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Simulated Annealing Approach to Solve Capacitated Vehicle Routing Problem

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Abstract Capacitated Vehicle Routing Problem (CVRP) is classified in the VRP to minimize the cost travelled when deliverymen want to deliver goods for customers to serve all customers' demand. This research presented Simulated Annealing (SA) for solving the CVRP. The aims are to identify a set of vehicle routes with the lowest total transportation cost between three types of customers such as random customer, cluster customer and mixed random cluster customer where these datasets are obtained from the Augerat Benchmark. This research's final result is the comparison between SA and Branch-Cut-Price to get the optimal solution.

Keywords Vehicle Routing Problem; Capacitated Vehicle Routing Problem; Simulated Annealing.

1 Introduction

Vehicle Routing Problem (VRP) is a combinatorial optimization and integer programming seeking to service a number of customers with fleet of vehicles. VRP aims to find a set of routes at a minimal cost beginning and ending the route at the depot, so that the known demand of all nodes is fulfilled [1]. Each node is visited only once, by only one vehicle, and each vehicle has a limited capacity. There are many applications of VRP such as garbage disposal, mail delivery, school bus routing, airline schedule and many more. VRP is one of the optimization problems that belong to NP- hard (Non-deterministic Polynomial-time hard) problem because it is difficult to solve [2]. It has also become one of the important topics to discuss and analyze. VRP has been researched in many ways and used in many formulations to solve all problems in the real world by using any method to get the best result [3]. There are many types of VRP; this research is focusing on Capacitated Vehicle Routing Problem (CVRP). CVRP is defined as the problem of determining optimal routes to be used by vehicles starting from one or more depots to serve all customers' demand, observing some constraints [4]. The CVRP was first formulated by Christofides et al. [5], a fixed capacity of vehicle serves a set of customers from a common point called warehouse. A customer is visited by the vehicle only once, the vehicle capacity cannot exceed the maximum capacity, and the model deserves to find minimum distance of vehicle route or minimum time to serve the customers. CVRP is like VRP with the additional constraint that every vehicle must have uniform capacity of single commodity. There are numbers of solution method for CVRP which are exact, heuristics and metaheuristics. To sum up, the VRP is an optimization problem that is flexible. It is a dynamic expandable problem that can be used to model any real-world situation and solve specific problems.

1.1 Problem Statement

CVRP is one of the most significant issues in the optimization of distribution networks. In transportation, one major area that has received lots of attention over the years is CVRP. This research will focus on CVRP where we want to find the optimal paths from a depot to the set of customers while also considering the vehicles' capacity to reduce the cost of transportation of goods and services. In this research, SA algorithm will be used in solving the CVRP by finding an optimal route through a given Cartesian of X and Y coordinates. The problem is to pick up or deliver the items for the least cost while never exceeding the capacity of the vehicles.

1.2 Objective of the Research

The objectives of this research are:

i. To minimize the transportation cost in finding the best route of CVRP using SA.
ii. To conduct an experimentation effect of changing the parameter values in SA such as initial temperature and cooling rate.
iii. To compare the performance of SA with Branch-Cut-Price Algorithm in solving CVRP.

2 Literature Review

2.1 Simulated Annealing (SA) Method

SA is an excellent example of incorporating ideas from a completely different field, and an unrelated area that deliver significant and unforeseen advantages at first glance. SA used the concept of annealing which is the method of heating a solid to eliminate strain and crystal imperfections and cooling it slowly [7]. The free energy of the solid has reduced during this process. To avoid being stuck in a local minimum, the initial heating is essential. It is possible to consider virtually every feature as the free energy of some device. Thus, observing and imitating how nature reaches a minimum should yield optimization algorithms during the annealing process [8].

Other than that, SA is a technique of algorithmic relaxation that finds its origins in statistical mechanics. The approach has the potential to achieve an almost optimal solution with a large search space of complex for different combinatorial optimization problems [9]. In this paper, the concept of CVRP by using SA method will be discussed. This method can be contributed to solve the minimization transportation cost to get near optima and travel distance. From the previous experiment, several researchers have proposed SA as an algorithm to solve CVRP. Experiments on literature sources show that SA is fast and outperforms current approaches [10].

SA's benefit can be solved by the arbitrary system and cost features. This approach statistically ensures that an optimal solution can be found. It is reasonably easy to code, even for complex problems [10]. It is a robust technique and can deal with a large amount of data. Besides, the versatility and ability to approach global optimality are its key advantages [11]. As it does not depend on any restrictive properties of the model, the algorithm is very adaptable. Techniques from SA are easily tuned. Using SA, therefore, offers the right solution and give optimal results in solving any problem of complexity [12]. Based on all the advantages of SA mentioned, we will use this powerful method in solving CVRP.

