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Numerical Integrations of Improper Integral with Microsoft Foundation Classes

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Abstract The purpose of this study is to investigate the numerical method in improper integral and its applications. Improper integral is the integral with unbounded domains, or the integrand has infinite discontinuity and numerical integration is the significant method to evaluating the definite integral when there are no antiderivative present, or the function is complicated to integrate. In this study, only improper integral with infinite limit as the upper limit is considered numerical integration that been used only trapezoidal rule. Significantly, by numerical integration this study will investigate the important role of a finite number replacing the infinite upper limit and the width of subinterval in the trapezium rule when approximating converge improper integral. In addition, Microsoft Foundation classes or MFC will implemented the same approach to determine the depolarization factor for ellipsoids.

Keywords depolarization factors; trapezoidal rule; MFC

1 Introduction

In the seventeenth century, the integration process had been developed by Isaac Newton and Gottfried Leibniz, but Benhard Riemann was the one who formulated the modern definition of the definite integral [1]. Integration is the process of finding the antiderivative of the integral which known as indefinite integral where it does not involve in approximating the integral by its limit. Definite limit is the integration process involve in finding the antiderivative and the limit integration that will resulting in some value of integrations.

In this study, the numerical method in improper integral investigated with the application that used the same approach. Improper integral is the type of integral with unbounded domain, or the integrand has infinite discontinuity [2]. While numerical integration is the method that involve in approximating the definite integral using the numerical technique. Significantly, numerical integration is used when there are exist of the tabulated functions, or the function complicated to integrate. The improper integral that been used in this research is only improper integral with infinite limit as the upper limit.

Trapezoidal rule is the numerical integration method to approximate the definite integral in this study. The composite trapezoidal rule is one of the methods that resulting value by dividing the interval into small number of interval when the interval of the integration is big. This method uses the polynomial of order one to approximate the area under the curve and the interval is divided by the small interval of trapezoid.

To be highlighted, in this study will investigate the important role of a finite number or a real number replacing the infinite upper limit and the width of the subinterval in trapezium rule that approximating converge improper integrals. Generally, the improper integral will converge to their analytical value when the infinite upper limit replaced with some significantly large number.

In other than, Microsoft Foundation Classed (MFC) will be implemented in this study to approximating the depolarization factor of ellipsoid using the same approach as the trapezoidal rule. This is the development of C++ language on window's environment to displaying text, graphic and image in late 1980's. By this application, the depolarization factor is easily computed using the same approach as the trapezoidal rule.

2 Literature Review

2.1 Functions Defined by Improper Integrals

The preparation of improper integral begins with two useful convergence criteria for improper integral that not involving criteria [6]. It stated that f is locally integrable on an interval I if it is integrable on every finite closed subinterval of I [6]. The two useful convergence criterion stated as in [6] below:

Theorem 1 Cauchy Criterion for Convergence of an Improper Integral I [6]
Suppose g is locally integrable on [a.b) and denoted that,

$$G(r) = \int_a^r g(x)\, dx\, , a \leq r < b$$

The improper integral $\int_a^r g(x)\ dx$ converges if and only if for each $\epsilon > 0$, there is an $r_0 \in [a, b)$ such that,

$$|G(r) - G(r_1)| < \epsilon, r_0 \leq r, r_1 < b$$

Theorem 2 Cauchy Criterion for Convergence of an Improper Integral I [6]
Suppose g is locally integrable on [a.b) and denoted that,

$$G(r) = \int_a^r g(x)\, dx\, , a \leq r < b$$

The improper integral $\int_a^r g(x)\ dx$ convergence if and only if for each $\epsilon > 0$, there is an $r_0 \in [a, b)$ such that [6],

$$|G(r) - G(r_1)| < \epsilon, a \leq r, r_1 < r_0$$

In the part of uniform convergence of improper integral as stated in [6], function $f = f(x, y)$ with domains $I \times S$ where S is and interval or a union of intervals and I is one of the following forms [6]:

1. $[a, b)\ with - \infty < a < b\ \leq\ \infty$
2. $(a, b] with - \infty < a < b\ \leq\ \infty$
3. $(a, b)\ with - \infty < a < b\ \leq\ \infty$

f is locally integrable with respect to *x* on *I*, the improper integral that has a stated property "on S" that mean it has the property for every $y \in S$ [6].

