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Nanofluid Flow and Heat Transfer Past a Permeable Shrinking Sheet with Convective Boundary Condition

[1]Muhammad Ikmal Syakir Bustamam and [2]Noraihan Afiqah Rawi

[1,2]Department of Mathematical Sciences
Faculty of Science, Universiti Teknologi Malaysia,
81310 Johor Bahru, Johor, Malaysia.

e-mail: [1]syakir1998@graduate.utm.my , [2]noraihanafiqah@utm.my

Abstract Nanofluid flow and heat transfer past a permeable shrinking sheet with convective boundary condition is studied. In this study, the Tiwari-Das nanofluid model has been considered with water as base fluid and copper nanoparticles is chosen. The partial differential equations of the problem are transformed to the ordinary differential equations using appropriate similarity transformation. Then, the ordinary differential equations are solved numerically using bvp4c built-in function by MATLAB software. The numerical results are presented graphically to investigate the influence of nanoparticles volume fraction and suction parameter on velocity and temperature profiles. The comparison of results with previous studies were made to validate the results. It is found that the rise of copper nanoparticles volume fraction led to the enhancement of the velocity as well as temperature profiles. This outcome indicate that the nanoparticles volume fraction can significantly affect the fluid motion and the heat transfer characteristics.

Keywords Nanofluid; shrinking sheet; steady flow; heat transfer

1 Introduction

Nanofluid can be described as a fluid containing nanometer sized particles. The nanoparticles consist of a substance with a much higher thermal conductivity, such as metal, metal oxide or carbon-based materials, and thus its addition significantly increases the fluid's heat transfer capability. Choi [1] was the first who introduced the term nanofluid. There are two popular models of heat transfer convection in nanofluids with the latest developments in nanotechnology. Buongiorno [2] considering seven slip mechanisms that can create a relative velocity between the nanoparticles and the base fluid. Among all the slip mechanisms, only Brownian motion and thermophoresis are important to develop the model for convective transport in nanofluids. Tiwari and Das [3] proposed the model which investigated the enhancement of thermal properties affected by the nanoparticle volume fraction. Abel *et al.* [4] reported that applications of nanofluids in engineering such as microelectronics, refrigerants, fuel cells, pharmaceutical processes, household refrigerators and chillers.

Stretching sheet existed when points get farther away from the axis whereas shrinking sheet existed when points get closer to the axis. In some applications for example, wire drawing, extrusion, metal spinning and hot rolling, the boundary layer flow over a stretching sheet is important. Crane [5] was the first who investigated the study of stretching sheets. The boundary layer flow caused due to a shrinking sheet was first discovered by Wang [6]. Since then, a wide range of concerns dealing with these subjects have been investigated. Ali *et al.* [7] measured MHD mixed convection of stagnation point flow on a vertical stretching sheet. Rashidi *et al.* [8] analyzed the free convective heat and mass transfer in a steady two-dimensional magnetohydrodynamic fluid flow over a stretching vertical surface in presence of thermal radiation and non-uniform magnetic field. Then, Rashidi *et al.* [9] investigated the combined heat and mass transfer of mixed convective flow over a moving vertical flat plate with hydrodynamic slip and thermal convective boundary condition.

The exact solution for some nanofluid flow past a porous shrinking/stretching surface has been found by Turkyilmazoglu [10]. The rate of heat transfer has been reported to be highly dependent on the density of the nanofluid considered and the heat capacitance. Ganvir *et al*. [11] analyzed the characteristics of nanofluid heat transfer and reported that heat transfer enhancement is greater for metallic nanoparticles, although the mechanism of thermal conductivity is still not very clear in terms of enhancement. Khan *et al.* [12] analytically examined the time-dependent mixed convection effect in a couple of nanofluid stress using an oscillatory stretching sheet and reported an increase in temperature distribution for Brownian and Hartman number.