2.2 Related Works on Simulated Annealing

Many researchers have researched the SA method based on previous research. This is because SA provides the best outcome and give more optimal solution. Perwira et al. [13] present an algorithm based on SA for a VRP version. The suggested SA performance is compared to the method's performance in prior works to solve CVRP.

Wei et al. [14] studied the CVRP with two-dimensional loading constraints. The study requires the creation of a set of low-cost routes that begin and end at the central depot in order to meet consumer demands for a set of two-dimensional and weighted products. To address the CVRP, a SA with a mechanism of repeatedly cooling and heating is proposed. Based on the findings, the SA algorithm is put to the test on the most common CVRP two-dimensional loading constraints. The results reveal that SA beats all other algorithms and, in most cases, SA improves on the best-known solutions.

The SA method good result compared against a frequently used heuristic known as the closest heuristics for the case study dataset [11]. Based on the results, the SA and closest neighbor algorithms outperform other methods. The SA outperforms the closest other heuristic approach [13]. Furthermore, the suggested SA approach consistently finds the same answer as the exact technique. As a result, the SA approach produces good solution quality for the problem.

3 Mathematics Formulation of CVRP

In this study, SA is considered to solve CVRP. CVRP defined as n customers with known demand $d_i (i = 1 \ldots n)$ are serviced with m vehicles start and end at depot with uniform capacity C. The objective of this CVRP is to minimize the distance travelled and the assumptions are to load of each vehicle should not exceed the given vehicle capacity. Next, each customer is served exactly once and each vehicle starts and ends at the depot.

CVRP is a problem of identifying the optimal routing for vehicles departing from a single or multiple depots in order to serve all customers. The objective may be to reduce the cost, time, or number of vehicles required to solve the problem. Moreover, the depot has several vehicles available to meet customer demand, and they must return to the depot. Each vehicle has a similar capacity. The mathematical formulation of the capacitated VRP will be presented below.

3.1 Notation of CVRP

Below are the notations involve in CVRP:

C : Capacity of the vehicles
d_i : Demand of customer i
C_{ij} : Distance between customer i to customer j
n : Number of customers
H : Set of vehicles
h : Number of vehicles

A complete graph where $G = (V, E)$, where $V = \{0,1, \ldots, n\}$ is the vertex set and E is the edge set. Vertices $i = 1,2, .. n$ correspond to the customers. Vertex 0 corresponds to depot and n is the number of customers. A set of vehicles $H = \{1,2, .. m\}$, where every vehicle capacity C, is available at the depot and must be return to depot after finishing their deliveries. A non-negative cost C_{ij} is associated with each edge $(i,j) \in E$ and represents distance from vertex i to vertex j for all $i \neq j$. $x_{ijk} \in \{0,1\}$ where 1 if the vehicle k travels from customer i to j and 0 otherwise. $x_{ik} \in \{0,1\}$ where 1 if the customer i is visited by vehicle k and 0 otherwise.

3.3 Model and Description of CVRP

The mathematical model that includes the objective function and constraints of CVRP is as follows:

$$\text{Minimize} \sum_{k-1}^{h}\sum_{i=0}^{n}\sum_{j=0}^{n} c_{ij}x_{ijk} \tag{1}$$

Subject to:

$$\sum_{i=0}^{n} x_{i0k} - \sum_{j=0}^{n} x_{0jk} = 0\,, \forall k = 1, \dots, h \tag{2}$$

$$\sum_{i=0}^{n}\sum_{k=1}^{h} x_{ijk} = 1, \forall j = 1,2, \dots, n \tag{3}$$

$$\sum_{j=0}^{n}\sum_{k-1}^{h} x_{ijk} = 1, \forall i = 1,2, \dots, n \tag{4}$$

$$\sum_{j=1}^{n} x_{0jk} \leq 1,\ \forall k = 1,2,.., h \tag{5}$$

$$\sum_{i=0}^{n} x_{ijk} = y_{jk}, \forall j = 0,1,.., n;\ k = 1,2 \dots, h \tag{6}$$

$$\sum_{j=0}^{n} x_{ijk} = y_{ik}, \forall i = 0,1,.., n;\ k = 1,2 \dots, h \tag{7}$$

$$\sum_{j=1}^{n} d_i\, y_{ik} \leq C,\ \forall k = 1,2,.., h \tag{8}$$

where

(1) represents for objective function of the minimize total travel distance.

