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Unsteady MHD Squeezing Flow of Jeffrey fluid Between Two Parallel Plates with Stretching Lower Plate

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Abstract The time-dependent squeezing flow of magnetohydrodynamic Jeffrey fluid between two horizontal parallel plates is investigated in this study. The upper plate is squeezed towards the lower permeable stretching plate. The fluid is electrically conducted in the presence of a variable magnetic field. The induced magnetic field is neglected for a small magnetic Reynolds number. The partial differential equations are reduced into ordinary differential equation (ODE) using dimensionless variables. The ODE is solved numerically using finite different scheme of Keller-box method. The effect of Deborah number, squeezing and magnetic parameter of the flow for suction and blowing cases on the velocity profiles are presented graphically and analysed. The velocity profiles increase initially with the increasing value of Deborah number for suction cases. An increase in the squeezing parameter enhances the velocity profiles for both suction and blowing cases. As the magnetic parameter increasing, the velocity profiles decrease for $0 \leq \eta \leq 0.4$ and it increases for $0.4 \leq \eta \leq 1$.

Keywords Squeezing flow; Jeffrey fluid; Suction and blowing; Magnetic field; Keller-box method.

1 Introduction

The well-known area of magnetohydrodynamic (MHD) is the interaction of fluids with electromagnetic fields. The flow of fluid under the influence of an electromagnetic field, such as the fluid of the MHD between moving parallel plates contributes to a squeezing flow. According to Siddiqui *et al*. [1] the use of MHD fluid as a lubricant is of interest since it avoids unintended changes in lubricant viscosity with temperature under some severe operating conditions. MHD lubrication has been investigated both numerically and practically in an externally high pressure thrust bearing. The fluid under consideration is electrically conducting in many physical conditions, and even a mild magnetic or electromagnetic field presence can affect the flow behavior. Therefore, it was necessary to discuss the flow under the influence of the magnetic field to see how it affects the conduct of the flow.

In many medical and industrial processes, the squeezing flow between parallel plates had been used to solve problems [1]. A better understanding of such flow models, explaining the squeezing flow between parallel walls, is often required to improve these equipment and machines. Some researchers made attempts to ensure the squeezing flow model become more understandable. Some of the application in industrial have different behaviour of conducting fluid when there exists the influence of magnetic fields. A porous medium always characterized by properties such as porosity and permeability. Porosity describes the volume of fluid that the material can bear, whereas permeability is the volume of fluid that can move through it [2].

This study focuses about unsteady MHD squeezing flow of Jeffrey fluid between two parallel plates with stretching lower plate. The governing equations such as continuity, momentum and energy equations are included. After using a similarity transformation, the governing equations are reduced to a first order nonlinear ordinary differential equation. This problem is solved numerically using finite difference scheme of Keller-box method. Numerical solution is solved using MATLAB software.

2 Literature Review

2.1 Types of Fluid

Any material that flows or deforms under applied shear stress is called as a fluid. Fluids contain liquids, gases and plasma, and form a subset of the states of matter. Both liquid and gases are fluid such as water, oil and air. Fluids can be classified based on their viscosity, conductivity, density and compressible or not.

2.1.1 Properties of Newtonian Fluid

Newtonian fluids are fluids that obey the Newton's law of viscosity. Newtonian fluid is defined as a true fluid whose shear stress is directly proportional to the shear strain rate. According to Afifah [3] Newtonian fluid is a fluid that as forces act on the fluid, does not affect viscosity. For a Newtonian fluid, the viscosity depends entirely on the fluid's temperature and pressure. Newtonian fluids are incompressible.

Newtonian fluids are characterized as those materials that have a linear relationship in laminar flow between the shear stress imposed and the resulting shear rate [4]. At a fixed temperature, the viscosity of Newtonian fluid remains constant. The viscosity decreases as the temperature of the fluid increases. This kind of fluid's viscosity is inversely proportional to the increase in its temperature.