The heat transfer of unsteady boundary layer flow of a Maxwell fluid over a permeable shrinking surface with convective boundary conditions was demonstrated by Mondal *et al.* [13]. In another study, Tian *et al*. [14] provided a numerical study on the nanofluid MHD convective stagnation-point flow near the stretching surface, considering the effects of thermophoresis and Brownian motion. They presented an investigation on the influences of the magnetic field, Prandtl number, viscous dissipation, Lewis number, heat source/sink with suction/injection, Brownian motion parameter and thermophoresis parameters on the thermal boundary layer and flow field, along with Nusselt number, skin-friction coefficient, and Sherwood number. Waini *et al.* [15] investigated the flow and heat transfer over a nonlinear permeable stretching/shrinking surface of hybrid nanofluids. Abu Bakar *et al.* [16] investigated the stability analysis on the flow and heat transfer of nanofluid past a stretching/shrinking cylinder with suction effect. Mixed convection heat transfer in a square porous cavity filled with a nanofluid with suction/injection effect has been demonstrated by Sheremet *et al*. [17]. Rohni et al. [18] implemented the flow and heat transfer over an unsteady shrinking sheet with suction in a nanofluid using Buongiorno's model. MHD flow over a permeable stretching/shrinking sheet of a nanofluid with suction/injection was investigated by Naramgari and Sulochana [19].

Boundary layer flows have been studied using either a constant surface temperature or a constant heat flux boundary condition, but only few researches on nanofluid flow considered a convective boundary condition. The convective boundary condition is more general and realistic especially with respect to several engineering and industrial processes like transpiration cooling process, material drying and many more. Therefore, it seems appropriate to use the convective boundary condition to study other boundary layer flow situations. Ahmad and Mustafa [20] implemented the convective boundary conditions on the rotating flow of nanofluid induced by an exponentially stretching sheet. In the same year, Junaid *et al.* [21] considered the three-dimensional rotating flow of nanofluid induced by a convectively heated deformable surface. They used the shooting approach combined with fifth-order Runge–Kutta method to determine the velocity and temperature distributions above the sheet. Hayat *et al*. [22] presented a simple isothermal nanofluid flow model through a porous space of homogeneous–heterogeneous reactions under the physically acceptable convective type boundary conditions.

Motivated by the important of works above, the objective of this research is to study the nanofluid flow past a permeable shrinking sheet with the convective boundary condition. The basic governing equations are closely followed the previous study done by Waini *et al*. [15]. In addition, the behavior of fluid velocity and temperature profiles for different value of embedded parameters will be graphically analyzed.

2 Mathematical Formulation

A steady flow and heat transfer past a vertical permeable shrinking sheet is considered in this study. In Figure 1 below, the x-axis is measured along the plate and the y-axis is perpendicular to the plate where there is a plate at $y = 0$.

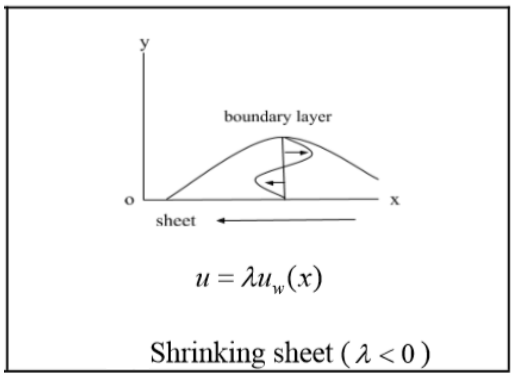

Figure 1: The physical model and coordinate system

The surface is assumed to be shrunk with the velocity $u_w(x) = ax$ where a is a constant and the wall mass flux velocity is v_0. It is presumed that the bottom surface of the plate is heated by convection from a hot fluid of uniform temperature T_f which supplies a heat transfer coefficient h_f. The surface temperature T_w is the result of a convective heating process which is characterized by the hot fluid and T_∞ is the ambient fluid temperature, thus, we have $T_f > T_w > T_\infty$. Here, nanofluid is considered where the nanoparticles size is assumed to be uniform and the nanoparticles agglomeration effects on thermophysical properties is ignored due to the nanofluid is synthesized from the nanoparticles and the base fluid as a stable mixture. The governing equations for nanofluid flow past a vertical permeable shrinking sheet are written as follows:

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \qquad (1)$$

Momentum equation:

$$u\frac{\partial y}{\partial x} + v\frac{\partial y}{\partial x} = \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2}, \qquad (2)$$

Energy equation:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial x}=\frac{k_{nf}}{(\rho C_p)_{nf}}\frac{\partial^2 T}{\partial y^2}\,. \tag{3}$$

