The Application of Linear Programming in Bakery Shop for Profit Maximization

[1]Nor Faiezzatie Shahira Mohamad and [2]Ong Chee Tiong

[1,2]Department of Mathematical Sciences
Faculty of Science, Universiti Teknologi Malaysia,
81310 Johor Bahru, Johor, Malaysia.

e-mail: [1]faiezzatie@graduate.utm.my, [2]ongutm@gmail.com

Abstract The linear programming model has been used by many businesses to determine the best products combination to be produced. Because linear programming is one of the best methods used in this field, this research will focus on how to apply the linear programming technique to maximize the profit of the bakery shop. The purpose of this study is to use a linear programming model with the primary goal of identifying and justifying which products can maximize profit. The methods highlighted are the Simplex method and Excel's Solver, both of which are simple to follow and apply in real life, particularly in business.

Keywords Linear Programming; maximize the profit; Simplex Method; Excel's   Solver.

1   Introduction

Many companies have used the linear programming model to determine the best combination of different products that must be manufactured in order to maximize profit. Optimization methods are commonly used as the most relevant theories for selecting the best option from a set of alternatives based on defined objective criteria. Mathematical programming, also known as constrained optimization, is a mathematical procedure for determining the best resource allocation. The model also includes structural constraints, which are a set of conditions that must be met for the best solution to be found. It is called a linear programming model if the model consists of a linear objective function and linear constraints in decision variables [1].

Many situations, such as the availability of resources and the possibility of uncertainties during the production process, make it difficult to measure actual output productivity and efficiency in the manufacturing industry [2]. Production input shortages are a constant problem for industries all over the world, resulting in the use of low capacity and, as a result, low outputs. But an economy is capable of grow only if management decisions at the company level lead to increased performance through either minimization of costs or maximization of profit resulting in increased output in the real sector.

The objectives of the research are:
   a) Examine and explore the application of the linear programming in profit maximization.
   b) Adopt the technique of linear programming applications to ensure maximum profit in bakery manufacturing.
   c) To identify and justify which products that can maximize the profit.

2   Literature Review

In practice, linear programs can have thousands of variables and constraints. Many businesses have been established, and continue to be established, in order to generate financial profit. In this regard, the main goal of such businesses is to maximize (optimize) profit. These resources could include men, materials, and money. The problem revolves around determining how to allocate resources to achieve the best result, which can be related to profit, cost, or both [3]. Many problems can therefore be described as Linear Programming problems.

Most researchers have used linear programming to maximize profit and reduce transportation costs in various fields [4]. Balogun used the linear programming method to optimize production in the Coca-Cola bottling company for maximum profit. Anieting used the linear programming technique to optimize production. In order to achieve an optimal profit in local soap production, Igbinehi used linear programming.

The goal of this study is to use linear programming to maximize profit from soft drink production at the Nigeria Bottling Company's Ilorin plant [5]. The optimum results derived from the model, based on the data collected, show how much Coke and Fanta Orange should be produced for two products, according to this method. As a result, a specific number of crates should be their production quantities. The Nigeria Bottling Company will make the most money this way.

2.1   Solving Linear Programming Using Simplex Method

The Simplex method is a pivot algorithm that transforms the fundamental solutions through possible solutions while enhancing the objective function [3].

2.2   Solving Linear Programming using Excel's Solver

It can contain 30 to 1000 variables in a linear program and it is nearly impossible to solve either graphically or algebraically. In order to address these real-world issues, companies generally use Excel's Solver. Excel's Solver is Microsoft Excel's open-source linear optimizer [6].

3   Methodology

We focus on the Simplex method and the Excel Excel's Solver technique to solve the problem to maximize the profit.

3.1   The process to formulate a Linear Programming problem

Steps for generically defining a Linear Programming problem:

1.   Identify the decision variables.
2.   Write the objective function.
3.   Mention the constraints.
4.   Explicitly state the non-negativity restriction.

The decision variables, objective function, and constraints all must be linear functions for a problem to be a linear programming problem.

3.2 Simplex Method

3.2.1 Steps for Simplex Method

Step 1: Standard form

❖ Before solving for the optimal solution, all linear programs must be in standard form, which has three requirements:

      1. There must be a problem with maximization.
      2. All linear constraints must be in the inequality less-than-or-equal-to.
      3. There are no negative values in any of the variables.

❖ Simply multiply both the left and right sides of the objective function by -1 to convert a minimization linear program model to a maximization linear program model.

❖ If it seeks to maximize the objective function, a linear programming problem is in standard form.