2.2 Notes on the Convergence of Trapezoidal Rule Quadrature

The other name of numerical integration is numerical quadrate that refer to the approximation of an integral of some function $f(x)\,dx$ by a discrete summation $\sum w_i f(x_i)$ over point x_i with some weight w_i [7]. The accuracy of the trapezoidal rule with uniformly spaced points examined in these notes. The convergence rate of this method determined by the smoothness of properties of the function and usually it at the endpoint [7].

The integral to be for $x \in [0,2\pi]$ where the integral

$$I = \int_0^2 f(x)\,dx$$

The approximation in the trapezoidal rule by the summation:

$$I_N = \frac{f(0)\Delta x}{2} + \sum_{n=}^{N-1} f(n\Delta x)\Delta x + \frac{f(2\pi)\Delta x}{2}$$

Where $\Delta x = \frac{2\pi}{N}$

This paper wanted to analyze how fast the error $E_N = |I - I_N|$ decrease with N. The estimate the error as being $O\Delta x^2 = O(N^{-2})$, assuming $f(x)$ is twice differentiable on $(2,2\pi)$ the estimation is correct only for the upper bound [7]. The error can decrease much for many interesting functions faster that as discussed in simple, pessimistic upper bound, quadrature error via Fourier analysis, convergence rate from the Fourier Series and the Clenshaw-Curtis quadrature.

2.3 The Depolarization Factors for Ellipsoids and Some of Their Properties

This paper study about the problem involving magnetic which initially used to describe the magnetic properties of material [3]. Demagnetizing factor or known as depolarization factor used in potential problem. There might be some difficulty in measuring the magnetivity of the material when the total field inside the material and the surrounding change. Then, to correct the data on certain magnetic material, depolarization factors were studied [3]. The depolarization factors used by Milton [5] to study composite and showed some useful properties of depolarization factor where it is reviewed in this paper. Formulation used in the paper as follows:

$$H_d = H_a - d_i M$$

Magnetization denoted as M, magnetic field denotes as H_a, demagnetizing field denoted as H_d . d_i for $i = 1,2,3$ denoted as depolarization factor for ellipsoid in x, y, and z direction where a, b, and c be the semi principal exes of an ellipsoid as below:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

Milton [5] stated the definition of depolarization factors as follow:

$$d_1 = \frac{abc}{2} \int_0^\infty \frac{dy}{(y+a^2)^{\frac{3}{2}} \sqrt{(y+b^2)(y+c^2)}} \qquad (5.3)$$

$$d_2 = \frac{abc}{2}\int_0^{\infty} \frac{dy}{(y+b^2)^{\frac{3}{2}}\sqrt{(y+a^2)(y+c^2)}} \quad (5.4)$$

$$d_2 = \frac{abc}{2}\int_0^{\infty} \frac{dy}{(y+c^2)^{\frac{3}{2}}\sqrt{(y+b^2)(y+c^2)}} \quad (5.5)$$

3 Composite Trapezoidal Rule

The numerical method used in this study is trapezoidal rule to approximate the definite integral. The general formula of trapezoidal rule as below,

$$\int_a^b f(x)dx \approx \frac{h}{2}[f(a)+f(b)] \ where\ h = b - a \quad (3.1)$$

