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Intuitionistic Fuzzy Bézier Curve Approximation Model for Uncertainty Data

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Abstract
The uncertainty data problem with intuitionistic data is difficult to deal since some data are being ignored because they are affected by noise. To overcome this problem, approximation of Bézier curve by using intuitionistic fuzzy set approach is introduced in this paper. Firstly, intuitionistic fuzzy control point is defined based on intuitionistic fuzzy set and its properties. Then, this control point is blended with Bernstein basis function and intuitionistic fuzzy Bézier is constructed. Next, this curve is visualized by using approximation method which consists of membership, non-membership, uncertainty curves. Some numerical examples and an algorithm to obtain the intuitionistic fuzzy Bézier curve also shown.
Keywords: Intuitionistic Fuzzy Set; Intuitionistic Fuzzy Number; Intuitionistic Fuzzy Control Point, Intuitionistic Fuzzy Bézier Curve.

1 Introduction

The uncertainty in the data set has become a big concern in a variety of field. It is difficult for the data analyst to deal with ambiguity and fuzziness of data because the data may be affected by noise. In 1965, Zadeh [1] has introduced fuzzy set theory approach to handling problem with uncertainty and fuzzy data that occur in daily life. However, fuzzy set theory only considers full membership data and non-full membership data and uncertainty is ignoring and not analyzed. Therefore, Atanassov [2] has introduced generalization of fuzzy set theory which known as intuitionistic fuzzy set theory in 1986 involving membership degree, non-membership degree and uncertainty degree. It is very compatible to deal with uncertainty. The concept of intuitionistic fuzzy set is an alternate way to defining a fuzzy set when the available data is inadequate to describe and process since the fuzzy set theory only consider full membership data.

The data set is an important element and play important role in data visualization in term of curve or surface. In most cases, uncertainty data will be removed or ignored from collection data, regardless it will effect on generating curve of surface. Therefore, the analysis and visualization process will be incomplete. Hence, if a data set contains an element of uncertainty, the data should be filtered so it can be applied to generate curves and surfaces model. Developing geometric models with an intuitionistic fuzzy set is one of effective approach to deal with difficulty of visualization data if there is uncertainty data. Geometric modeling is a method of developing mathematical representations in the form of geometry, whereas intuitionistic fuzzy set theory is a mathematical representation focused on concepts and strategies for dealing with uncertain problems. Many researchers have used geometric modelling with intuitionistic fuzzy set approach to deal with uncertainty data [3], [4], [5].

The aim of this paper is to generate and visualized geometric model that can handle uncertainty data which focused on intuitionistic fuzzy Bézier curve approximation is introduced. Before, the intuitionistic fuzzy Bézier curve can be generated, the intuitionistic fuzzy control point must be defined through using intuitionistic fuzzy set and its properties. These control points are blending with Bernstein basis function to generate intuitionistic fuzzy Bézier curve models and visualized through approximation method. This paper is organized as follows. Section 1 discussed some introduction and previous works related to this research. In section 2, intuitionistic fuzzy point relation (IFPR), its properties and IFCP is shown. Section 3 introduces approximation of IFBC by using IFCP. Section 4 shows a numerical example and visualization of IFBC. The properties of the curve and the algorithm to obtain the curve are also shown. Finally, section 5 will conclude this research.

2 Preminilaries

This section shows some basic definition of intuitionistic fuzzy set consists of intuitionistic fuzzy number, intuitionistic fuzzy relation and intuitionistic fuzzy point.

Definition 2.1. Let a (crisp) set X is fixed and let $A \subset X$ be a fixed set. An intuitionistic fuzzy set (IFS) A^* in X is an object of the following [2]:

$$A^* = \{(x, \mu_A(x), v_A(x), \pi_A(x)) | x \in X\} \tag{1}$$

where functions $\mu_{A^*} : X \to [0,1]$, $v_{A^*} : X \to [0,1]$, $\pi_{A^*} : X \to [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ to the set A^* respectively and for every $x \in X$, $0 \le \mu_A(x) + v_A(x) \le 1$. If $\pi_A(x) = 1 - (\mu_A(x) + v_A(x))$, then $\pi_A(x)$ is the degree of non-determinacy (uncertainty) or intuitionistic index of membership of element $x \in X$ to set *A* where $0 \le \pi_A(x) \le 1$.

