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Mixed Convection Boundary Layer Flow of Viscoelastic Nanofluid Past Over a Sphere

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Abstract In this study, the investigation about mixed convection boundary layer flow of viscoelastic nanofluid past over a sphere has been carried out. The chosen nanoparticle is cooper while the base fluid is Carboxymethyl cellulose solution (CMC).The governing partial differential equations are transformed into differential equations using suitable transformation and then will be solved numerically solved using Keller box method. The effect of viscoelastic parameters, mixed convection parameters and Prandtl number has been plotted graphically.

Keywords Mixed convection, nanofluid, viscoelastic, sphere

1 Introduction

Nanofluids are being used in human life every day because of its application in industrial field such as engineering, biomedical, and energy mechanical. Nanofluids are a new type of fluid created by dispersing nanometer sized particles in any base fluid. As a result, the presence of nanoparticles in a base fluid can alter the physical and chemical properties such as viscosity, thermal conductivity, thermal diffusivity and heat transfer. However, the effect of several properties such as nanoparticles particle size, concentration and nanoparticles types can affect the stability of the nanofluid. In many industrial applications heat transfers are one of the most crucial topics either in cooling or heating process. One of the methods that have been implemented in these crucial topics is by using nanofluid.

During past years many researchers do their finding about the nanofluid, by testing different types of nanofluids, incorporate mixed convection in their experiment or implement geometry in research. These elements are tested along with nanofluid to identify the changes in term of physical or chemical properties of the fluids.Choi and Eastmen [1] were the first to adopt the term ‘nanofluid’. Nanofluids expected to outperform traditional heat transfer fluids and micro – sized metallic particles in terms of performance. The heat transmission process is usually carried out the particles surface, so particles with a larger surface have a bigger potential in heat transfer. To identify the flow and heat exchanged process an experiment was conducted by Alwawi *et al.* [2] using water-γAl_2O_3 and ethylene glycol-γ Al_2O_3 nanofluid flow inside a channel. Other than that, hybrid nanofluid has received many researchers attention. So, Saba *et al.* [3] tested hybrid ($CNT - Fe3O4$ $/H2O$) nanofluid to study about the heat transfer phenomena for hybrid nanoparticles in a squeezing wall.

Extensive research has been conducted by the scientist by adding mixed convection to test the nanofluid characteristics because of a higher usage in science and technology fields. A study has been conducted by Abu – Nada and Chamkha *et al.* [4] to identify the effect of a steady mixed convection for the nanoparticles volume fraction and also the angle of heat transfer. Other than that, Job and Gunakala [5] have tested the convection flow of nanofluid through grooved channels with two heat generating solid cylinder. The surface roughness from the object can be used to test the micro-irregularities that present at the object. Apart from cylinder, other geometries such sphere also has been tested by the researchers. This concept was used by Patil *et al.*[6] to investigate the non-linear mixed convection nanofluid flow about a rough sphere with the presence of liquid hydrogen.

2 Mathematical Model

The mixed convection boundary layer flow over a sphere with radius a was placed in a viscoelastic nanofluid. It is assumed that the temperature of the ambient fluid is T_∞ .Using the model of the nanofluid suggested by Tiwari- Das [7], T_w is the constant temperature of the surface of the sphere, g is the gravity acceleration and $\frac{1}{2}U_\infty$. The governing equations of the problem are as flow:

Continuity equation:

$$\frac{\partial}{\partial \bar{x}}(\bar{r}\,\bar{u}) + \frac{\partial}{\partial \bar{y}}(\bar{r}\,\bar{v}) = 0. \quad (2.1)$$

Momentum equation:

$$\begin{aligned}\rho_{nf}\left(\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right) &= (\rho_{nf})\bar{u}_e\frac{\partial \bar{u}_e}{\partial \bar{x}} + (\mu_{nf})\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} \\ &+k_0\left(\frac{\partial}{\partial \bar{x}}\left(\bar{u}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right) + \bar{v}\frac{\partial^3 \bar{u}}{\partial \bar{y}^3} + \frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{v}}{\partial \bar{y}^2}\right) \\ &+g(\rho\beta)_{nf}\ (T - T_\infty)\sin\left(\frac{\bar{x}}{a}\right). \end{aligned} \quad (2.2)$$