From the above equation, a represent lower limit, b is the upper limit, and the h is the width of the interval. Then, from this general trapezoidal rule equation it can obtain the composite trapezoidal rule. Composite trapezoidal rule is where the interval is dividing into subinterval and applying the rule to each of the subinterval. For example, the number of subintervals given by n where n = 4. Then,

$$\int_a^b f(x)dx = \int_{a\,=\,x_0}^{x_1} f(x)dx + \int_{x_1}^{x_2} f(x)dx + \int_{x_2}^{x_3} f(x)\,dx + \int_{x_3}^{x_4} f(x)dx$$
$$\approx\ Area\ A1\ +\ Area\ A2\ +\ Area\ A3\ +\ Area\ A4$$
$$=\frac{h}{2}(f_0+f_1)+\frac{h}{2}(f_1+f_2)+\frac{h}{2}(f_2+f_3)+\frac{h}{2}(f_3+f_4)$$
$$=\ \frac{h}{2}[(f_0+f_4)\ +\ 2(f_1+f_2+f_3)$$

Then the interval of the integration of [a,b] can be divided into n subinterval of the size h where $h\ =\ \frac{b-a}{n}$

Thus, the general equation of composite rule as below,

$$\int_a^b f(x)dx = \frac{h}{2}[(f_0+f_n)\ +\ 2(f_1+f_2+\cdots+f_{n-1}) \quad (3.2)$$

4 Result and Discussion

4.1 Numerical Examples and Discussion

There are three types of improper integral that been used in this study which is:

1. $\int_1^{\infty} \frac{1}{x}dx$ (4.1.1)
2. $\int_1^{\infty} e^{-x^2}dx$ (4.1.2)

3. $\int_0^\infty \frac{1}{x^2+1} dx$ (4.1.3)

By solving the function using the analytical integration method it resulting in the antiderivative but the function 4.1.2 is complicated to integrate and the antiderivative is unknown. Thus, value of function 4.1.1 and 4.1.3 is 1 and 1.57079. The figures below shown the result of numerical integration by using the trapezoidal rule method.

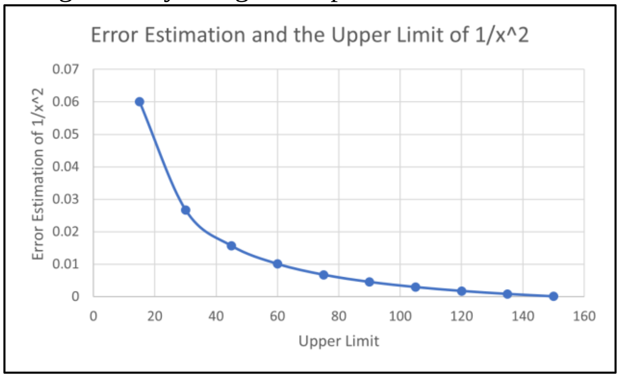

Figure 1: Error Estimation and The Upper Limit

From the above figure, the error estimation of the function 4.1.1 is decreasing when the upper limit replacing into significant large number. The value of trapezoidal rule convergence to their analytical value when the upper limit is increase and the error is small.

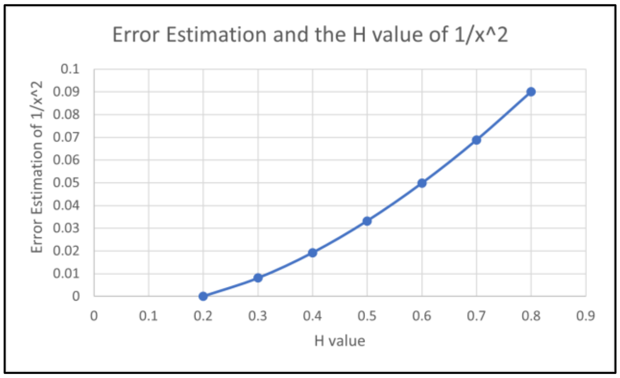

Figure 2: Error Estimation and the width value h

The figure above shown the difference when the width value is decreasing and the error estimation also decreasing. By changing the h value into the smaller values, the subinterval of the trapezium will divide into many subintervals compared to when the h value is bigger. The approximation of trapezoidal rule convergence to their analytical limit when the h value is smaller.

