on the temperature profile corresponds to fixed value Pr = 0.7 referred to a study done by Zakaria *et al.* (2013). From the figure above, it is demonstrated that the temperature gradually in time, *t*. It is noted that the temperature decreases with increasing of time, *t* to its free stream value.

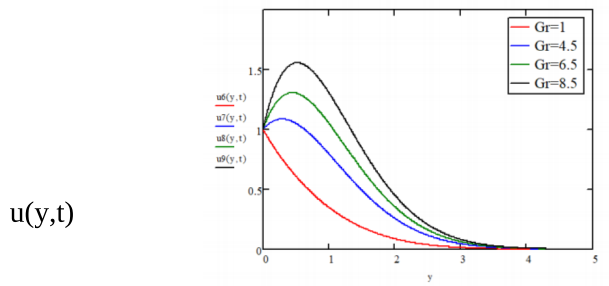

Figure 5.3 Velocity profiles for different values of *Gr* when Pr = 1.5, β_1= 0.8 and *t* = 1.0.

Figure 5.3 indicates that the effect of different values of *Gr* on the velocity profile with constant number of Pr, β_1 and *t.* It shows that the higher the value of *Gr*, the higher the velocity as *Gr* is a ratio of bouyancy force to viscous force.

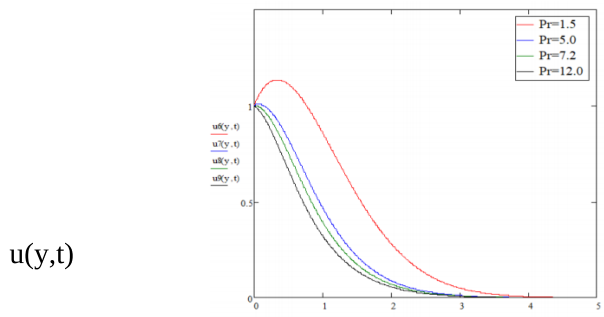

Figure 5.4 Velocity profiles for different values of Pr when *Gr* = 5.0, β_1= 0.8 and *t* = 1.0.

Figure 5.4 shows that when the values of Pr increases, the velocity decreases as Pr is a ratio of momentum diffusivity to the thermal diffusivity. Momentum diffusivity is also known as viscous force.

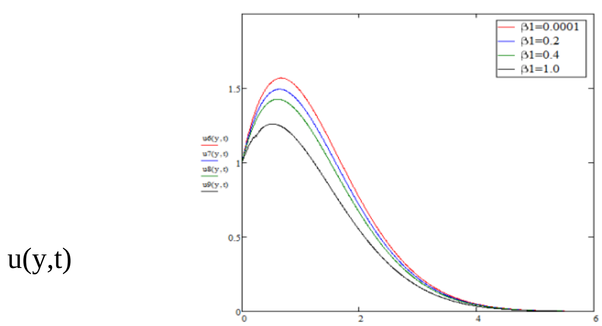

Figure 5.5 Velocity profiles for different values of β_1 when Pr = 0.7, *Gr* = 5.0 and *t* = 1.0.

Figure 5.5 shows that the velocity profile with different value of β_1. It is observed that the higher the Brinkman fluid parameter in the boundary region gives out the lower the velocity of fluid.

u(y,t)

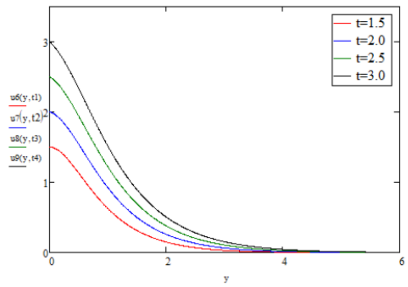

Figure 5.6 Velocity profiles for different values of *t* when Pr = 14, β_1= 0.8 and *Gr* = 7.0.

Figure 5.6 shows that the velocity of Brinkman fluid increases with the increasing in time, *t*. It is also to be noted that for very small values of *t*, the velocity profiles are nearly flat, but assume parabolic shapes near the plate as *t* increases.