(2) is the number of vehicles that arrive at depart from depot is the same.

(3) represents for each customer for location i is visited exactly once.

(4) represents for each customer for location j is visited exactly once.

(5) is defines that at most h vehicles are to be used.

(6) express the relation between two decision variables where is represents for location i and vehicle k .

(7) express the relation between two decision variables where is represents for location j and vehicle k .

(8) represents for guarantees that vehicle capacity is not exceeded.

3.4 Simulated Annealing Implementation

The implementation of SA for CVRP is as follows. It is designed to solve the proposed CVRP efficiently and has been suggested to solve the Vehicle Routing Vehicle (VRP) and its variants. The core function of SA that divides it from other approaches is its mechanism for exploring worse solutions, even infeasible ones with a low probability to escape from the local optimal [10]. The probability of accepting a worse solution is calculated using a formula that considers the difference between solutions and a temperature parameter. This section identifies the implementation of SA for CVRP in-depth, including the solution representation, the parameters used, the SA technique, and the neighborhood move. For temperature schedule, we must choose an appropriate temperature schedule when designing the SA algorithm for an optimization problem. The effectiveness of the SA algorithm can be used. The initial, relatively large value of T must be defined in the schedule and the increasingly smaller values in the subsequence. It is also essential to define the number of iterations of each temperature that should be generated. There two three parameters to be specified for the temperature schedule: initial temperature and cooling rate [11].

3.5 Initial Temperature

A temperature parameter is used to handle the acceptance of adjustment. The initial temperature value, T_0 need to be high enough to achieve a large number of acceptances at the initial stages. Depending on the cooling rate, it is gradually reduced over time. As the algorithm continues and the temperature becomes cooler, unfavorable solutions are less likely to be selected. A stopping criterion the choice of final temperature. Usually, a proper small temperature is being set as the final temperature to become the stopping condition. In this study, the algorithm will stop when temperature drops down to a pre-selected final temperature 0.0000 [14].

3.6 Cooling Rate

The rate of cooling is the factor that is used to lower the temperature at the end of each temperature change counter. The chance of becoming stuck in a local minimum is larger when the faster decrement rate is employed. The slower the decrement rate, on the other hand, the more processing time is required.

$$T_t = \alpha T_{t-1} \tag{9}$$

where

T_t is represent the temperature at the end.
T_{t-1} is represent the temperature at the beginning.

α is the represent the cooling rate which can range from 0 to 1. The value of α is accomplished when using in the ranges between 0.8 and 0.99 [12].

3.7 Simulated Annealing Flowchart

Figure 1 displays the SA flowchart which is the process involved in solving CVRP.