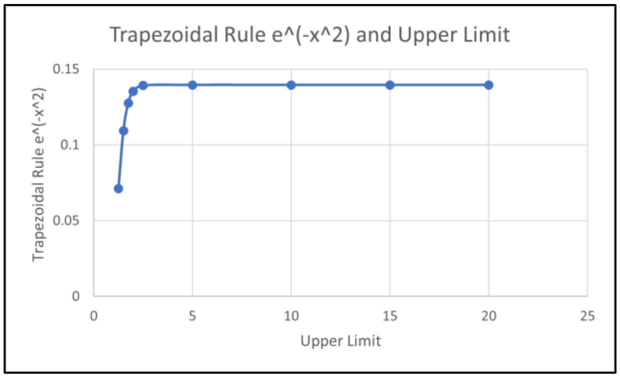

Figure 3: Trapezoidal Rule and The Upper Limit

The figure 3 shown that the graph is converge when the upper limit is increasing, this show that the trapezoidal rule values is approximating the integral accurately.

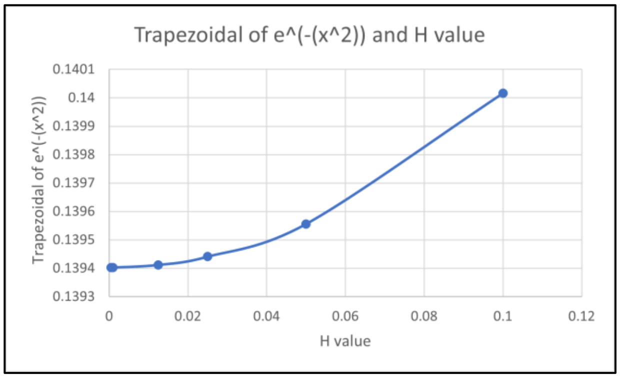

Figure 4: Trapezoidal rule and width value h

Figure 4 above shown that the value of trapezoidal rule converge when the h value small. As the h subinterval value become smaller then, the area under the curve will divided into may subinterval and resulting accurate value of the calculation. Then the value of trapezoidal rule approximating function 4.1.2 is accurate.