Energy equation:

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = (\alpha_{nf})\frac{\partial^2 T}{\partial \bar{y}^2}, \quad (2.3)$$

subjected to the boundary conditions,

$$\begin{aligned}&\bar{u} = 0\,, \quad \bar{v} = 0, \quad T = T_w, \quad at \quad \bar{y} = 0, \quad \bar{x} \geq 0, \\ &\bar{u} = \bar{u}_e(\bar{x}), \quad \frac{\partial \bar{u}}{\partial \bar{y}} = 0, \quad T = T_\infty, \ as \ \bar{y} \to \infty, \ \bar{x} \geq 0,\end{aligned} \quad (2.4)$$

where,

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}\,, \qquad \alpha_f = \frac{k_f}{(\rho C_p)_f}\,, \qquad \alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}},$$

$$
\begin{gathered}
(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s, \\
(\rho\beta)_{nf} = (1-\phi)(\rho\beta)_f + \phi(\rho\beta)_s, \\
\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s, \\
k_{nf} = k_f \frac{(k_s+2k_f) - 2\phi(k_f-k_s)}{(k_s+2k_f)+\phi(k_f-k_s)},
\end{gathered} \tag{2.5}
$$

$\bar{x}$ and $\bar{y}$ represent the Cartesian coordinates along the surface of the sphere. While, the $\bar{u}$ and $\bar{v}$ are the velocity components, T is the temperature of the fluid, $k_0 > 0$ is the constant of the viscoelastic material (Walter's Liquid-B model), Q_0 is the heat generation constant, ϕ is the nanoparticle volume fraction, $(\rho C_P)_{nf}$ is the heat capacitance of nanofluid, $(\beta)_{nf}$ is the coefficient of thermal expansion of nanofluid, k_{nf} is the thermal conductivities of the nanofluid, k_f and k_s are the thermal conductivities of the fluid and of the solid fractions, ρ_{nf} and μ_{nf} are the density and dynamic viscosity of nanofluid, μ_f is the viscosity of the fluid fraction, and α_{nf} and α_f is the thermal diffusivity of the nanofluid. Lastly, velocity outside the boundary is $\bar{u}_e(\bar{x})$ and $\bar{r}(\bar{x})$ is the radial distance from the symemetrical axis of the sphere which is given by:

$$
\bar{r}(x) = a\sin\left(\frac{\bar{x}}{a}\right), \quad \bar{u}_e(\bar{x}) = \frac{3}{2}U_\infty \sin\left(\frac{\bar{x}}{a}\right). \tag{2.6}
$$

Then, the dimensionless variables are introduced:

$$
\begin{gathered}
x = \frac{\bar{x}}{a}, \quad y = Re^{\frac{1}{2}}\left(\frac{\bar{y}}{a}\right), \quad r(x) = \frac{\bar{r}(\bar{x})}{a}, \quad u = \frac{\bar{u}}{U_\infty}, \\
v = Re^{\frac{1}{2}}\left(\frac{\bar{v}}{U_\infty}\right), \quad u_e(x) = \frac{\bar{u}_e(x)}{U_\infty}, \quad \theta = \frac{T-T_\infty}{T_w - T_\infty},
\end{gathered} \tag{2.7}
$$

$Re = U_\infty a/v$ is the Reynold number.