2.1.2 Properties of Non-Newtonian Fluid

Non-Newtonian fluids is a fluid that affects viscosity as forces act on the fluid [3]. Non-Newtonian fluid does not follow the Newton's law of viscosity. Non-Newtonian fluids is a fluid in which shear stress is not directly proportional to the degree of shear strain. According to Kahshan *et al.* [5] the majority of industrial and biological fluids are non-Newtonian, and the Newton's law of viscosity does not explain the dynamic rheological properties of non-Newtonian fluids. In order to research the non-Newtonian flow of fluids in porous-walled channels and tubes, relatively little work has been performed.

The Jeffrey fluid model is one of many non-Newtonian fluid models that has attracted many researchers because of its consideration as a great physiological fluid model [5]. Since its constitutive equation can be reduced to that of the Newtonian model as a special case, the Jeffrey model is known as a generalization of the commonly used Newtonian fluid model. The Jeffrey fluid model is capable of describing the non-Newtonian fluids' stress relaxation property, which cannot be defined by the normal viscous fluid model. As the plate is squeezed, the fluid speed and wall shear tension intensify [6]. According to Kahshan *et al.* [5] the Jeffrey fluid model can well define the class of non-Newtonian fluids with the characteristic memory time scale, also known as the relaxation time.

Khan *et al.* [7] have considered of using Casson fluid. Unlike other simplistic models, it is able to capture complex rheological properties of a fluid. Casson fluid is other example of non-Newtonian fluid. Casson fluid can be characterized as a shear thinning liquid that is assumed to have an infinite viscosity at zero shear rate, a yield stress below which no flow occurs, and a zero viscosity at an infinite shear rate [7].

2.2 Unsteady MHD squeezing flow between two parallel plates

The action of electrically conducting fluids in a magnetic field is described by MHD. In a flowing conductive fluid, a magnetic field causes currents. A current that passes through a conductive fluid will generate forces and influence the magnetic field on the fluid [2]. MHD effects can be represented by fluid dynamics equations from Navier-Stokes. The study of MHD in engineering and industrial fields is important. Through research on MHD, we can achieve the properties of fluid flow that can be used to reduce its negative impact on a material. In various geometry forms and fluid types, MHD has been widely studied.

The MHD flow between two parallel plates is known as Hartaman flow. Hartaman and Lazarus, have been studied about the influence of magnetic field towards flowing fluid between two parallel infinite plates. According to [8] MHD is referred to as electrically conducting liquids, some of which contain liquid metals such as mercury, molten iron and ironized plasma gases. If an electrically conducting fluid is put in a continuous magnetic field, the fluid's motion causes current producing forces on the fluid [9]. MHD lubrication has been investigated both theoretically and experimentally in an externally pressurized thrust bearing by [10].

The 2-dimensional movement of an MHD fluid between parallel plates that travel symmetrically along the axial symmetry line is considered, giving rise to the squeezing flow [1]. The primary interest is in the effects of rotation and magnetic force on squeezing. The effects of squeezing and magnetic force on rotation will also be taken into consideration. In the meantime, speaking of fluid flow across a parallel plate involving a gripping force that exists around it, this event was known by the recognition of flow separation and an unstable flow occurs [11].

3 Problem Formulation

Consider the unsteady two-dimensional flow of an incompressible Jeffrey fluid separated by a distance between the two parallel walls at $y = h(t) = \sqrt{v(1-\gamma t)/a}$. The upper plate at $y = h(t) = \sqrt{v(1-\gamma t)/a}$ moving with velocity $v_w(t) = -\frac{\gamma}{2}\sqrt{v/a(1-\gamma t)}$ towards the lower porous plate at $y = 0$. The lower plate is stretching with velocity $u_w(t) = ax/(1-\gamma t)$ where a is constant and $\gamma(>0)$ represents the nonlinearly stretching sheet parameter. An electrically conducting fluid is applied with the magnetic field $M = B_0/(1-\gamma t)$ applied in the y-direction. The induced magnetic field is not considered for a small magnetic Reynolds number. Hence,