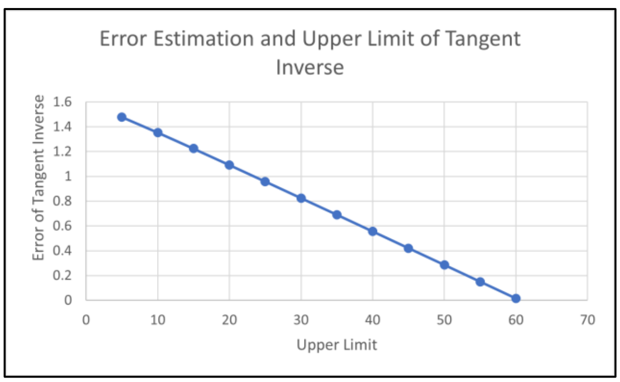

Figure 5: Error estimation and the upper limit

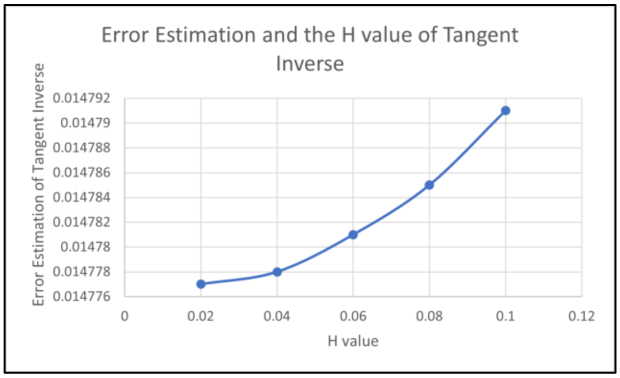

Figure 6: Error estimation and width value h

The graph of function 4.1.3 is resulting in the figure 5 and figure 6. When solving the function by analytical method it will resulting into tangent inverse as the antiderivative. From the figure 5, error estimation of trapezoidal rule is decreasing as the upper limit increasing. This shown that the value of trapezoidal rule is converge to their analytical value when the upper limit is significantly change into large numbers. By figure 6, the error also decreasing when the h value decreasing. This due to the many subintervals of trapezium dividing under the area of curve resulting in the accurate value of the integration. Significantly, the value of trapezoidal rule is convergence to their analytical value when the upper limit is change into significantly large number and the accuracy if the trapezoidal rule is depended on the size of subinterval when approximating the functions.

5 Applications

Microsoft Foundation Classes or MFC is the development on C++ language on windows environment. MFC is the type of library that easy to use function that have purposed displaying like text, graphic and image. This application used the same approach method in trapezoidal rule but using the depolarization function of ellipsoid.

The depolarization factor were studies to correct the data on certain magnetic material [4]. This is due to the magnetization there might be some difficulty to measuring the magnetivity of the material where the total field inside the material of irregular shape magnetize by a uniformly applied field and its surrounding [3].

The mathematical formulation in the study [3] of the depolarization factor or demagnetizing factor where the magnetization denoted as *M*, magnetic field denotes as H_a, demagnetizing field denoted as H_d in equation 5.1, and d_i for *i = 1,2,3* denoted as depolarization factor for ellipsoid in x, y, and z direction where a, b, and c be the semi principal exes of an ellipsoid in 5.2 as below:

$$H_d = H_a - d_i M \qquad (5.1)$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \qquad (5.2)$$

Milton [5] stated the definition of depolarization factors as follow:

$$d_1 = \frac{abc}{2}\int_0^{\infty} \frac{dy}{(y+a^2)^{\frac{3}{2}}\sqrt{(y+b^2)(y+c^2)}} \quad (5.3)$$

$$d_2 = \frac{abc}{2}\int_0^{\infty} \frac{dy}{(y+b^2)^{\frac{3}{2}}\sqrt{(y+a^2)(y+c^2)}} \quad (5.4)$$

$$d_2 = \frac{abc}{2}\int_0^{\infty} \frac{dy}{(y+c^2)^{\frac{3}{2}}\sqrt{(y+b^2)(y+c^2)}} \quad (5.5)$$

By using this definition of depolarization factor then by using MFC and the same method as trapezoidal rule, the application created as the figure below.

