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Perfect Codes of Non-commuting Graphs of Dihedral Groups of Order at Most 18

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Abstract A code C of graph Γ is called a perfect code if the intersection of the neighborhood of code C are empty set and union of the neighborhood of C are same with vertex set of the Γ. The non-commuting graph denoted by Γ_G is a graph with the non-central vertex set such that two distinct vertices x and y are adjacent whenever they did not commute in G, i.e., $xy \neq yx$. This study is focused on the interplay between dihedral group, non-commutating graph and perfect code for order at most 18. Therefore, the value of n for dihedral group in this study is $3 \leq n \leq 9$.

Keywords Perfect code; Non-commuting graph; Dihedral group; Non-commutative element; Coding Theory.

1 Introduction

The interplay between group theory and graph theory has become a focus of research over the last decades. There are several works that have assigned a group or a ring to a graph and studied the properties of the associated graph [1,2,3,4]. However, this research focusses on group theory, more precisely on dihedral groups while for the graph theory focusses on non-commuting graph. In addition, this research shown the interplay between both graph and coding theory where perfect code of non-commuting graphs has been determined.

Dihedral groups are among the examples of finite groups. Dihedral groups are called a group of symmetries of a regular polygon which include rotation and reflection. In addition, dihedral groups are also known as a non-abelian permutation group. Dihedral group, D_n $(n \geq 3)$ is defined as the rigid motions taking a regular n-gon back to itself, with the operation being composition, where a rigid motion is a distance-preserving transformation, such as a rotation and a reflection. Jafarpour *et al.* [5] stated that dihedral groups play a significant role in group theory, geometry, and chemistry.

The idea of graphs come from a real situation that can easily described by drawing diagram consists of a set of points and lines linking these points. The concept was introduced by Leonard Euler in 1736 [6]. In 1975, the idea of non-commuting graph comes from the Endrős problem [1].

From that time on, non-commuting graphs have been widely studied. The concept of the non-commuting graph of a finite group has been introduced by Abdollahi *et al.* in 2006 [7]. They studied the effect of graph theoretical properties of Γ_G on the group theoretical properties of G and the properties of two non-abelian groups with isomorphic to non-commuting graph is always the same. Later, in 2008, Darafsleh [8] studied the certain of simple group H. The non-commuting graph, Γ_G isomorphic with the non-commuting graph of simple group H, Γ_H implies the group G is isomorphic with the simple group H.

The idea of coding theory was introduced by Claude Shannon in 1948 [9]. Coding theory is a branch of mathematics that deals with the transmission and retrieval of data through noisy network [10]. Perfect code on graph was implemented by Biggs [11]. Perfect code are error-correcting codes in Hamming spheres surrounding the codewords entirely fill the Hamming space without overlap.

Next, there are no nontrivial 1-perfect codes over complete bipartite graphs with at least three vertices proven by Kratochvil in 1986 [12]. Moreover, Mollard (2011) have generalized the perfect codes in Cartesian products of graphs [13]. In 2015, Krotov [14] proven the existence of 1-perfect codes in Doob graph without admitting some group structure. Meanwhile, Ma *et al.* [15] stated the full overview of finite groups that embrace a perfect code with enhance power graphs. Recently, Ma [16] studied all perfect codes of a proper reduced power graph as determined, given that a perfect code is recognized by the right reduced power graph. In 1972, Lenstra proved two theorems of perfect codes which are the Lloyd's theorem and assertion of impossibility of perfect group codes over non-prime power alphabet [17].

2 Preliminaries

In this section, some concepts and previous research findings that are used in this study are presented.

2.1 Dihedral Group

Group theory is the study of groups which its system consists of a set of elements with a binary operation that combines any two elements of the set and satisfy some axioms. The group need to be closed under the binary operation and satisfy the associative law, contain an identity, and have an inverse [18].

The elementary aspects of the dihedral groups are explained as follows [19]

Theorem 2.1 [19] D_n has a size of $2n$.

Theorem 2.2 [19] The n rotations in D_n are $1, a, a^2, \ldots, a^{n-1}$.

Theorem 2.3 [19] The n reflections in D_n are $b, ab, a^2b, \ldots, a^{n-1}b$.

Theorem 2.4 [19] The group D_n has $2n$ elements that consists of the n rotations and n reflections satisfying the relations: $a^n = 1, b^2 = 1$ and $bab = a^{-1}$, where $D_n = \{1, a, a^2, \ldots, a^{n-1}, b, ab, a^2b, \ldots, a^{n-1}b\}$.