The concept of fuzzy number has been developed through fuzzy set and possibility theory. The intuitionistic fuzzy number was introduced in [6] and they studied perturbations of intuitionistic fuzzy number and the first properties of the correlation between these numbers. Hence, the intuitionistic fuzzy number can be defined as follows:

Definition 2.2. Intuitionistic fuzzy set A^* is called intuitionistic fuzzy number (IFN) if A^* is an intuitionistic fuzzy subset of real line, normal i.e. there is any $x_0 \in R$ such that $\mu_{A^*}(x) = 1,\ v_{A^*}(x) = 0,$ convex for membership function, μ_{A^*} i.e. $\mu_{A^*}(\lambda x_1 + (1-\lambda)x_2) \ge \min^*(\mu_{A^*}(x_1), \mu_{A^*}(x_2)) \quad \forall x_1, x_2 \in {}^\sim, \lambda[0,1]$ and concave for non-membership function, $v_{A^*}(x)$ i.e. $v_{A^*}(\lambda x_1 + (1-\lambda)x_2) \geq \max^*(v_{A^*}(x_1), v_{A^*}(x_2))$ $\forall x_1, x_2 \in {}^{\sim}, \lambda[0,1]$.

Next, triangular intuitionistic fuzzy number (TIIFN) is intuitionistic fuzzy set in $^{\sim}$ with membership function and non-membership function was introduced by [7] is defined as follows:

Definition 2.3. Triangular intuitionistic fuzzy number (TIFN) is intuitionistic fuzzy set in $^{\sim}$ with membership function and non-membership function as:

$$\mu_{A^*}(x) = \begin{cases} \dfrac{x-L}{M-L}, & L \leq x < M \\ \dfrac{R-x}{R-M}, & M < x \leq \beta' \\ 0, & \text{otherwise} \end{cases} \text{ and } v_{A^*}(x) = \begin{cases} \dfrac{M-x}{M-L'}, & L' \leq x < M \\ \dfrac{x-M}{R'-M}, & M < x \leq R' \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

where $L' < L < M < R < R'$ and $\mu_A(x), v_A(x) \leq 0.5$ for $\mu_A(x) = v_A(x) \forall x \in R$ and value of both membership is 0 and 1 otherwise respectively. The symbolic representation of triangular intuitionistic fuzzy number is $A^*_{IFN} = (L', L, M, R, R')$. L and R are called left and right spreads of membership function $\mu_A(x)$ respectively. L' and R' are represented as left and right spreads of non-membership function $v_A(x)$ respectively. Membership function and non-membership function is shown in Figure 1.

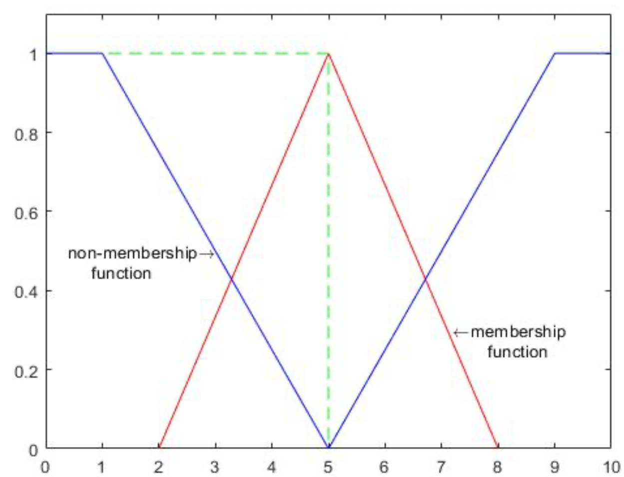

Figure 1 Membership function and non-membership function of triangular intuitionistic fuzzy number