6 Conclusion

It is observed that, the velocity increases when the values of Grashof number and time increase while decreases for higher values of Prandtl number and Brinkman type fluid parameter. Temperature profiles also increase for higher values of time but decrease for higher values of Prandtl number. As a conclusion, it is clear that there are a few factors that effects the temperature and velocity of Brinkman type fluid which are Prandtl number (Pr), Grashof number (Gr), Brinkman type fluid parameter (β_1) and time (*t*). The aim of this research is to provide the solutions for the convection flow of Brinkman type fluid over an accelerated plate.

7 References

[1] Gaggiol, R. A. (1969). More on generalizing the definitions of "heat" and "entropy." *International Journal of Heat and Mass Transfer*, *12*(5), 656–660. https://doi.org/10.1016/0017-9310(69)90048-9

[2] Aamina, F., Khan, I., & Sheikh Muhammad Saqib, N. (2017). Magnetohydrodynamic flow of brinkman-type engine oil based MoS2-nanofluid in a rotating disk with hall effect. *International Journal of Heat and Technology*, *35*(4), 893–902. https://doi.org/10.18280/ijht.350426

[3] Dörner, P., Schröder, W., & Klaas, M. (2021). Experimental quantification of Oscillating flow in Finite-length STRAIGHT Elastic vessels for Newtonian and non-Newtonian fluids. *European Journal of Mechanics - B/Fluids*, *87*, 180–195. https://doi.org/10.1016/j.euromechflu.2021.02.001

[4] Maroofiazar, R., & Fahimi Farzam, M. (2021). Experimental investigation of energy harvesting from sloshing phenomenon: Comparison of Newtonian and non-Newtonian fluids. *Energy*, *225*, 120264. https://doi.org/10.1016/j.energy.2021.120264

[5] Fetecau, C., Vieru, D., & Fetecau, C. (2008). A note on the second problem of Stokes for Newtonian fluids. *International Journal of Non-Linear Mechanics*, *43*(5), 451–457. https://doi.org/10.1016/j.ijnonlinmec.2007.12.022

[6] Dong, X., & Liu, X. (2021). Multi-objective optimization of heat transfer in microchannel for non-Newtonian fluid. *Chemical Engineering Journal*, *412*, 128594. https://doi.org/10.1016/j.cej.2021.128594

[7] Ali, F., Aftab Alam Jan, S., Khan, I., Gohar, M., & Ahmad Sheikh, N. (2016). Solutions with special functions for time fractional free convection flow of Brinkman type fluid. *The European Physical Journal Plus*, *131*(9). https://doi.org/10.1140/epjp/i2016-16310-5

[8] Saqib, M., Khan, I., & Shafie, S. (2019). Generalized magnetic blood flow in a cylindrical tube with magnetite dusty particles. *Journal of Magnetism and Magnetic Materials*, *484*, 490–496. https://doi.org/10.1016/j.jmmm.2019.03.032

[9] Natural Convection Heat and Mass Transfer Flow with Hall Current, Rotation, Radiation and Heat Absorption Past an Accelerated Moving Vertical Plate with Ramped Temperature. (2015). *Journal of Applied Fluid Mechanics*, *8*(01). https://doi.org/10.36884/jafm.8.01.22600

[10] Das, S., Tarafdar, B., & Jana, R. N. (2018). Hall effects on unsteady MHD rotating flow past a periodically accelerated porous plate with slippage. *European Journal of Mechanics - B/Fluids*, *72*, 135–143. https://doi.org/10.1016/j.euromechflu.2018.04.010

[11] Zheng, L., Zhao, F., & Zhang, X. (2010). Exact solutions for generalized Maxwell fluid flow due to oscillatory and constantly accelerating plate. *Nonlinear Analysis: Real World Applications*, *11*(5), 3744–3751. https://doi.org/10.1016/j.nonrwa.2010.02.004