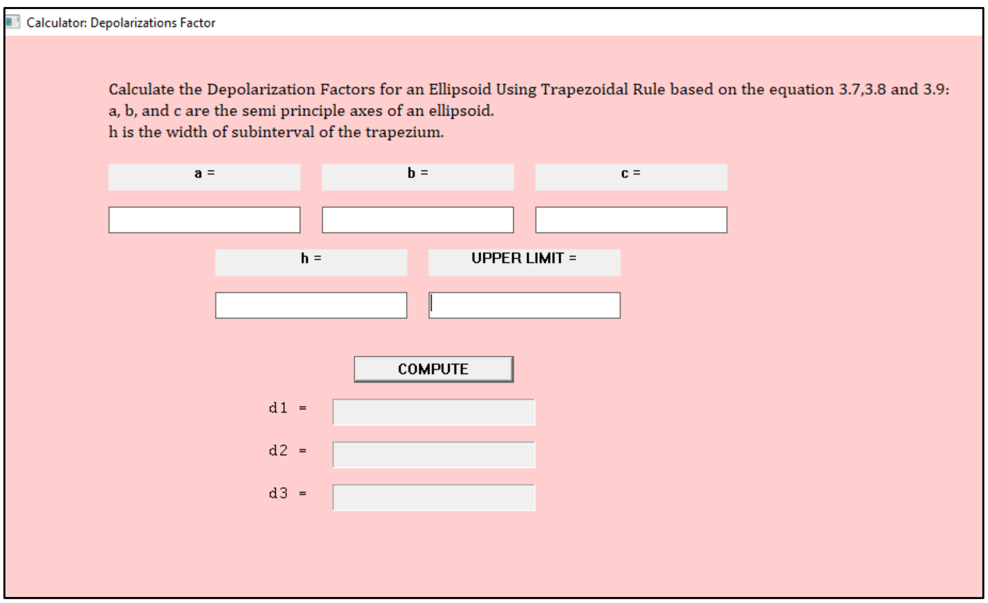

Figure 7: MFC of depolarization Factors

Based on the figure 7 above, windows display the common mechanism such as keyboard keys, edit box for input, button for processing and main window, static box, and list view window for output.

In the figure 7, the push button is the button that processing the calculation of depolarization factor which is the "COMPUTE" boxes based on the figure. This rectangular button active when the user clicked it with the mouse. Edit box is the mechanism that collect input directly from the window such as the rectangular box in upper limit that display "50". The static box is the rectangular box that displaying the fixed message or the output. For examples from the figure the static box that displaying the fixed message is "a", "b", "c", "h", "UPPER LIMIT" and the equations. The static box that displaying the output is the rectangular box under the rectangular equation boxes. The result of the MFC of depolarization factor as present as below when changing the value of semi principles axes where;