Generally, fuzzy relations are fuzzy sets defined on universal sets, which are Cartesian products of $X \times Y$ that describe the strength of association between two sets' components. Fuzzy relation has been studied in [8]. The concept of intuitionistic fuzzy relation is based on the definition of intuitionistic fuzzy set. Before the IFCP is defined, intuitionistic fuzzy relation (IFR) must be defined first which used as a converter from the definition of intuitionistic fuzzy number to the definition of intuitionistic fuzzy data points. IFR is shown in Definition 4 and Definition 5.

Definition 2.4. Let $X, Y \subseteq \sim$ be universal sets, then

$$R^* = \{((x,y), \mu_{R^*}(x,y), v_{R^*}(x,y), \pi_{R^*}(x,y)) | (x,y) \in X \times Y\} \tag{3}$$

where $\mu_R : X \times Y \to [0,1]$, $v_R : X \times Y \to [0,1]$ and $\pi_R : X \times Y \to [0,1]$ where $\pi_A(x,y) = 1 - (\mu_A(x,y) + v_A(x,y))$ and Equation (3) is satisfy the condition $0 \le \mu_R(x,y) + v_R(x,y) \le 1, \quad \forall (x,y) \in X \times Y$.

Definition 2.5. Let $X, Y \subseteq \sim$ and $Q^* = \{x, \mu_{Q^*}(x), v_{Q^*}(x) | x \in X\}$ and $W^* = \{y, \mu_{W^*}(y), v_{W^*}(y) | y \in Y\}$ are two intuitionistic fuzzy sets. Then $R^* = \{(x,y), \mu_R(x,y), v_R(x,y), \pi_R(x,y) | (x,y) \in X \times Y\}$ is a intuitionistic fuzzy relation on Q^* and W^*.

Intuitionistic fuzzy relation takes the values in the interval [0,1]. Figure 2 is the transformation and formation of intuitionistic fuzzy relation through intuitionistic fuzzy number and intuitionistic fuzzy relation.

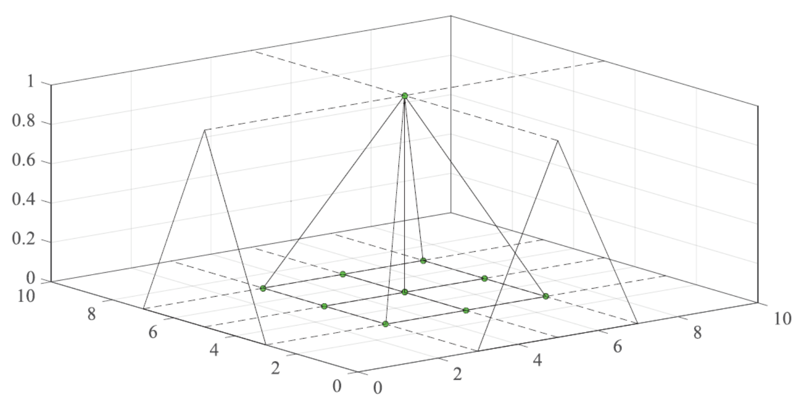

Figure 2 Transformation and formation of intuitionistic fuzzy relation

Next, the intuitionistic fuzzy point was introduced by [9]. Hence, the intuitionistic fuzzy point (IFP) is defined as:

Definition 2.6. Let $L, R \in (0,1)$ and $L + R \leq 1$. An intuitionistic fuzzy point, $T^*_{(L,R)}$ of X is an intuitionistic fuzzy set of X defined by $T^*_{(L,R)} = \langle x, \mu_T, \gamma_T \rangle$ where $y \in X$

$$\mu_T(y) = \begin{cases} L & if \quad y = x \\ 0 & if \quad y \neq x \end{cases} \text{ and } \lambda_T(y) = \begin{cases} R & if \quad y = x \\ 0 & if \quad y \neq x \end{cases} \tag{4}$$