Definition 2.1 [19] Conjugate
Let a and b be two elements in finite group G, then a and b are called conjugate if there exists an element g in G such that $gag^{-1} = b$.

Definition 2.2 [19] Conjugacy class
Let a and b be two elements in finite group G, then a and b are called conjugate in G (and call b a conjugate of a) if $x^{-1}ax = b$ for some $x \in G$. The conjugacy class of a is the set $cl(a) = \{x^{-1}ax | \, x \in G\}$.

2.2 Non-commutating Graph

At 2008, Talebi [3] studied the non-commuting graph of dihedral groups D_{2n} and the independent number, vertex chromatic number, clique number and minimum size of vertex cover of dihedral groups were found. Recently, Khasraw *et al.* [20] studied the detour index, eccentric connectivity, and total eccentricity polynomials of non-commuting graph of dihedral group, D_n.

Definition 2.3 [7] Let G be a non-abelian group and let $Z(G)$ be the center of G. The non-commuting graph denoted by Γ_G is a graph with vertex set $G \backslash Z(G)$ and two distinct vertices x and y are adjacent whenever they did not commute in G, $xy \neq yx$.

2.3 Perfect Code

The perfect code is defined by Kratochvil in [9] as follows:

Definition 2.4 [9] Neighbourhood
Let $\boldsymbol{X}$ be an element of $\mathbb{F}_{\boldsymbol{q}^{\boldsymbol{n}}}$ and $\boldsymbol{r} \geq \boldsymbol{0}$. Then the neighbourhood of $\boldsymbol{X}$ is denoted as

$$S_r(X) = \left\{Y \in \mathbb{F}_{q^n} |\, d(x, y) \leq r\right\}$$

Definition 2.5 [12] Let Γ be a graph with vertex set $V(\Gamma)$ and edge set $E(\Gamma)$. A subset C of $V(\Gamma)$ is called a perfect code if it satisfies the following conditions:

i. $S_1(X) \cap S_1(Y) = \emptyset$ for all $X, Y \in C, X \neq Y$
ii. $V(\Gamma) = \cup \; S_1(X)$, for all , $X \in C$

Theorem 2.5 [12]
Let Γ be a graph and C is a subset of $V(\Gamma)$ is a code. Then C is a perfect code whenever it satisfies the following conditions:

i. C is an independent set.
ii. Every vertex in $V(\Gamma)$ is either in C or it is adjacent to exactly one element in C.

3 Methodology

In this study, the value of n for dihedral group involves is in $3 \leq n \leq 9$ that gives the range of order is between 6 and 18. To achieve the results, the non-commutative elements of dihedral groups of order at most 18 are determined by using definition of non-commutative elements, $xy \neq yx \ni \forall x, y \in D_n$. After that, the non-commuting graph of these dihedral groups of order at most 18 have been obtained and illustrated. The non-commuting graph denoted by Γ_{D_n} is a graph with vertex set $G \backslash Z(D_n)$ and two distinct vertices x and y are adjacent whenever they did not commute in D_n, $xy \neq yx$.

Later, the perfect codes of dihedral groups of order at most 18 are determined by using the definition of perfect code such as $S_1(X) \cap S_1(Y) = \emptyset$ for all $X, Y \in C, X \neq Y$ and $V(\Gamma) = \cup$

$S_1(X)$, for all $X \in C$. Lastly, the observation and analysis of the pattern of perfect code of non-commutating graph of order at most 18 after all the perfect code has been found.

4 Results and Discussion

4.1 The Non-commutative Elements of Dihedral Groups of Order at Most 18

The non-commutative elements for dihedral group for $n = 3, 4, 5, 6, 7, 8$ and 9 have been determined by using the definition of non-commutative and the results obtained shows as the proposition. Proposition 4.1 show the non-commutative elements of dihedral group of order 6.

Proposition 4.1 Let $n = 3$, D_3 be a dihedral group of order 6. D_3 is represented as $D_3 = \langle a, b: a^3 = b^2 = 1, ba = a^{-1}b \rangle$. Then, the non-commutative elements of D_3 is $\{a, a^2, b, ab, a^2b\}$.

Proof The non-commutative elements of D_3 are found and defined as $xy \neq yx$.