In order to form the dimensionless equation, substitute equation (2.7) to equation (2.1) until (2.3). The resulting equations are given below:

Continuity equation:

$$
\frac{\partial}{\partial x}(r\,u) + \frac{\partial}{\partial y}(r\,v) = 0. \tag{2.8}
$$

Momentum equation:

$$
\begin{aligned}
\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) &= \left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)u_e\frac{\partial u_e}{\partial x} \\
&+ \frac{1}{(1-\phi)^{2.5}}\frac{\partial^2 u}{\partial y^2} + K\left(\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right) + v\frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2}\right) \\
&+ \left((1-\phi)+\phi\frac{(\rho\beta)_s}{(\rho\beta)_f}\right)\lambda\sin(x).
\end{aligned} \tag{2.9}
$$

Energy equation:

$$\left((1-\phi)+\phi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right)\left(u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}\right)$$

$$=\frac{1}{Pr}\frac{(k_s+2k_f)-2\phi(k_f-k_s)}{(k_s+2k_f)+\phi(k_f-k_s)}\left(\frac{\partial^2\theta}{\partial y^2}\right), \tag{2.10}$$

and the boundary condition (4) will become,

$$\bar{u}=0,\quad v=0,\quad \theta'=-1 \text{ at } y=0,\quad x\geq 0,$$
$$\bar{u}=\bar{u}_e(\bar{x}),\quad \frac{\partial u}{\partial y}=0,\quad \theta=0,\quad \text{as } y\to\infty,\quad x\geq 0, \tag{2.11}$$

while,

$$\mathrm{Pr}=\frac{v}{\alpha},\quad K=\frac{k_0U_\infty}{a\rho_f v},$$
$$\lambda=\frac{Gr}{Re^2},\quad Gr=\frac{g\beta(T_w-T_\infty)a^3}{v^2{}_f}. \tag{2.12}$$

Pr represent the Prandtl number, while K is the dimensionless viscoelastic parameter, λ is the mixed convection parameter and Gr is the Grashof number. It should be mention that $\lambda>0$ is used for aiding flow while $\lambda<0$ for opposing flow. K = 0 represent Newtonian fluids.

In order to solve the equation from (2.8) until (2.10) with boundary condition (2.11), the following variables are introduced:

$$\Psi=xr(x)f(x,y),\quad \theta=\theta(x,y), \tag{2.13}$$

while, Ψ is known as stream function that can be defined as:

$$u=\frac{1}{r}\frac{\partial\Psi}{\partial y},\quad v=-\frac{1}{r}\frac{\partial\Psi}{\partial x}. \tag{2.14}$$

Then, equation (2.14) is applied into equation (2.8) to (2.11), by considering $u_e(x)=\frac{u_e(x)}{U_\infty}=\frac{3}{2}\sin x$. The following transformed equations are:

Continuity equation:

$$\frac{\partial^2\Psi}{\partial x\partial y}-\frac{\partial^2\Psi}{\partial x\partial y}=0. \tag{2.15}$$

Momentum equation:

$$\left((1-\phi)+(\phi)\frac{\rho_s}{\rho_f}\right)\left(-\frac{\cos x}{\sin x}\left(\frac{\partial f}{\partial y}\right)^2+x\left(\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}-\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}\right)\right.$$
$$\left.+\left(x\frac{\cos x}{\sin x}+1\right)\left(\left(\frac{\partial f}{\partial y}\right)^2-\frac{\partial^2 f}{\partial y^2}f\right)\right)$$

$$
\begin{aligned}
&= \frac{9}{4}\frac{\sin x \cos x}{x}\left((1-\phi)+(\phi)\frac{\rho_s}{\rho_f}\right)+\frac{1}{(1-\phi)^{2.5}}\left(\frac{\partial^3 f}{\partial y^3}\right) \\
&+K\left(x\frac{\partial^2 f}{\partial x \partial y}\frac{\partial^3 f}{\partial y^3}+2\left(x\frac{\cos x}{\sin x}+1\right)\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+x\frac{\partial^4 f}{\partial x \partial y^3}\frac{\partial f}{\partial y}-2\left(x\frac{\cos x}{\sin x}\right)\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}-x\frac{\partial f}{\partial x}\frac{\partial^4 f}{\partial y^4}\right. \\
&\left.-\left(x\frac{\cos x}{\sin x}+1\right)\frac{\partial^4 f}{\partial y^4}f-x\frac{\partial^3 f}{\partial x \partial y^2}\frac{\partial^2 f}{\partial y^2}\quad-\left(x\frac{\cos x}{\sin x}+1\right)\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial y^2}\right) \\
&+\left((1-\phi)+\phi\frac{(\rho\beta)_s}{(\rho\beta)_f}\right)\lambda\theta \sin(x). \qquad (2.16)
\end{aligned}
$$