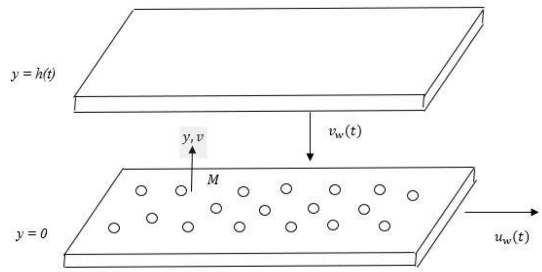

Figure 1: Schematic Diagram of the flow.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\begin{aligned}\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=&-\frac{\partial p^*}{\partial x}-\frac{\sigma {B_0}^2}{(1-\gamma t)}u+\frac{\mu}{1+\lambda_1}\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right]\\&+\frac{\mu\lambda_2}{1+\lambda_1}\left[\frac{\partial^3 u}{\partial t\partial x^2}+\frac{\partial^3 u}{\partial t\partial y^2}+u\left(\frac{\partial^3 u}{\partial x^3}+\frac{\partial^3 u}{\partial x\partial y^2}\right)\right.\\&+v\left(\frac{\partial^3 u}{\partial y^3}+\frac{\partial^3 u}{\partial x^2\partial y}\right)+\frac{\partial u}{\partial y}\left(\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 v}{\partial x^2}\right)\\&\left.+\frac{\partial u}{\partial x}\left(2\frac{\partial^2 u}{\partial x^2}-\frac{\partial^2 u}{\partial y^2}\right)+2\frac{\partial v}{\partial x}\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial v}{\partial y}\frac{\partial^2 v}{\partial x\partial y}\right],\end{aligned} \tag{2}$$

$$\begin{aligned}\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=&-\frac{\partial p^*}{\partial y}+\frac{\mu}{1+\lambda_1}\left[\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial x^2}\right]\\&+\frac{\mu\lambda_2}{1+\lambda_1}\left[\frac{\partial^3 v}{\partial t\partial x^2}+\frac{\partial^3 v}{\partial t\partial y^2}+u\left(\frac{\partial^3 v}{\partial x^3}+\frac{\partial^3 v}{\partial x\partial y^2}\right)\right.\\&+v\left(\frac{\partial^3 v}{\partial y^3}+\frac{\partial^3 v}{\partial x^2\partial y}\right)+\frac{\partial u}{\partial x}\left(\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 v}{\partial x^2}\right)\\&\left.+\frac{\partial v}{\partial x}\left(\frac{\partial^2 v}{\partial x\partial y}+\frac{\partial^2 u}{\partial y^2}\right)+2\frac{\partial v}{\partial y}\frac{\partial^2 v}{\partial y^2}\right],\end{aligned} \tag{3}$$

where u and v are the velocities in x and y direction respectively, p^* is the pressure, ρ is the fluid density, σ the electrical conductivity, μ the dynamic viscosity, λ_1 the ratio of relaxation to retardation times and λ_2 the retardation time.
The boundary conditions are prescribed in the following forms

$$u = U_0 = \frac{ax}{1-\gamma t}, \qquad v = -\frac{V_0}{1-\gamma t}, \qquad \text{at } y = 0,$$

$$u = 0, \qquad v = V_h = -\frac{\gamma}{2}\sqrt{\frac{v}{a(1-\gamma t)}}, \qquad \text{at } y = h(t). \tag{4}$$

Here $U_0 = ax/1-\gamma t$ at $y = 0$ denotes the velocity of stretching lower plate with the stretching rate a, $V_0 > 0$ indicates the suction and $V_0 < 0$ for the injection velocity.
The similarity transformation adopted from Muhammad *et al*, (2017) are given as follows

$$u = U_0 F'(\eta), \qquad v = -\sqrt{\frac{av}{1-\gamma t}}\,F(\eta), \qquad \eta = \frac{y}{h(t)},$$

$$U_0 = \frac{ax}{1-\gamma t}, \qquad h(t) = \sqrt{v(1-\gamma t)/a} \tag{5}$$

Substituting the dimensionless variables from Eq. (5) into Eqs. (1) to (3), the following dimensionless momentum equation is obtained

$$F^{IV} + (1+\lambda_1)\left[FF''' - F'F'' - \frac{S_q}{2}(\eta F''' + 3F'')\right]$$
$$+\beta\left[2F''F''' - F'F^{IV} - FF^{V} + \frac{S_q}{2}(\eta F^{V} + 5F^{IV})\right] - M^2(1+\lambda_1)F'' = 0. \tag{6}$$