First, let $x = e$.
If $y = e$, thus $(e)(e) = e$ and $(e)(e) = e$. Then, $(e)(e) = (e)(e)$.
If $y = a$, thus $(e)(a) = a$ and $(a)(e) = a$. Then, $(e)(a) = (a)(e)$.
If $y = a^2$, thus $(e)(a^2) = a^2$ and $(a^2)(e) = a^2$. Then, $(e)(a^2) = (a^2)(e)$.
If $y = b$, thus $(e)(b) = b$ and $(b)(e) = b$. Then, $(e)(b) = (b)(e)$.
If $y = ab$, thus $(e)(ab) = ab$ and $(ab)(e) = ab$. Then, $(e)(ab) = (ab)(e)$.
If $y = a^2b$, thus $(e)(a^2b) = a^2b$ and $(a^2b)(e) = a^2b$. Then, $(e)(a^2b) = (a^2b)(e)$.
Hence, $(e)(y) = (y)(e)$.

Next, let $x = a$.
If $y = e$, thus $(a)(e) = a$ and $(e)(a) = a$. Then, $(a)(e) = (e)(a)$.
If $y = a$, thus $(a)(a) = a^2$ and $(a)(a) = a^2$. Then, $(a)(a) = (a)(a)$.
If $y = a^2$, thus $(a)(a^2) = e$ and $(a^2)(a) = e$. Then, $(a)(a^2) = (a^2)(a)$.
If $y = b$, thus $(a)(b) = ab$ and $(b)(a) = a^2b$. Then, $(a)(b) \neq (b)(a)$.
If $y = ab$, thus $(a)(ab) = a^2b$ and $(ab)(a) = b$. Then, $(a)(ab) \neq (ab)(a)$.
If $y = a^2b$, thus $(a)(a^2b) = b$ and $(a^2b)(a) = ab$. Then, $(a)(a^2b) \neq (a^2b)(a)$.
Hence, $(a)(y) \neq (y)(a)$ is $\{b, ab, a^2b\}$.

In a similar way for $x = a^2, b, ab$ and a^2b.

Therefore, the non-commutative elements of D_3 are $\{a, a^2, b, ab, a^2b\}$.

Proposition 4.2 Let $n = 4$, D_4 be a dihedral group of order 8. D_4 is represented as $D_4 = \langle a, b: a^4 = b^2 = 1, ba = a^{-1}b \rangle$. Then, the non-commutative elements of D_4 is $\{a, a^3, b, ab, a^2b, a^3b\}$.

Proposition 4.3 Let $n = 5$, D_5 be a dihedral group of order 10. D_5 is represented as $D_5 = \langle a, b: a^5 = b^2 = 1, ba = a^{-1}b \rangle$. Then, the non-commutative elements of D_5 is $\{a, a^2, a^3, a^4, b, ab, a^2b, a^3b, a^4b\}$.

Proposition 4.4 Let $n = 6$, D_6 be a dihedral group of order 12. D_6 is represented as $D_6 = \langle a, b: a^6 = b^2 = 1, ba = a^{-1}b \rangle$. Then, the non-commutative elements of D_6 is $\{a, a^2, a^4, a^5, b, ab, a^2b, a^3b, a^4b, a^5b\}$.

Proposition 4.5 Let $n = 7$, D_7 be a dihedral group of order 14. D_7 is represented as $D_7 = \langle a, b: a^7 = b^2 = 1, ba = a^{-1}b \rangle$. Then, the non-commutative elements of D_7 is $\{a, a^2, a^3, a^4, a^5, a^6, b, ab, a^2b, a^3b, a^4b, a^5b, a^6b\}$.

Proposition 4.6 Let $n = 8$, D_8 be a dihedral group of order 16. D_8 is represented as $D_8 = \langle a, b: a^8 = b^2 = 1, ba = a^{-1}b \rangle$. Then, the non-commutative elements of D_8 is $\{a, a^2, a^3, a^5, a^6, a^7, b, ab, a^2b, a^3b, a^4b, a^5b, a^6b, a^7b\}$.

Proposition 4.7 Let $n = 9$, D_9 be a dihedral group of order 18. D_9 is represented as $D_9 = \langle a, b: a^9 = b^2 = 1, ba = a^{-1}b \rangle$. Then, the non-commutative elements of D_9 is $\{a, a^2, a^3, a^4, a^5, a^6, a^7, a^8, b, ab, a^2b, a^3b, a^4b, a^5b, a^6b, a^7b, a^8b\}$.