The appropriate boundary conditions for the above boundary layer equations are:

$$v=v_0\,,\;\; u=\lambda u_w(x)\,,\;\; -k_{nf}\left(\frac{\partial T}{\partial y}\right)=h_f(T_f-T)\;\;\text{at}\;\; y=0\,,$$
$$u\to 0\,,\;\; T\to T_\infty\;\;\text{as}\;\; y\to\infty\;\;, \tag{4}$$

where u and v are the velocity components of the nanofluid along the x- and y- axes and T is the nanofluid temperature. Further, ρ_{nf}, μ_{nf}, $(\rho C_p)_{nf}$ and k_{nf} represents the density, dynamic viscosity, heat capacity and thermal conductivity of the nanofluid. Here, the subscripts f, nf and n represent the fluid, nanofluid and copper (Cu) nanoparticles solid components, respectively.

Following Devi and Devi [23], we are looking for a similarity solution of equations (1) - (4) by using the following similarity variables:

$$\psi=x\sqrt{a\nu_f}\,f(\eta)\,,\;\; \theta(\eta)=\frac{T-T_\infty}{T_f-T_\infty}\,,\;\; \eta=y\sqrt{\frac{a}{\nu_f}}\,, \tag{5}$$

where the stream function denoted by ψ with $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$ so that the continuity equation (1) is satisfied identically. The velocities are expressed as

$$u=axf'(\eta)\;\text{and}\;\; v=-\sqrt{a\nu_f}\,f(\eta)\,. \tag{6}$$

Then the wall mass transfer velocity becomes

$$v_0=-\sqrt{a\nu_f}\,S\,, \tag{7}$$

where $S=f(0)$ is the mass flux parameter with $S>0$ represent fluid suction while $S<0$ represent fluid injection and ν_f represent the base fluid kinematic viscosity.

Following Devi and Devi [23], and Oztop and Abu-Nada [24], the equations of nanofluid constants is given in Table 1 and Table 2 provides the thermophysical properties of the nanoparticles and the base fluid.

Table 1: Nanofluid thermophysical properties

Properties	Nanofluid
Heat capacity	$(\rho C_p)_{nf} = (1-\varphi)(\rho C_p)_f + \varphi(\rho C_p)_n$
Density	$\rho_{nf} = (1-\varphi)\rho_f + \varphi\rho_n$
Dynamic viscosity	$\mu_{nf} = \dfrac{\mu_f}{(1-\varphi)^{2.5}}$
Thermal conductivity	$k_{nf} = \dfrac{k_n + 2k_f - 2\varphi(k_f - k_n)}{k_n + 2k_f + \varphi(k_f - k_n)} \times (k_f)$

Table 2: Nanoparticles and fluid thermophysical properties

Properties	Cu	Water
$k(W/mK)$	400	0.613
$C_p(J/kgK)$	385	4179
$\rho(kg/m^3)$	8933	997.1

Substituting (5) into equations (2) and (3), we obtained the ordinary differential equations and then the equations are modified by using the equation of nanofluid constant in Table 1 to form:

$$\frac{1}{(1-\varphi)^{2.5}} f''' + C1 ff'' - C1 f'^2 = 0, \qquad (8)$$

$$\frac{k_{nf}}{k_f}\theta'' + C2 \Pr f\theta' = 0, \qquad (9)$$

where,

$$C1 = (1-\varphi) + \frac{\varphi \rho_n}{\rho_f} \text{ and } C2 = (1-\varphi) + \varphi \frac{(\rho C_p)_n}{(\rho C_p)_f},$$

subjected to,

$$f(0) = S, \quad f'(0) = \lambda, \; \theta'(0) = -\frac{k_f}{k_{nf}} Bi(1-\theta(0)),$$
$$f'(\eta) \to 0, \; \theta(0) \to 0 \text{ as } \eta \to \infty \tag{10}$$

where prime denotes the differentiation with respect to η , Pr = $\frac{v_f}{a_f}$ represent the Prandtl number, Bi = $\left(\frac{h_f}{k_f}\right)\sqrt{\frac{v_f}{a}}$ represents the Biot number and λ represents the parameter of stretching or shrinking sheet with $\lambda > 0$ corresponds to a stretching sheet, $\lambda < 0$ to a shrinking sheet and $\lambda = 0$ to a static sheet. Besides, φ represent the volume fraction of copper nanoparticles. Note that, Bi $\to \infty$ refers to the constant wall temperature condition, $\theta(0) = 1$.