The corresponding boundary conditions are

$$F(0) = S, \qquad F'(0) = 1, \qquad F(1) = \frac{S_q}{2}, \qquad F'(1) = 0, \tag{7}$$

where S_q is the squeezing parameter, S is the suction/blowing parameter, β is the Deborah number and M is the magnetic parameter. The physical parameters involved in the dimensionless equations is defined as

$$S_q = \frac{\gamma}{a}, \qquad \beta = \frac{a\lambda_2}{1-\gamma t}, \qquad M^2 = \frac{\sigma B_0^2}{\rho a}, \qquad S = \frac{V_0}{ah}. \tag{8}$$

4 Keller-box Procedure

The problem is solved by using Keller-box method. The ODEs are first converted into a system of first-order equations. By letting new dependent variables $f(\eta), g(\eta), u(\eta), v(\eta)$ and $w(\eta)$, the transformed equation can be written as

$$F = f,\ F' = g,\ F'' = u,\ F''' = v,\ F^{IV} = w. \tag{9}$$

By substitute Eq. (9) into Eq. (6), the ODEs become

$$w + (1+\lambda_1)\left[fv - gu - \frac{S_q}{2}(\eta v + 3u)\right]$$
$$+\beta\left[2uv - gw - fw' + \frac{S_q}{2}(5w + \eta w')\right] - M^2(1+\lambda_1)u = 0, \tag{10}$$

with a new form of boundary conditions;

$$f(0) = S, \qquad g(0) = 1, \qquad w(0) = 0, \qquad f(1) = \frac{S_q}{2}, \qquad g(1) = 0. \tag{11}$$

Then, the ODEs system is approximated by a finite difference method which is the central difference scheme. Therefore, equation (9) and (10) becomes,

$$\frac{1}{h_j}\left(f_j^i - f_{j-1}^i\right) = \frac{1}{2}\left(g_j^i + g_{j-1}^i\right), \tag{12}$$

$$\frac{1}{h_j}\left(g_j^i - g_{j-1}^i\right) = \frac{1}{2}\left(u_j^i + u_{j-1}^i\right), \tag{13}$$

$$\frac{1}{h_j}\left(u_j^i - u_{j-1}^i\right) = \frac{1}{2}\left(v_j^i + v_{j-1}^i\right), \tag{14}$$

$$\frac{1}{h_j}\left(v_j^i - v_{j-1}^i\right) = \frac{1}{2}\left(w_j^i + w_{j-1}^i\right), \tag{15}$$

$$\begin{aligned}
&\frac{1}{2}\left(w_j^i + w_{j-1}^i\right) + (1+\lambda_1)\Bigg[\frac{1}{4}\left(f_j^i + f_{j-1}^i\right)\left(v_j^i + v_{j-1}^i\right) \\
&-\frac{1}{4}\left(g_j^i + g_{j-1}^i\right)\left(u_j^i + u_{j-1}^i\right) - \frac{S_q}{2}\Bigg(\frac{1}{4}\left(\eta_j^i + \eta_{j-1}^i\right)\left(v_j^i + v_{j-1}^i\right) \\
&+\frac{3}{2}\left(u_j^i + u_{j-1}^i\right)\Bigg)\Bigg] + \beta\Bigg[\frac{1}{4}\left(g_j^i + g_{j-1}^i\right)\left(w_j^i + w_{j-1}^i\right) \\
&-\frac{1}{2h_j}\left(f_j^i + f_{j-1}^i\right)\left(w_j^i - w_{j-1}^i\right) + \frac{S_q}{2}\Bigg(\frac{5}{2}\left(w_j^i + w_{j-1}^i\right) \\
&+\frac{1}{2h_j}\left(\eta_j^i + \eta_{j-1}^i\right)\left(w_j^i - w_{j-1}^i\right)\Bigg)\Bigg] - \frac{M^2}{2}(1+\lambda_1)\left(u_j^i + u_{j-1}^i\right) = 0.
\end{aligned} \tag{16}$$