4.2 The Non-commutating Graph of Dihedral Groups of Order at Most 18

Determination of the non-commutating graph for dihedral group for order at most 18 is using by the definition of non-commutating graph and the graph have been illustrated as partite graph. The results obtained shows as the lemma. Lemma 4.1 show the non-commuting graph of dihedral group of order 6.

Lemma 4.1 Let D_3 be a dihedral group represented as $\{e, a, a^2, b, ab, a^2b\}$. Then, the non-commuting graph of D_3 is a complete 4-partite graph, $K_{1,1,1,2}$.

Proof According to Definition 2.3 and Proposition 4.1, vertex set of D_3 denoted as $V(\Gamma_{D_3})$ is $\{a, a^2, b, ab, a^2b\}$ and the center of D_3, $Z(D_3)$ is $\{e\}$. Then, the connected vertices in $\Gamma(D_3)$ are combination of two 4-star graph where both nodes are a and a^2. Hence, two vertices are not having any edge which are a and a^2 while other vertices are connected. Therefore, the non-commuting graph of D_3 pictured as a complete 4-partite graph, $K_{1,1,1,2}$ in Figure 1.

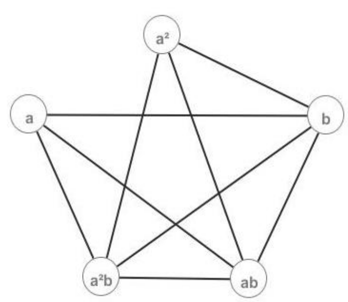

Figure 1: The non-commuting graph of D_3

Lemma 4.2 Let D_4 be a dihedral group represented as $\{e, a, a^2, a^3, b, ab, a^2b, a^3b\}$. Then, the non-commuting graph of D_4 is a complete 5-partite graph, $K_{1,1,1,1,2}$.

Lemma 4.3 Let D_5 be a dihedral group represented as $\{e, a, a^2, a^3, a^4, b, ab, a^2b, a^3b, a^4b\}$. Then, the non-commuting graph of D_5 is a complete 6-partite graph, $K_{1,1,1,1,1,4}$.

Lemma 4.4 Let D_6 be a dihedral group represented as $\{e, a, a^2, a^3, a^4, a^5, b, ab, a^2b, a^3b, a^4b, a^5b\}$. Then, the non-commuting graph of D_6 is a complete 4-partite graph, $K_{2,2,2,4}$.

Lemma 4.5 Let D_7 be a dihedral group represented as $\{e, a, a^2, a^3, a^4, a^5, a^6, b, ab, a^2b, a^3b, a^4b, a^5b, a^6b\}$. Then, the non-commuting graph of D_7 is a complete 8-partite graph, $K_{1,1,1,1,1,1,1,6}$.

Lemma 4.6 Let D_8 be a dihedral group represented as $\{e, a, a^2, a^3, a^4, a^5, a^6, a^7, b, ab, a^2b, a^3b, a^4b, a^5b, a^6b, a^7b\}$. Then, the non-commuting graph of D_8 is a complete 5-partite graph, $K_{2,2,2,2,6}$.

Lemma 4.7 Let D_9 be a dihedral group represented as $\{e, a, a^2, a^3, a^4, a^5, a^6, a^7, a^8, b, ab, a^2b, a^3b, a^4b, a^5b, a^6b, a^7b, a^8b\}$. Then, the non-commuting graph of D_9 is a complete 10-partite graph, $K_{1,1,1,1,1,1,1,1,1,8}$.

4.3 The Perfect Codes of Non-commutating Graph of Dihedral Groups of Order at Most 18

The perfect code of the non-commutating graph of dihedral group for $n = 3,4,5,6,7,8$ and 9 have been determined by using the definition of perfect code (Definition 2.3) and the results obtained shows as the following theorems. Theorem 4.1 shows the perfect code of the non-commutating graph of dihedral group of order 6.

Theorem 4.1 Let $V(\Gamma_{D_3}) = \{a, a^2, b, ab, a^2b\}$. Then, a perfect code of non-commutating graph of dihedral group of order 6, D_3 are $\{b\}, \{ab\}$ and $\{a^2b\}$.