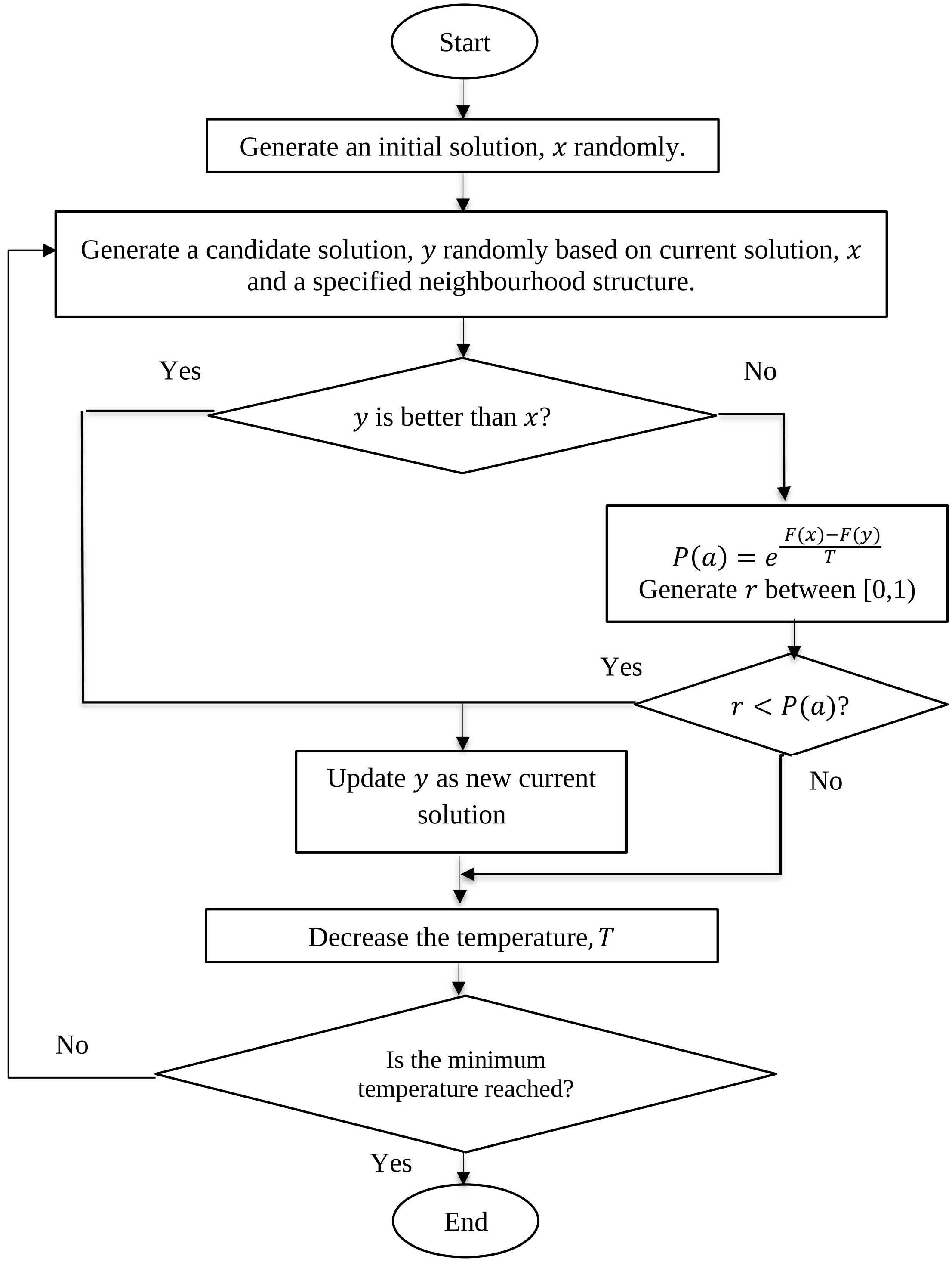


Figure 1: Simulated Annealing Flowchart

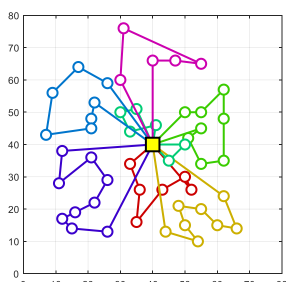
4 Result and Analysis

The results obtained by solving CVRP using SA are shown and analyzed. The data were run 20 times in MATLAB and the result of best, worst and average were recorded. In this problem, we want to determine the best route for each vehicle that give the minimum cost travelled. The best is obtained by comparing the 20 results that give the lowest cost while the worst is the result that give the highest cost. The aims are to identify a set of vehicle routes with the lowest total transportation cost between three types of customers such as random customer, cluster customer and mixed random cluster customer where these datasets are obtained from the Augerat Benchmark.

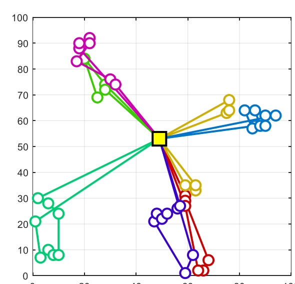
4.1 Parameter Estimation

Several parameters have been experimented to solve the CVRP by using SA. The proposed SA use parameters such as T_0 and α where T_0 denotes the initial temperature while α denotes cooling rate that control the cooling schedule. To end the annealing process, a stopping criterion is used. In this research, the iterations are set to be 500 where after reach this iteration the algorithm will stop and final best cost is obtained. Besides, at 500th iterations, the final temperature drops to 0 and there is no improvement on the result obtained.

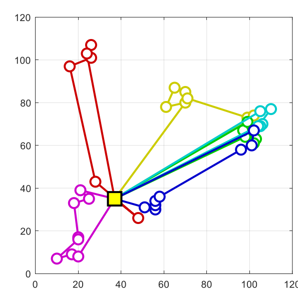
4.2 Best Route of three sets of customer location



Random Location



Cluster Location



Random-Cluster Location

Figure 2: Three Types of Location for data sets

Figure 2 shows three types of the location. In this research, investigations were done through three different cases; random, cluster and random-cluster locations. The test instances are generated based on the CVRP dataset proposed by Augerat Benchmark. Random location involves 50 customers visited by 7 vehicles, cluster location contains 50 customers visited by 7 vehicles and random-cluster location involves 41 customers visited by 6 vehicles. We will assume only homogeneous vehicle type in this research. The location points are based on the Cartesian coordinate and the demands for each customer are presented.