The nonlinear system is linearized using Newton's method. Substituting into the nonlinear algebraic system equations (12) to (16) and neglecting higher order of δ yields

$$\delta f_j - \delta f_{j-1} - \frac{h_j}{2}\left(\delta g_j + \delta g_{j-1}\right) = (r_1)_{j-\frac{1}{2}}, \tag{17}$$

$$\delta g_j - \delta g_{j-1} - \frac{h_j}{2}\left(\delta u_j + \delta u_{j-1}\right) = (r_2)_{j-\frac{1}{2}}, \tag{18}$$

$$\delta u_j - \delta u_{j-1} - \frac{h_j}{2}\left(\delta v_j + \delta v_{j-1}\right) = (r_3)_{j-\frac{1}{2}}, \tag{19}$$

$$\delta v_j - \delta v_{j-1} - \frac{h_j}{2}\left(\delta w_j + \delta w_{j-1}\right) = (r_4)_{j-\frac{1}{2}}, \tag{20}$$

$$\begin{aligned}
&(a_1)\delta w_j + (a_2)\delta w_{j-1} + (a_3)\delta v_j + (a_4)\delta v_{j-1} + (a_5)\delta u_j + (a_6)\delta u_{j-1} \\
&+(a_7)\delta g_j + (a_8)\delta g_{j-1} + (a_9)\delta f_j + (a_{10})\delta f_{j-1} = (r_5)_{j-\frac{1}{2}},
\end{aligned} \tag{21}$$

where

$$a_1 = h_j + \beta\left[\frac{h_j}{2} g_{j-\frac{1}{2}} - \frac{1}{2} f_{j-\frac{1}{2}} + \frac{S_q}{2}\left(\frac{5h_j}{2} + \frac{1}{2}\eta_{j-\frac{1}{2}}\right)\right],$$

$$a_2 = h_j + \beta\left[\frac{h_j}{2}g_{j-\frac{1}{2}} + \frac{1}{2}f_{j-\frac{1}{2}} + \frac{S_q}{2}\left(\frac{5h_j}{2} - \frac{1}{2}\eta_{j-\frac{1}{2}}\right)\right],$$

$$a_3 = h_j(1+\lambda_1)\left[\frac{1}{2}f_{j-\frac{1}{2}} - \frac{S_q}{2}\left(\frac{1}{2}\eta_{j-\frac{1}{2}}\right)\right] + \beta\left[h_j v_{j-\frac{1}{2}}\right],$$

$$a_4 = a_3,$$

$$a_5 = h_j(1+\lambda_1)\left[-\frac{1}{2}g_{j-\frac{1}{2}} - \frac{3S_q}{4}\right] - \frac{h_j M^2}{2}(1+\lambda_1) + \beta\left[h_j w_{j-\frac{1}{2}}\right],$$

$$a_6 = a_5,$$

$$a_7 = h_j(1+\lambda_1)\left[-\frac{1}{2}u_{j-\frac{1}{2}}\right] + \beta\left[\frac{h_j}{2}w_{j-\frac{1}{2}}\right],$$

$$a_8 = a_7,$$

$$a_9 = h_j(1+\lambda_1)\left[\frac{1}{2}v_{j-\frac{1}{2}}\right] - \beta\left[\frac{w_j - w_{j-1}}{2}\right],$$