Objective function $z = f(x_1, x_2, x_3, x_4) = c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4$

where:
$z = maximum\ profit$
$c_j = product\ profit\ contribution\ to - j$
$x_j = product\ group\ to - j$ (Bread, Cakes, Cookies and Muffins)
subjected to the constraints

$a_{11}x1 + a_{12}x2 + a_{13}x3 + a_{14}x4 \leq b_1,$

$a_{21}x1 + a_{22}x2 + a_{23}x3 + a_{24}x4 \leq b_2,$

$a_{31}x1 + a_{32}x2 + a_{33}x3 + a_{34}x4 \leq b_3,$

$a_{41}x1 + a_{42}x2 + a_{43}x3 + a_{44}x4 \leq b_4,$ (1)

where $x_i, b_i \geq 0$.

Step 2: Determine the slack variables

❖ In order to transform the constraints into solvable equalities with a single definite answer, slack variables are required.

After adding slack variables, the corresponding system of constraint equations are;
$a_{11}x1 + a_{12}x2 + a_{13}x3 + a_{14}x4 + s_1 = b_1,$

$a_{21}x1 + a_{22}x2 + a_{23}x3 + a_{24}x4 + s_2 = b_2,$

$a_{31}x1 + a_{32}x2 + a_{33}x3 + a_{34}x4 + s_3 = b_3,$

$a_{41}x1 + a_{42}x2 + a_{43}x3 + a_{44}x4 + s_4 = b_4,$ (2)

where $s_i \geq 0$.

Step 3: Setting up The Tableau

❖ A Simplex tableau is used to perform row operations on the linear programming model and to check for optimality of a solution.

Table 1 The Simplex Tableau for the model.

Basic Variable	x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	NK
s_1	a_{11}	a_{12}	a_{13}	a_{14}	1	0	0	0	b_1
s_2	a_{21}	a_{22}	a_{23}	a_{24}	0	1	0	0	b_2
s_3	a_{31}	a_{32}	a_{33}	a_{34}	0	0	1	0	b_3
s_4	a_{41}	a_{42}	a_{43}	a_{44}	0	0	0	1	b_4
z	c_1	c_2	c_3	c_4	0	0	0	0	0

Information:
$z = the\ objective\ function\ is\ the\ optimal\ value\ (maximum)$
$c_n = coefficient\ of\ profit\ per\ product\ for\ each$
$x_n = decision\ variables\ to - n$
$s_m = slack\ variable\ to - m$
$a_{mn} = resource\ requirements\ for\ each\ x_n$
$b_m = the\ number\ of\ resources\ available$
$n = the\ number\ of\ decision\ variables\ starts\ from\ 1,2,3,4$
$m = the\ number\ of\ types\ of\ resources\ used\ from\ 1,2,3,4$

For this initial simplex tableau, the basic variables are s_1, s_2, s_3, s_4 and the nonbasic variables (which have a value of zero) are x_1, x_2, x_3, x_4.

Step 4: Check Optimality

1) To use the tableau to check for optimality, all values in the last row must be greater than or equal to zero.
2) If a variable's value is less than zero, it hasn't reached its optimal value.

Step 5: Identify Pivot variable

1) Looking at the bottom row of the tableau and the indicator will reveal the pivot variable.
2) The pivot variable will be the intersection of the row with the smallest non-negative indicator and the smallest negative value in the bottom row.

Pivoting

1. The entering variable corresponds to the smallest (most negative) input in the table's bottom row.
2. In the column determined by the entering variable, the departing variable corresponds to the smallest nonnegative ratio.
3. The entry in the simplex table in the column of the entering variable and in the row of the departing variable is called the pivot.

Finally, we apply Gauss-Jordan elimination to the column containing the pivot to form the enhanced solution. (This process is called pivoting.)

Step 6: Create the new tableau

Use row operations to optimize the pivot variable while keeping the rest of the tableau equivalent.
1) Convert the pivot variable into a unit value in order to optimize it (value of 1). Multiply the row containing the pivot variable by the reciprocal of the pivot value to transform the value.
2) The other values in the column containing the unit value will become zero once the unit value has been determined.
3) The other variables not contained in the pivot column or pivot row must be calculated using the new pivot values.

This step can be summarized using the equation on the following page:
New tableau value = (Negative value in old tableau pivot column) x (value in new tableau pivot row) + (Old tableau value)

Step 7: Check Optimality
Step 8: Identify new pivot variable

If the solution is found to be not optimal, a new pivot variable must be determined.