In this case, x is called the support of $T^*_{(L,R)}$. An intuitionistic fuzzy point is said to belong to and intuitionistic fuzzy set $A^* = \langle x, \mu_A, \lambda_A \rangle$ of X, denoted by $T^*_{(L,R)} \in A^*$, if $L \leq \mu_A(x)$ and $R \leq \lambda_A(x)$. Next section will introduce intuitionistic fuzzy control point, P^* and some of its properties.

3 Approximation of Intuitionistic Fuzzy Bézier Curve models

In this section, some of definition and theorem to construct IFBC models is shown. IFBC is obtained by blending IFCP with Bernstein polynomial or basis function. Finally, IFBC is visualized using approximation method. The degree of polynomial of Bézier curve depends on the number of IFCP to define the curve. First of all, IFCP was introduced by [10] is defined through concept of intuitionistic fuzzy set and its properties that have been defined from previous section. Thus, IFCP is defined as follows:

Definition 3.1. IFCP is defined as set of points $n+1$ that indicates the positions and coordinates of a location and is used to describe the curve and is denoted by

$$P_i^* = \left\{ P_0^*, P_1^*, \ldots, P_n^* \right\} \tag{5}$$

where the control polygon vertices or the control point is numbered from 0 to n.

Next, the control polygon formed by connecting all the IFCP with lines, starting with P_0^* and end with P_n^* is called control polygon or Bézier polygon. The convex hull of the Bézier polygon contains the Bézier curve. Thus, intuitionistic fuzzy control polygon can be defined as:

Definition 3.2. Assuming P^* is intuitionistic fuzzy control point as in Definition 4, then linear curve is formed by the line segment through P_i^* and P_{i+1}^* for $i = 0, 1, \ldots, n-1$ is called as intuitionistic fuzzy control polygon. Figure 3 below show intuitionistic fuzzy control polygon with their respective intuitionistic fuzzy control points. These intuitionistic fuzzy control polygon consists of membership, non-membership and ambiguity function. Thereby, Figure 3 illustrate intuitionistic fuzzy control polygon as below:

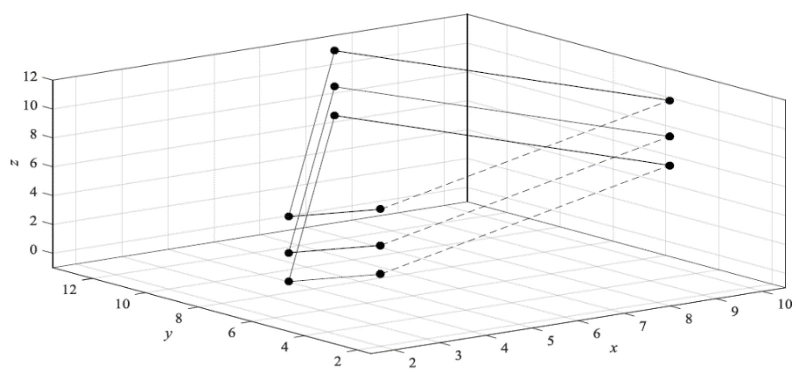

Figure 3 Intuitionistic fuzzy control polygon with respective intuitionistic fuzzy control point

Later, the intuitionistic fuzzy Bézier curve was introduced by [10] can be defined by using IFCP from Definition 3.1 and control polygon from Definition 3.2.