$$a_{10} = a_9 \tag{22}$$

and

$$(r_1)_{j-\frac{1}{2}} = f_{j-1} - f_j + h_j g_{j-\frac{1}{2}},$$

$$(r_2)_{j-\frac{1}{2}} = g_{j-1} - g_j + h_j u_{j-\frac{1}{2}},$$

$$(r_3)_{j-\frac{1}{2}} = u_{j-1} - u_j + h_j v_{j-\frac{1}{2}},$$

$$(r_4)_{j-\frac{1}{2}} = v_{j-1} - v_j + h_j w_{j-\frac{1}{2}},$$

$$\begin{aligned}(r_5)_{j-\frac{1}{2}} = {} & -h_j(w_j + w_{j-1}) - h_j(1+\lambda_1)\Big[f_{j-\frac{1}{2}} \cdot v_{j-\frac{1}{2}} \\ & -g_{j-\frac{1}{2}} \cdot u_{j-\frac{1}{2}} - \frac{S_q}{2}\left(\eta_{j-\frac{1}{2}} \cdot v_{j-\frac{1}{2}} + 3u_{j-\frac{1}{2}}\right)\Big] \\ & -\beta\Big[h_j g_{j-\frac{1}{2}} \cdot w_{j-\frac{1}{2}} - f_{j-\frac{1}{2}} \cdot (w_j - w_{j-1}) + \frac{S_q}{2}(5w_{j-\frac{1}{2}} \\ & +\eta_{j-\frac{1}{2}}(w_j - w_{j-1})) - 2h_j v_{j-\frac{1}{2}} \cdot w_{j-\frac{1}{2}}\Big] + \frac{h_j M^2}{2}(1+\lambda_1)u_{j-\frac{1}{2}}.\end{aligned} \tag{23}$$

The linear system (17) to (21) is solved using the block-elimination method, forward and backward sweeps.

5 Results and Discussion

The governing equations (1) to (3) together with boundary conditions (4) are solved numerically using Keller-box method. Here the influence of embedding parameters to see the influence on the velocity profile, $f'(\eta)$ for suction and blowing cases are discussed. Computations are carried out for various values of $S(-0.5 \leq S \leq 0.5)$, $S_q(0.0 \leq S \leq 1.5)$, $\beta(0.0 \leq S \leq 0.6)$, $\lambda_1(0.0 \leq S \leq 3.0)$ and $M(0.0 \leq S \leq 3.0)$.

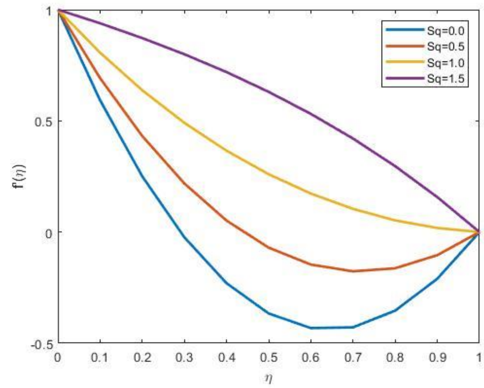

Figure 2: Impact of Sq on $f'(\eta)$ for suction case with S = 0.5, M = 1.0, β = 0.1 and $\lambda 1$ = 0.5.

Figures 2 illustrates the graph of the impact of squeezing parameter, S_q on the velocity profile, $f'(\eta)$ for suction cases. The other parameters such as S, β, λ_1 and M are keep constant. It can be seen that the velocity profile, $f'(\eta)$ decreases near the porous wall where the suction effect is dominant. Pressure is generated because of the squeezing at the upper wall towards a stretching porous wall. The pressure enhances the flow and causes the velocity profile, $f'(\eta)$ near the upper wall increases to satisfy the mass conservation constrain.

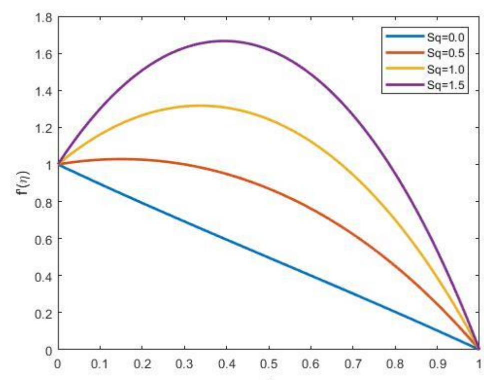

Figure 3: Impact of S_q on $f'(\eta)$ for blowing case with $S = 0.5, M = 1.0, \beta = 0.1$ and $\lambda_1 = 0.5$.