4.3 Comparison of 3 Types of Location

Table 1: Comparison of three sets of customer location

No	Types Of Data	Initial Temperature, T_0	Alpha	Best	Worst	Average	Time taken to reach optimal	Ranking
1	Random Location	110	0.84	501.6874	581.6780	558.455	1 min 31 s	3
2	Cluster Location	120	0.82	720.8999	755.6809	737.171	1 min 27 s	2
3	Random-Cluster Location	110	0.83	786.3697	864.5673	824.586	1 min 22 s	1

Comparison and analysis are made by taking the time for each type of data to reach the optimal solution. Based on the results found in table 1, the random-cluster location reached the optimal first with 1 minute and 22 seconds, followed by the cluster location that takes 1 minute and 27 seconds to achieve the optimal solution. However, random location takes the longest among other location to reach the optimal solution. This happen probably due to the distance of each location that is random and far among each other. Thus, the swapping of locations take longer time for the random location to reach the optimal solution compared to cluster and random-cluster location.

4.4 Comparison Optimal Solution Between algorithm

To check the performance of the proposed SA algorithm, it is necessary to compare it with another method which is Branch-Cut-and-Price algorithm. Therefore, the experiment was conducted on benchmark instances consists of the datasets for CVRP. Table 4.23, Table 4.24 and Table 4.25 show the comparison of optimal solution for random, cluster and random-cluster locations respectively.

Table 2: Comparison of Optimal Solution for Random Location

Algorithm	Scale	Vehicle	Capacity Limit	Optimal Solution
Branch-Cut-and-Price	50	7	150	554
Simulated Annealing	50	7	150	501.6874

Table 3: Comparison of Optimal Solution for Cluster Location

Algorithm	Scale	Vehicle	Capacity Limit	Optimal Solution
Branch-Cut-and-Price	50	7	100	741
Simulated Annealing	50	7	100	720.8999

Table 4: Comparison of Optimal Solution for Random-Cluster Location

Algorithm	**Scale**	**Vehicle**	**Capacity Limit**	**Optimal Solution**
Branch-Cut-and-Price	41	6	100	829
Simulated Annealing	41	6	100	786.3697

Based on the result from Table 2, Table 3 and Table 4, we make a comparison between SA method and Branch-Cut-and-Price method. The optimal solution for Branch-Cut-and-Price are obtained from Augerat Benchmark. From the comparison of both methods, SA can provide a better result compared to Branch-Cut-and-Price as it gives the lowest optimal solution for the random, cluster and random-cluster locations. The SA method shows the effectiveness and promising a better result compared to the Branch-Cut-and-Price method. Furthermore, the computational time of using SA approach is reported to be faster and quickly in getting the optimal solution as this method is a metaheuristic.

5 Conclusion

In conclusion, investigations were done through three different cases; random, cluster and random-cluster locations where the data are collected from Augerat Benchmark with different types of customer's location. The parameters estimation before implementing SA were done which is initial temperature and cooling rate for different types of location. In this chapter, the data were run 20 times in MATLAB and the result of best, worst and average were recorded. Comparison and analysis are made by taking the time for each type of data to reach the optimal solution. In getting the optimal solution, random-mixed location takes the shortest computational time followed by cluster and random location. Apart from that, the performance of SA is being compared with another method which is Branch-Cut-and-Price algorithm where the optimal solution for Branch-Cut-and-Price are obtained from Augerat Benchmark. From the comparison of both methods, SA provide a better result compared to Branch-Cut-and-Price as it gives the lowest optimal solution for the random, cluster and random-cluster locations.

For the recommendation there are various ideas for future research. We may try to solve SA on other types of VRP such as Vehicle Routing Problem with Time Windows (VRPTW), Vehicle Routing Problem with Pick-Ups and Deliveries (VRPPD), Multiple Depot Vehicle Routing Problem (MDVRP) and Vehicle Routing Problem with Stochastic Dynamic (SDVRP) to see the effectiveness of SA algorithm. Besides, upgrading or improve the SA method by creating a hybrid of SA with other metaheuristic methods might produce a better result to solve the CVRP. This is because many sophisticated metaheuristics are available for the further research that enables us to find the best optimal solution.

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