Definition 3.3. Let $P^* = \left\{P_i^*\right\}_{i=0}^{n}$ as IFCP and these points will formed intuitionistic control polygon to generate IFBC that have degree of *n*. Thus, a parametric Bézier curve can be defined as follows:

$$S^*(t) = \sum_{i=0}^{n} P_i^* B_{n,i}(t), 0 \le t \le 1 \tag{6}$$

where the Bézier blending function is

$$B_{n,i}(t) = \binom{n}{i} t^i (1-t)^{n-i} \tag{7}$$

with binomial coefficients are

$$\binom{n}{i} = \frac{n!}{i!(n-1)!} \tag{8}$$

The intuitionistic fuzzy Bézier curve equation can also be expressed in matrix multiplication based on research in [11]. IFBC can be represented as a matrix by extending the analytic formulation of the curve into its Bernstein polynomial coefficients and then expressing these coefficients using the polynomial power basis, as shown below:

$$S^*(t) = [T][N][G] = [H][G] \tag{9}$$

where

$$[H] = \begin{bmatrix} B_{n,0} & B_{n,1} & \ldots B_{n,n} \end{bmatrix} \tag{10}$$

$$[G]^T = \begin{bmatrix} P_0 & P_1 & \ldots & P_n \end{bmatrix} \tag{11}$$

Therefore, IFBC is constructed and visualized by approximation method.

4 Numerical example and Algorithm of Intuitionistic Fuzzy Bézier Curve Model

In order to illustrate IFBC approximation, let consider IFBC with IFBC with five IFCP and degree of four $(n = 4)$ as in Table 4.1.

Table 4.1 Intuitionistic fuzzy control point with its respective degrees

IFCP, P_i^*	Membership, $\mu(P_i^*)$	Non-Membership, $v(P_i^*)$	Uncertainty, $\pi(P_i^*)$
$P_1^* = (2,2)$	0.6	0.3	0.1
$P_2^* = (7,8)$	0.5	0.3	0.2
$P_3^* = (11,13)$	0.7	0.2	0.1
$P_4^* = (17,18)$	0.5	0.1	0.4
$P_5^* = (25,23)$	0.2	0.3	0.5

Value of IFCP from Table 4.1 can be written as $P_1^* = \langle (2,2); 0.6, 0.3 \rangle, P_2^* = \langle (7,8); 0.5, 0.3 \rangle, P_3^* = \langle (11,13); 0.7, 0.2 \rangle, P_4^* = \langle (17,18); 0.5, 0.1 \rangle$ and $P_5^* = \langle (25,23); 0.2, 0.3 \rangle$ for $n = 4$. First of all, based on the algorithm that has been shown from chapter 3, Bernstein polynomials need to be calculate as Equation (7) as follows:

$$B^{*}_{4,0}(t)=\binom{4}{0}t^{0}(1-t)^{4-0}=(1-t)^{4}$$

$$B^{*}_{4,1}(t)=\binom{4}{1}t^{1}(1-t)^{4-1}=4t(1-t)^{3}$$

$$B^{*}_{4,2}(t)=\binom{4}{2}t^{2}(1-t)^{4-2}=4t^{2}(1-t)^{2}$$

$$B^{*}_{4,3}(t)=\binom{4}{3}t^{3}(1-t)^{4-3}=4t^{3}(1-t)^{1}$$

$$B^{*}_{4,4}(t)=\binom{4}{4}t^{4}(1-t)^{4-4}=t^{4}$$

Next, IFBC for membership function is calculated based on IFCP for membership degree which are $P_1^{*}=(2,2,0.6), P_2^{*}=(7,8,0.5), P_3^{*}=(11,13,0.7)$, $P_4^{*}=(17,18,0.5)$ and $P_5^{*}=(25,23,0.2)$
Thus $B^{*}(t)$ for membership degree can be written as Equation (6) as follows:

$$\begin{aligned}B^{*}(t)&=P_1^{*}B_{4,0}^{*}+P_2^{*}B_{4,1}^{*}+P_3^{*}B_{4,2}^{*}+P_4^{*}B_{4,3}^{*}+P_5^{*}B_{4,4}^{*}\\&=(2,2,0.6)(1-t)^{4}+4t(7,8,0.4)(1-t)^{3}+4t^{2}(11,13,0.7)(1-t)^{2}+\\&\quad 4t^{3}(17,18,0.5)(1-t)+(25,23,0.2)(t^{4})\\&=(2,2,0.6)(x^{4}-4x^{3}+6x^{2}-4x+1)+(7,8,0.4)(-x^{3}+3x^{2}-3x+1)+\\&\quad(11,13,0.7)(4x^{4}-8x^{3}+4x^{2})+(17,18,0.5)(-4x^{4}+4x^{3})+(25,23,0.2)(t^{4})\end{aligned}$$