Figure 3 showed that blowing from the lower wall cause a retarding force which decrease the fluid velocity. However, the fluid velocity increases in the upper half of the channel due to the squeezing effects are dominant in the upper half of the channel.

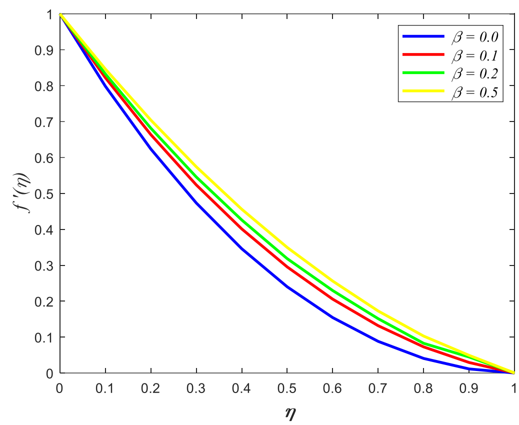

Figure 4: Impact of β on $f'(\eta)$ for suction case with $S = 0.5, S_q = M = 1.0$, and $\lambda_1 = 0.5$.

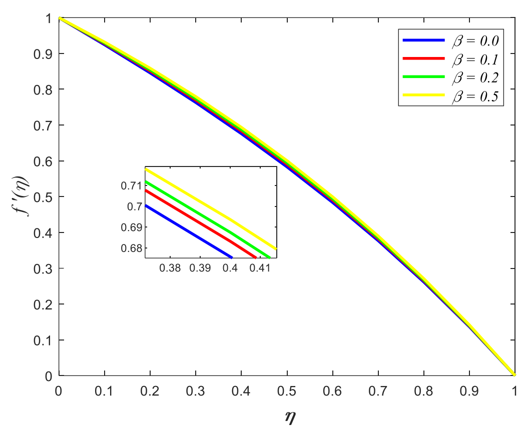

Figure 5: Impact of β on $f'(\eta)$ for blowing case with $S = 0.5, S_q = M = 1.0$, and $\lambda_1 = 0.5$.

From Figures 4, the velocity profile, $f'(\eta)$ increases initially with the increasing value of Deborah number, β. However, there is a decrease of the velocity profile, $f'(\eta)$ for $\eta \geq 0.35$. As expected, the magnitude of the velocity field is increasing function of β. Figure 5 showed the opposite results in the blowing situation.

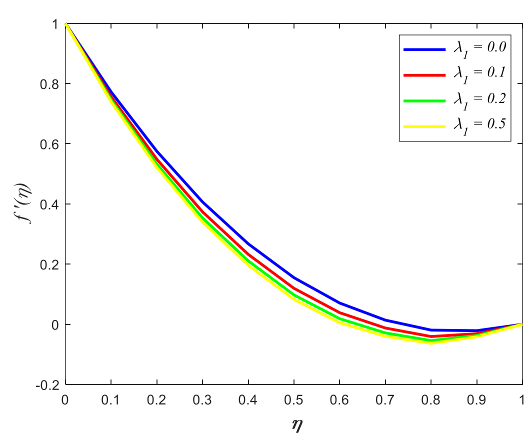

Figure 6: Impact of λ_1 on $f'(\eta)$ for suction case with $S = 0.5, S_q = M = 1.0$, and $\beta = 0.1$.