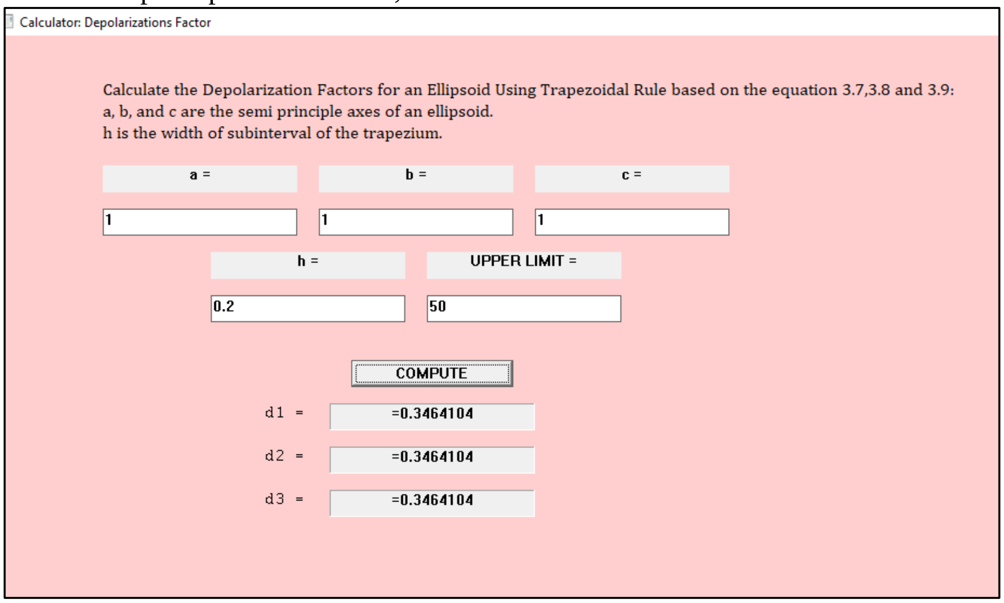

Figure 8: MFC of Depolarization Factor a=b=c

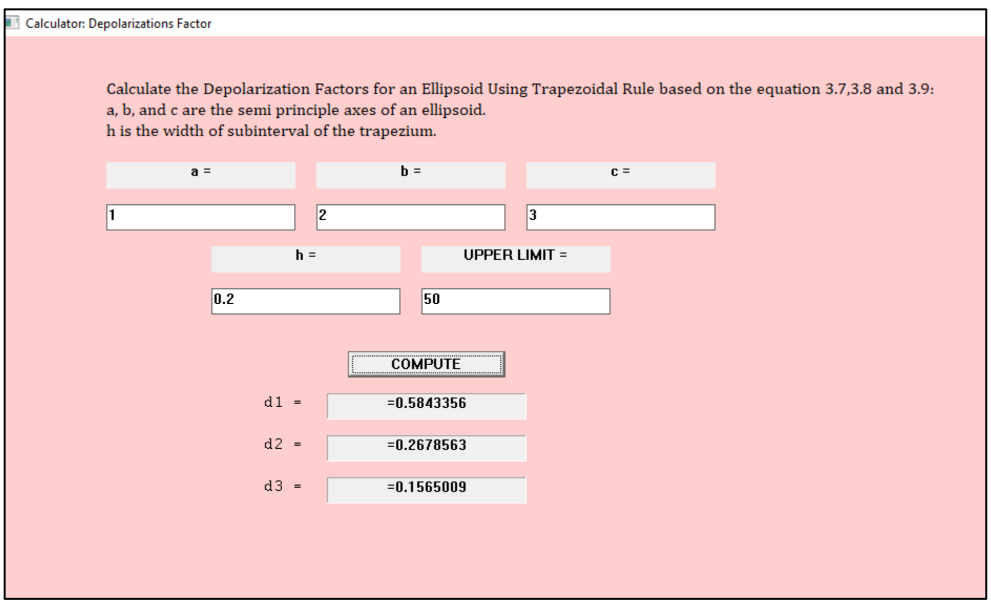

Figure 9: MFC of depolarization Factor of a, b and c different value

MFC displaying the friendly user in the windows by using three resources that had been mentioned before which is static box, edit box and the button. Mostly windows application will have these three resources in the presentation on windows.

6 Conclusion and Recommendations

In this study the only improper integral with the infinite upper limit as the upper limit is considered and the numerical integration that had been used is only trapezoidal rule. From the numerical example shows the improper integral with infinite upper limit has been studied. This study also investigates the important of changing the upper limit into significantly large number and the width of subinterval when it is small.

The previous numerical examples indicate that when changing the upper limit into the significant large number it will converge to their analytical value. Even when the analytical method cannot be carried out, the graph indicates that the value from the trapezoidal rule calculation is converge. Besides, when the width of the subinterval is small divided into many subintervals and the value of the improper integral become more accurate. Accordingly, the upper limit and the width of the subinterval resulting in the accuracy of the trapezoidal rule to be used in approximating the improper integral.

References

[1] Rosenthal, A. (1951). The History of Calculus. *The American Mathematical Monthly*, 58(2), 75-86. Doi:10.2307/2308368

[2] Abdul Sathar, M., Nurullah Rasedee, A., Mokhtar, N., & Ahmedov, A. A. (2019, December). Approximation of Improper Integral Based on Haar Wavelets. *Multidisciplinary Engineering Science and Technology (JMEST),* 6(12).

[3] Mohamad Yunos, N., Ahmad Khairuddin, T.K., Shafie, S., Ahmad, T., & Lionheart, W. (2019). The Depolarization Factors for Ellipsoids and Some of their Properties. *Malaysian Journal of Fundamental and Applied Sciences.*

[4] Chen, D. X., Pardo, E., and Sanchez, A. 2005. Demagnetizing factor for rectangular prisms. *IEEE Transactions on Magnetics*, 41(6): 2077-2088.

[5] Milton, G. (2002). Theory of Composite. *The Depolarization Factors for Ellipsoid and Some of Their Properties.*

[6] Trench, W. F. (2012). Functions defined by improper integrals. *Trinity University (Department of Mathematics), San Antonio, 3(3).*

[7] Johnson, S. G. (2010). Notes on the convergence of trapezoidal-rule quadrature.