The intuitionistic fuzzy quartic Bézier also can be represented in matrix form where,

$$\begin{aligned}B^{*}(t)&=\left[B_{4,0},B_{4,1},B_{4,2},B_{4,3},B_{4,4}\right]\left[P_1^{*},P_2^{*},P_3^{*},P_4^{*},P_5^{*}\right]\\&=\left[(1-t)^{4},\left(4t(1-t)^{3}\right),\left(4t^{2}(1-t)^{2}\right),\left(4t^{3}(1-t)\right),\left(t^{4}\right)\right]\begin{bmatrix}(2,2,0.6)\\(7,8,0.5)\\(11,13,0.7)\\(17,18,0.5)\\(25,23,0.2)\end{bmatrix}\end{aligned}$$

$$=\left[\left(t^4-4t^3+6t^2-4t+1\right),\left(-4t^4+12t^3-12t^2+4t\right),\left(4t^4-8t^3+4t^2\right)\right.$$

$$\left.\left(-4t^4+4t^3\right),(t)^4\right]\begin{bmatrix}(2,2,0.6)\\(7,8,0.5)\\(11,13,0.7)\\(17,18,0.5)\\(25,23,0.2)\end{bmatrix}$$

$$=\left[t^4,t^3,t^2,t^1,1\right]\begin{bmatrix}1 & -4 & 6 & -4 & 1\\-4 & 12 & -12 & 4 & 0\\4 & -8 & 4 & 0 & 0\\-4 & 4 & 0 & 0 & 0\\1 & 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}(2,2,0.6)\\(7,8,0.5)\\(11,13,0.7)\\(17,18,0.5)\\(25,23,0.2)\end{bmatrix}$$

Hence, the intuitionistic fuzzy quartic Bézier curve for membership degree is shown in Figure 4 and Figure 4.2 in front angle and side angle respectively.

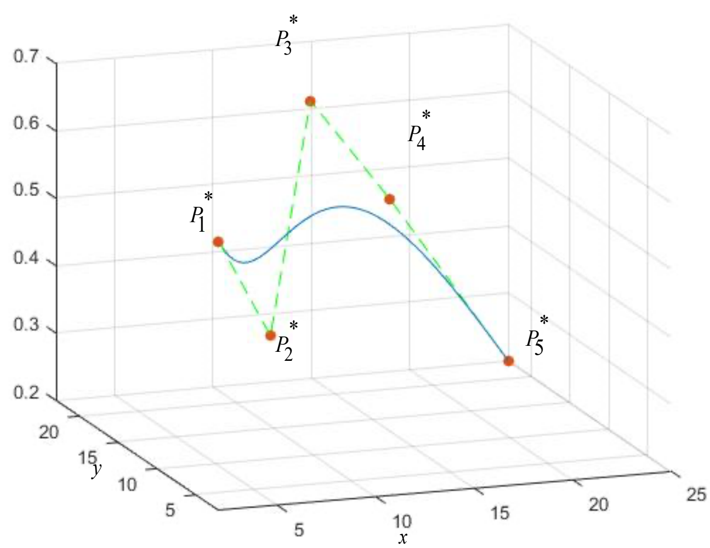

Figure 4Membership Bézier curve with respective control points and control polygon.