Step 9: Repeat the steps of create new tableau, check optimality, and identify and pivot variables if the tableau is still not in optimal condition.

Step 10: Identify optimal values

Once the tableau has been proven to be optimal, the best values can be determined.

3.3 Spreadsheet Modelling and Excel's Solver

3.3.1 The steps are:
    Step 1: User must familiarize his/herself with the LP data set.
    Step 2: Set up the optimization model (Model Construction).
    Step 3: Setting up Excel's Solver to solve LPs by the following sub steps:
        1. Open a new Excel spreadsheet and name it to "Maximize the bakery's profit".
        2. Lay out the problem data in Excel spreadsheet as follows:

A9	fx	Cons. No 4 (Milk in g)							
	A	B	C	D	E	F	G	H	I
1	Decision variables	X1	X2	X3	X4				
2	Initial Optimal no. of D.V	0	0	0	0				
3						total	<=,>=	limit	
4	Obj. Function (profit in RM)	No * X1	No * X2	No * X3	No * X4	0			
5	Constraints Set								
6	Cons. No 1 (Flour in g)	No * X1	No * X2	No * X3	No * X4	0		RHS L1	
7	Cons. No 2 (Sugar in g)	No * X1	No * X2	No * X3	No * X4	0		RHS L2	
8	Cons. No 3 (Butter in g)	No * X1	No * X2	No * X3	No * X4	0		RHS L3	
9	Cons. No 4 (Milk in g)	No * X1	No * X2	No * X3	No * X4	0		RHS L4	

Figure 1 Excel spreadsheet

Let x_1 = Bread, x_2 = Cakes, x_3 = Cookies and x_4 = Muffins.

3. Type the formula; F4 = SUMPRODUCT ($B$2: $C$2: …: $E$2; B4:C4:…: E4) and pull-down it to cells F6:F9.

4. Next, activate the Solver from Excel.

Specify the following in the Solver dialog box;

> Target Cell: F4
>
> Constraints: F6:F9 <=, =, or >= H6:H9
>
> Changing Cells: $B$2: $C$3: …: $E$2
>
> Equal To: Max or Min

5. While still in Solver, click Solve. This should return a dialog box with the notice: "Solver found solution".

4   Results and Analysis

Faisha Bakery is located at Pasir Mas, Kelantan and owned by the local. This bakery produced many kinds of desserts such as bread, cake, macarons, cookies and many more. By using the data below, we can find out which products need to be focus on to achieve the objective.

Table 2 The table consists of the list of raw materials and the profit for each type of products: bread, cakes, macarons and cookies.

Raw Materials	Products				Availability
	Bread	**Cake**	**Macarons**	**Cookies**	
Flour (kg)	0.6	0.25	0.15	0.23	160
Sugar (kg)	0.05	0.4	0.16	0.23	250
Butter (kg)	0.025	0.12	0.05	0.14	49
Chocolate (kg)	0.057	0.075	0.25	0.043	44
Profit (RM)	0.80	1.20	1.80	0.90	

By Simplex Method

1. Determine the decision variables

$$x_1 = \text{Bread}, x_2 = \text{Cake}, x_3 = \text{Macaron and } x_4 = \text{Cookies.}$$

2. Determine the constraints in the standard form.

$$0.6x_1 + 0.25x_2 + 0.15x_3 + 0.23x_4 \leq 160$$

$$0.05x_1 + 0.4x_2 + 0.16x_3 + 0.23x_4 \leq 250$$

$$0.025x_1 + 0.12x_2 + 0.05x_3 + 0.14x_4 \leq 49$$

$$0.057x_1 + 0.075x_2 + 0.25x_3 + 0.043x_4 \leq 44$$

where $x_1, x_2, x_3, x_4 \geq 0$ (3)

3. Determine the objectives function

$$Max\ Z = 0.8x_1 + 1.2x_2 + 1.8x_3 + 0.9x_4$$ (4)

4. Change the inequality by adding the slack variables to the left of obstacles.

Rewrite the objective functions

$$Max: Z - 0.8x_1 - 1.2x_2 - 1.8x_3 - 0.9x_4 = 0$$

$subjected\ to$

$$0.6x_1 + 0.25x_2 + 0.15x_3 + 0.23x_4 + S_1 = 160,$$

$$0.05x_1 + 0.4x_2 + 0.16x_3 + 0.23x_4 + S_2 = 250,$$

$$0.025x_1 + 0.12x_2 + 0.05x_3 + 0.14x_4 + S_3 = 49,$$

$$0.057x_1 + 0.075x_2 + 0.25x_3 + 0.043x_4 + S_4 = 44,$$ (5)