3 Result and Discussion

In this study, the Tiwari-Das nanofluid model has been considered with water as base fluid and copper nanoparticle is chosen. According to Oztop and Abu-Nada [24], the value of Prandtl number will be fixed at Pr=6.2 which represent water as a chosen base fluid. The effects of various parameters such as the nanoparticle volume fraction (φ) and suction parameter (*S*) on velocity and temperature profiles are discussed. All the plotted graphs below are for shrinking only and the result of stretching should be opposite with the result of shrinking.

Table 3 represents the comparison of -$\theta'(0)$ for regular fluid (φ=0) with published data for different values of Pr when λ=1, *S*=0, and Bi $\to \infty$. The present results show an excellent agreement to those obtained by Wang [6], Devi and Devi [23], and Waini *et al.* [15].

Table 3: Comparison of -$\theta'(0)$ for different values of Pr when λ=1, S=0, Bi $\to \infty$

Pr	Wang [6]	Devi and Devi [23]	Waini *et al.* [15]	Present Result
2	0.9114	0.91135	0.911357	0.911356
6.13	-	1.75968	1.759682	1.759681
7	1.8954	1.89540	1.895400	1.895402
20	3.3539	3.35390	3.353893	3.353904

The profiles of velocity, $f'(\eta)$ and temperature, $\theta(\eta)$ for the effect of φ are presented in Figures 1 and 2. Different values of φ are used while others parameters are fixed as S=2.2, Pr=6.2, $\lambda = -1$ and Bi=1. Based on Figure 1, it shows that $f'(\eta)$ increases as the parameter of nanoparticle volume fraction increases. This result shows the thinning of the momentum boundary layer. This is because the addition of copper nanoparticles in the base fluid provides resistance, regardless of their comparatively higher density than the base fluid and subsequently boosts the fluid velocity [25]. Furthermore, in Figure 2, it is observed that $\theta(\eta)$ enhances with the increment of nanoparticle volume fraction hence increase the boundary layer thickness. The increment of nanoparticles volume fraction may physically dissipate more energy (heat) which simultaneously, intensifies the temperature and enhanced the thermal conductivity of nanofluids [26]. As a result, an increase in the thermal conductivity of nanofluids has a positive effect on the fluid temperature as it increases with an increase in the nanoparticle volume fraction.