Energy equation:

$$
\begin{aligned}
&\left((1-\phi)+\phi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right)\left(\left(x\frac{\partial f}{\partial y}\right)\frac{\partial \theta}{\partial y}-\left(x\frac{\partial f}{\partial x}+\left(1+\frac{\cos x}{\sin x}\right)f\right)\frac{\partial \theta}{\partial y}\right) \\
&= \frac{1}{Pr}\frac{(k_s+2k_f)-2\phi(k_f-k_s)}{(k_s+2k_f)+\phi(k_f-k_s)}\left(\frac{\partial^2 \theta}{\partial y^2}\right), \qquad (2.17)
\end{aligned}
$$

along with boundary condition,

$$
\begin{aligned}
&f(0)=0, \quad f'(0)=0, \quad \theta'(0)=-\gamma_1(1-\theta(0)), \text{ on } y-0, \\
&\frac{\partial F}{\partial y}\to\frac{3}{2}\frac{\sin x}{x}, \quad \frac{\partial^2 f}{\partial y^2}=0, \qquad \theta\to 0 \;\; as\; y\to\infty. \qquad (2.18)
\end{aligned}
$$

The lower stagnation point of the sphere, $x \approx 0$, equation (2.16) and (2.17) will be reduced to:

Momentum equation:

$$
\begin{aligned}
&\left((1-\phi)+(\phi)\frac{\rho_s}{\rho_f}\right)\left(2ff''-f'^2+\frac{9}{4}\right)+\frac{1}{(1-\phi)^{2.5}}f''' \\
&+2K(f'f''''-f''^2)+\left((1-\phi)+(\phi)\frac{(\rho\beta)_s}{(\rho\beta)_f}\right)\lambda\theta=0. \qquad (2.19)
\end{aligned}
$$

Energy equation:

$$
\frac{1}{Pr}\frac{(k_s+2k_f)-2\phi(k_f-k_s)}{(k_s+2k_f)+\phi(k_f-k_s)}\theta''+2\left((1-\phi)+(\phi)\frac{(\rho\beta)_s}{(\rho\beta)_f}\right)f\theta', \qquad (2.20)
$$

with boundary conditions,

$$
\begin{aligned}
&f(0)=0, \quad f'(0)=0, \quad \theta'(0)=-1, \\
&f'\to\frac{3}{2}, \qquad f''=0, \qquad \theta\to 0, \qquad y\to\infty. \qquad (2.21)
\end{aligned}
$$

3 Research Methodology

Equation (2.19) and (2.20) along with boundary conditions (2.21) will be solved using Keller-box method in FORTRAN programming. The obtain data from FORTRAN will be used to plot graph in MATLAB.

4 Results and Discussion

The effect on skin friction (shear) and velocity for different positions, *Y* with various viscoelastic parameters *K* are illustrated in Figure 4.1 and Figure 4.2 respectively. It is observed that as the viscoelastic parameter increase, the value for skin friction (shear) and velocity are also decreasing.

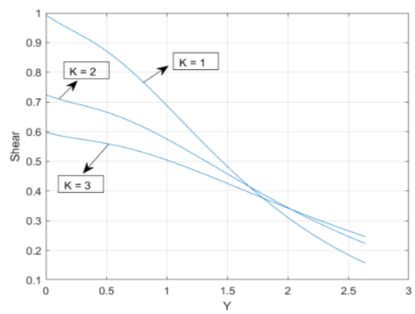

Figure 4.1 Shear at λ = 1 and *Pr* =1 for various value of *K*.