where $x_1, x_2, x_3, x_4, S_1, S_2, S_3, S_4 \geq 0$

5. Create the simplex table or tableau (in Excel)

Table 3 The Simplex table or tableau for the model

Basic variables	x_1	x_2	x_3	x_4	S_1	S_2	S_3	S_4	Nk
S_1	0.6	0.25	0.15	0.23	1	0	0	0	160
S_2	0.05	0.4	0.16	0.23	0	1	0	0	250
S_3	0.025	0.12	0.05	0.14	0	0	1	0	49
S_4	0.057	0.075	0.25	0.043	0	0	0	1	44
Z	-0.8	-1.2	-1.8	-0.9	0	0	0	0	0

6. Check for the optimality → 7. Pivoting the tableau. → 8. Create new tableau
9. Repeat the steps (step 6 – step 8) until the optimum solution is obtained.

Table 4 The last tableau that achieved the optimal solution

Multiply the entire row with the pivot value=0.105

Basic variables	x_1	x_2	x_3	x_4	S_1	S_2	S_3	S_4	Nk
S_1	1	0	0	-0.09707	1.854436	0	-3.62056	-0.38855	102.2059
S_2	0	0	0	-0.24114	0.059476	1	-3.4685	0.018015	90.35227
S_3	0	1	0	1.264001	-0.24019	0	9.992759	-1.85444	369.619
S_4	0	0	1	-0.18507	-0.35075	0	-2.17234	4.64492	41.81134
Z	0	0	0	0.206024	0.56396	0	5.184649	5.824694	600.568

By Excel Solver

Figure 2 The data are arranged as in the Excel sheet shown.

1. Type the formula; F4 = SUMPRODUCT ($B$4: $C$4: $D$4: $E$4) and pull-down it to cells F8:F11 or type F4 =(B4*B6) +(C4*C6) +(D4*D6) +(E4*E6)

2. Next, activate the Solver from Excel.

Specify the following in the Solver dialog box;
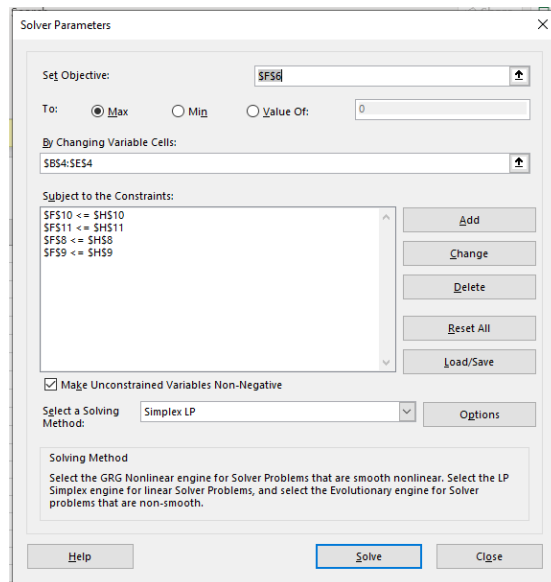
Figure 3 Fill in the Solver Dialog Box as shown.

Target Cell: F6
Constraints: F8:F11 <= H8:H11
Changing Cells: $B$4: $E$4
Equal To: Max

3. While still in Solver, click Solve.

Table 5 The results from the Excel.

Decision variables	x_1	x_2	x_3	x_4			
Initial Opt no of DV	102.2059	369.619	41.81134	0			
					total	<=	limit
Obj function	0.8	1.2	1.8	0.9	600.568		
flour	0.6	0.25	0.15	0.23	160		160
sugar	0.05	0.4	0.16	0.23	159.6477		250
butter	0.025	0.12	0.05	0.14	49		49
chocolate	0.057	0.075	0.25	0.043	44		44

The results for Faisha Bakery show that x_1(Bread)=102, x_2(Cake)=369 and x_3(Macaron)=41 in order to get a maximum profit of RM600.57. By having this result, we can conclude that Faisha Bakery can achieve its objective function, which is to get the maximum profit by producing 102 units of Bread, 369 units of Cakes and 41 units of Macaron while there is no need to produce Cookies as it does not contribute to the objective function.

Conclusion

By having these three types of results, we can say that by using Linear Programming, it can help us in figuring out the ways to have a maximum profit in our business as profit is one of our main goal in doing business. To be compared between both methods which are the Simplex method and Excel's Solver method, the Excel's Solver is easy to use but need to take very good care with the data and the formula involved, so that we do not miscalculate it.

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