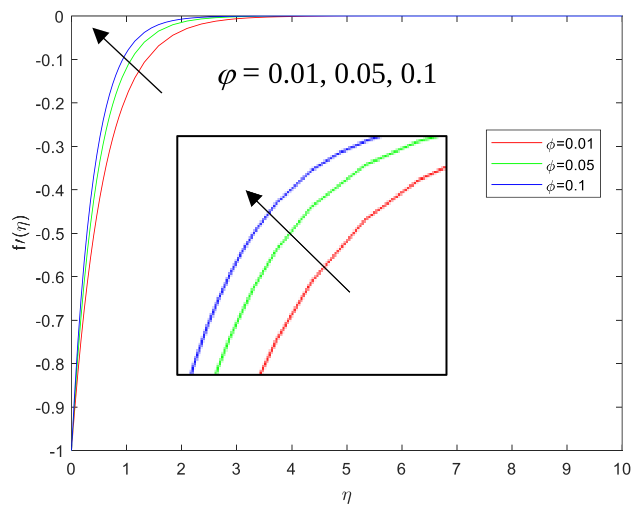

Figure 1: Effects of φ on $f'(\eta)$

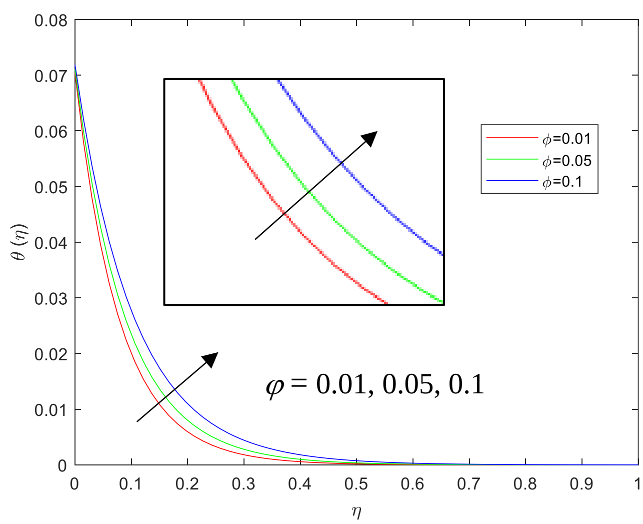

Figure 2: Effects of φ on $\theta(\eta)$

The effect of S on the profiles of $f'(\eta)$ and $\theta(\eta)$ are presented in Figures 3 and 4. Different values of S are used while others parameters are fixed as Bi=1, φ=0.1, Pr=6.2 and $\lambda=-1$. The increasing values of suction parameter led the velocity profile to decrease as presented in Figure 3. This is due to the effect of suction cause the reduction of the momentum boundary layer thickness and hence enhances the flow near the surface of the plate [16]. Furthermore, Figure 4 show the temperature profile decreases due to increasing values of suction parameter. The fluid suction may assist the heated fluid towards the surface, boost the thermal boundary layer thickness and subsequently, reduce the fluid temperature [26].

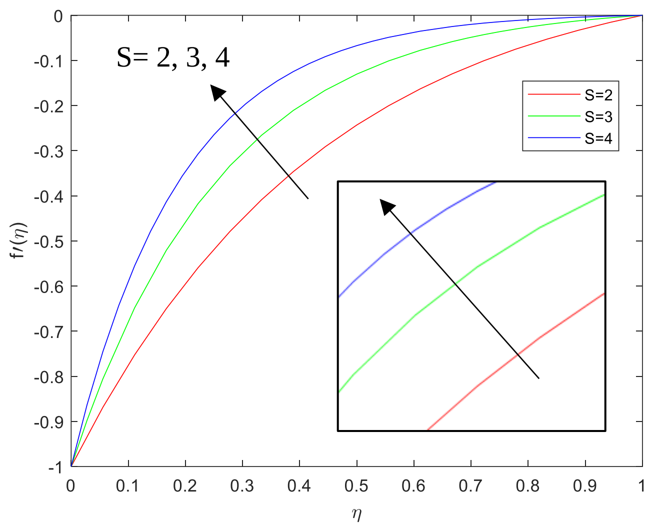

Figure 3: Effects of S on $f'(\eta)$

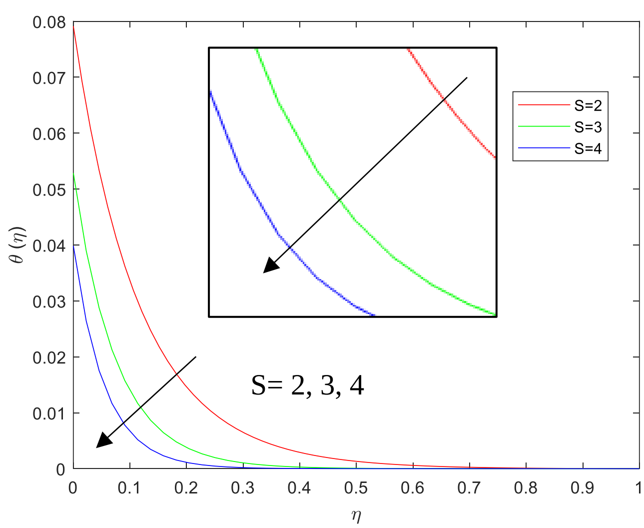

Figure 4: Effects of S on $\theta(\eta)$

4 Conclusion

In this paper, nanofluid flow and heat transfer past a permeable shrinking sheet with convective boundary condition was studied. It is noticed that the rise in φ led to the enhancement of the fluid velocity as well as temperature profile. By enhancing the values of suction, the velocity profile increases and the temperature profile decreases. This outcome indicates that the nanoparticles volume fraction and suction parameter can significantly affect the fluid motion and the heat transfer characteristics.

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