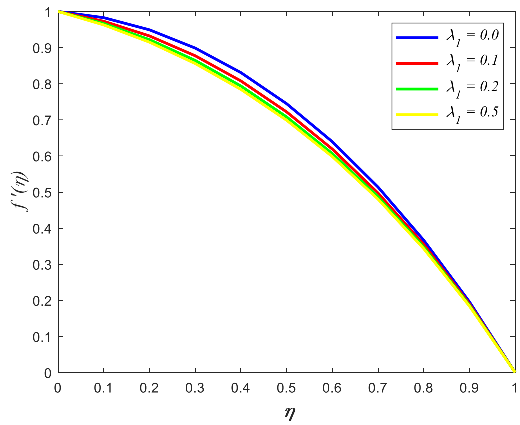

Figure 7: Impact of λ_1 on $f'(\eta)$ for blowing case with $S = 0.5, S_q = M = 1.0$, and $\beta = 0.1$.

Figure 6 presented the effect of λ_1 on the velocity profile, $f'(\eta)$ for suction case. It is found that the increase of λ_1, the velocity profile, $f'(\eta)$ decreases for $0 \leq \eta \leq 0.4$ and increases for $0.4 \leq \eta \leq 1$. Figure 7 showed the result for blowing case. From the graph it is seen an opposite behavior compared to the suction case.

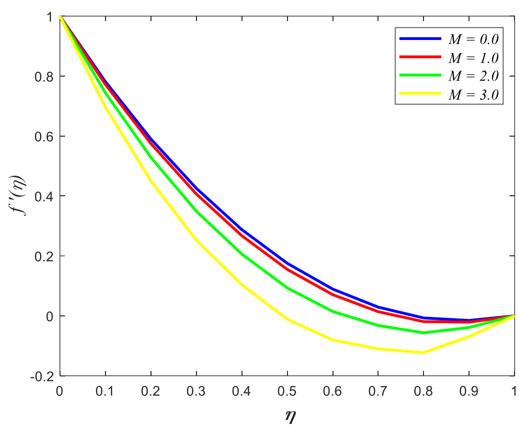

Figure 8: Impact of M on $f'(\eta)$ for suction case with $S = 0.5, S_q = 1.0, \lambda_1 = 0.5$, and $\beta = 0.1$.

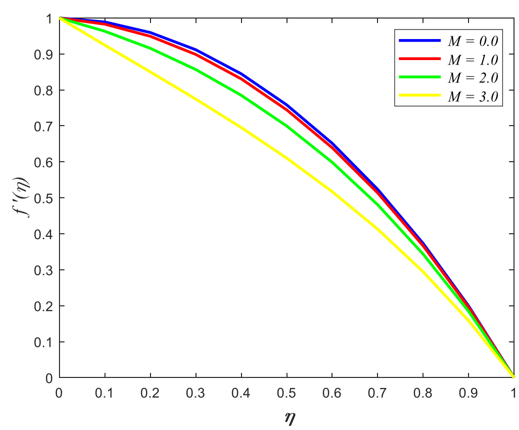

Figure 9: Impact of M on $f'(\eta)$ for blowing case with $S = 0.5, S_q = 1.0, \lambda_1 = 0.5$, and $\beta = 0.1$.

The influence of magnetic parameter, M on the velocity profile, $f'(\eta)$ for suction case is shown in Figure 8. It is observed that when M increases the velocity profile, $f'(\eta)$ decreases for $0 \leq \eta \leq 0.4$ and it increases for $0.4 \leq \eta \leq 1$. Figure 9 showed the opposite result for the blowing case. When the magnetic parameter, M increases, the velocity is decrease so does the velocity gradient since the same mass flow rate is imposed in order to satisfy the mass conservation constraint.

6 Conclusion

The problem of MHD unsteady squeezing flow of Jeffrey fluid over a porous stretching wall is examined. We have to pursue to determine how the Deborah number, β, squeezing parameter, S_q, and magnetic parameter, M affect the velocity profile, $f'(\eta)$. Solutions of the governing equation are obtained numerically using the Keller-box method. This study summarizes the following conclusion:

- An increase in the squeezing parameter, S_q enhances the velocity profile, $f'(\eta)$ for both suction and blowing cases.
- Influence of λ_1 and M on the velocity profile, $f'(\eta)$ are both similar for suction and blowing cases. The velocity profile, $f'(\eta)$ in suction is lower than blowing.
- The impact of Deborah number, β is opposite to λ_1 and M.

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