Later, this step is repeated to visualized non-membership and uncertainty curve for respective degree and the graph is shown as follows:

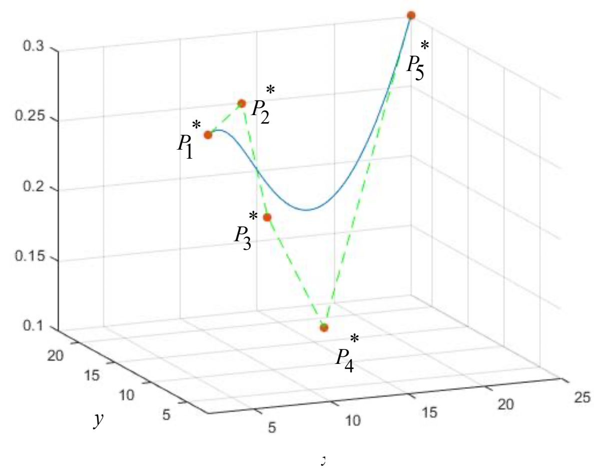

Figure 5 Non-membership Bézier curve with respective control points and control polygon

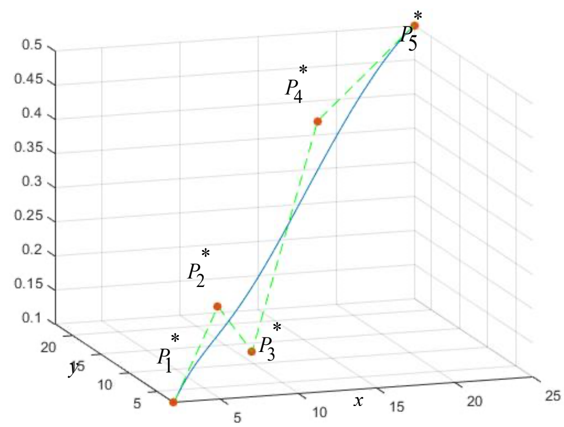

Figure 6 Uncertainty Bézier curve with respective control points and control polygon

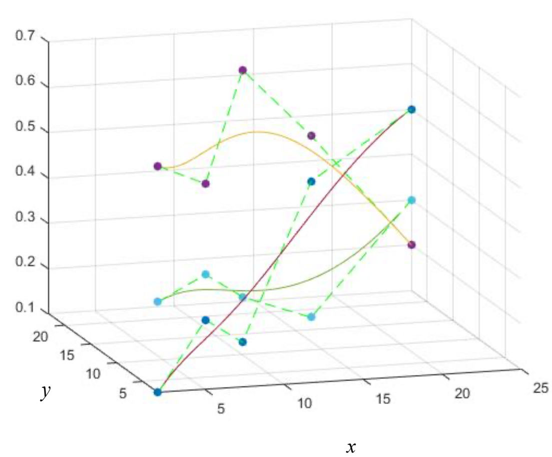

Figure 7 Intuitionistic fuzzy Bézier curve with respective control points and control polygon

An algorithm to construct and illustrate IFBC is simplified in matrix form as below:

Step 1: IFCP can be define from Equation (5).
Step 2: Degree of polynomials *n* can be determined by number of IFCP from Step 1.
Step 3: Calculate Bernstein basis function as in Equation (7).
Step 4: Coefficient of parameter terms are collected and rewrite as Equation (9).
Step 5: Step 1 until Step 4 is repeated for membership, non-membership and uncertainty curve.

5 Conclusion

This paper has introduced intuitionistic fuzzy Bézier curve approximation by defining IFCP. Approximation of IFBC model is an ideal approach in modelling data involving intuitionistic features because it is characterized by membership function, non-membership function and uncertainty function. With these functions, all data can be analyzing and fully process. The benefit of this model is its ability to visualize intuitionistic data in form of Bézier curve and it is easy to understand and analyze by data analyst. Based on Figure 4 until Figure 7, the intuitionistic fuzzy data problem can be solved through IFBC model. This model also can be extended to surface with interpolation method to solve intuitionistic fuzzy data problem.

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