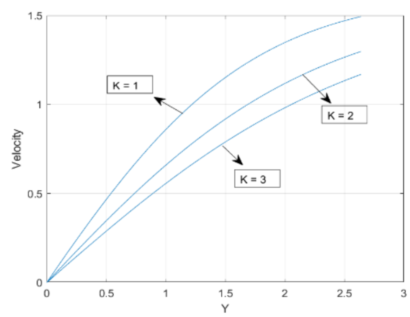

Figure 4.2 Velocity at λ = 1 and *Pr* = 1 for various value of *K*.

Figure 4.3 and Figure 4.4 presented the variation of the skin friction (shear) and velocity for different positions, *Y* with various value of mixed convection parameter λ. From the observation, when the value of λ increases, the skin friction (shear) and velocity profile also increase.

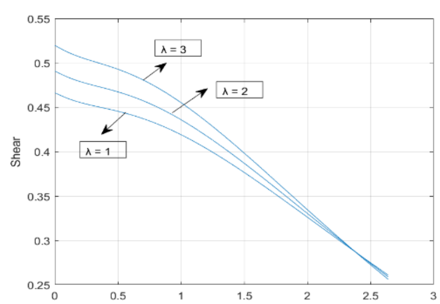

Figure 4.3 Shear at *K* = 1 and *Pr* = 1 for various value of λ.

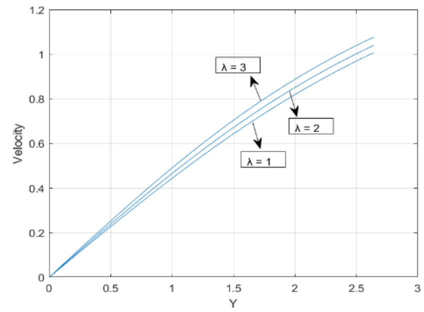

Figure 4.4 Velocity at K = 1 and Pr = 1 for various value of λ.

Figure 4.5 and Figure 4.6 illustrated the graph for shear and velocity for different positions, Y for various Prandtl number. The figure shows a decreasing trend for velocity and constant trend for shear as the Prandtl number increase.

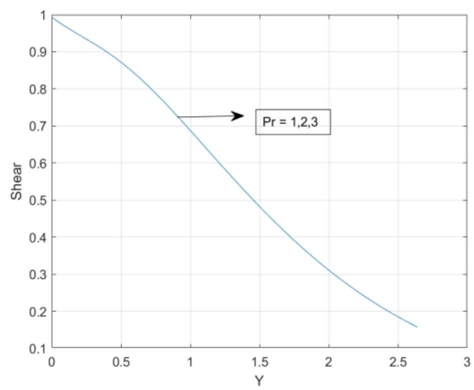

Figure 4.5 Shear at K = 1 and λ = 1 for various value of Pr.

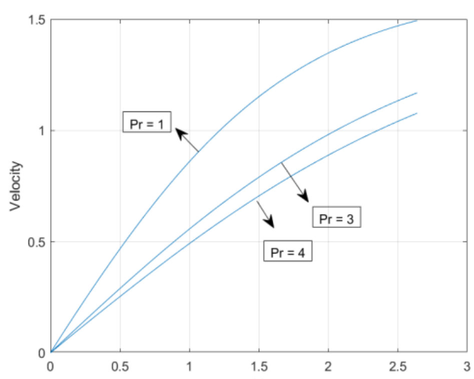

Figure 4.6 Velocity at K = 1 and λ = 1 for various value of Pr.

5 Conclusion

In this problem, the study about mixed convection of viscoelastic nanofluid past over a sphere has been carried out theoretically and graphically. It can be clearly see that the effect of viscoelastic parameter, mixed convection parameter and Prandtl number affect the fluid motion in terms of velocity and skin friction (shear).

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