Proof Based on Lemma 4.1, vertex set of D_3, $V(\Gamma_{D_3}) = \{a, a^2, b, ab, a^2b\}$. According to Definition 2.4, the neighborhood of vertex set of D_3 are,

$S_1(a) = \{a, b, ab, a^2b\}$
$S_1(a^2) = \{a^2, b, ab, a^2b\}$
$S_1(b) = \{a, a^2, b, ab, a^2b\}$
$S_1(ab) = \{a, a^2, b, ab, a^2b\}$
$S_1(a^2b) = \{a, a^2, b, ab, a^2b\}$

First, let $C = \{b\}$ be a code. Then,

i. $S_1(b) \cap S_1(\emptyset) = \{a, a^2, b, ab, a^2b\} \cap \emptyset = \emptyset$
ii. $S_1(b) = \{a, a^2, b, ab, a^2b\} = V(D_3)$

Thus, $C = \{b\}$ is a perfect code.

Next, let $C = \{ab\}$ be a code. Then,

i. $S_1(ab) \cap S_1(\emptyset) = \{a, a^2, b, ab, a^2b\} \cap \emptyset = \emptyset$
ii. $S_1(ab) = \{a, a^2, b, ab, a^2b\} = V(D_3)$

Thus, $C = \{ab\}$ is a perfect code.

Next, let $C = \{a^2b\}$ be a code. Then,

i. $S_1(a^2b) \cap S_1(\emptyset) = \{a, a^2, b, ab, a^2b\} \cap \emptyset = \emptyset$
ii. $S_1(a^2b) = \{a, a^2, b, ab, a^2b\} = V(D_3)$

Thus, $C = \{a^2b\}$ is a perfect code.

Next, for $C = \{a\}$ and $\{a^2\}$, they did not satisfy the second condition of perfect code in Definition 2.3 such that,

i. $S_1(a) = \{a, b, ab, a^2b\} \neq V(\Gamma_{D_3})$
ii. $S_1(a) = \{a, b, ab, a^2b\} \neq V(\Gamma_{D_3})$

Next, for $C = \{a, a^2\}, \{a, b\}, \{a, ab\}, \{a, a^2b\}, \{a^2, b\}, \{a^2, ab\}, \{a^2, a^2b\}, \{b, ab\}, \{b, a^2b\}$ and $\{ab, a^2b\}$ are not a perfect code because it does not satisfy the first condition in Definition 2.3 such that,

i. $S_1(a) \cap S_1(a^2) = \{a, b, ab, a^2b\} \neq \emptyset$
ii. $S_1(a) \cap S_1(b) = \{a, b, ab, a^2b\} \neq \emptyset$
iii. $S_1(a) \cap S_1(ab) = \{a, b, ab, a^2b\} \neq \emptyset$
iv. $S_1(a) \cap S_1(a^2b) = \{a, b, ab, a^2b\} \neq \emptyset$
v. $S_1(a^2) \cap S_1(b) = \{a, b, ab, a^2b\} \neq \emptyset$
vi. $S_1(a^2) \cap S_1(ab) = \{a, b, ab, a^2b\} \neq \emptyset$
vii. $S_1(a^2) \cap S_1(a^2b) = \{a, b, ab, a^2b\} \neq \emptyset$
viii. $S_1(b) \cap S_1(ab) = \{a, a^2, b, ab, a^2b\} \neq \emptyset$
ix. $S_1(b) \cap S_1(a^2b) = \{a, a^2, b, ab, a^2b\} \neq \emptyset$
x. $S_1(ab) \cap S_1(a^2b) = \{a, a^2, b, ab, a^2b\} \neq \emptyset$

Next, for $C = \{a, a^2, b\}, \{a, a^2, ab\}, \{a, a^2, a^2b\}, \{a, b, ab\}, \{a, b, a^2b\}, \{a, ab, a^2b\}, \{a^2, b, ab\}, \{a^2, b, a^2b\}, \{a^2, b, a^2b\}$ and $\{b, ab, a^2b\}$, they are not a perfect code because it does not satisfy the first condition in Definition 2.3 such that,

i. $S_1(a) \cap S_1(a^2) \cap S_1(b) = \{a, b, ab, a^2b\} \neq \emptyset$
ii. $S_1(a) \cap S_1(a^2) \cap S_1(ab) = \{a, b, ab, a^2b\} \neq \emptyset$
iii. $S_1(a) \cap S_1(a^2) \cap S_1(a^2b) = \{a, b, ab, a^2b\} \neq \emptyset$
iv. $S_1(a) \cap S_1(b) \cap S_1(ab) = \{a, b, ab, a^2b\} \neq \emptyset$
v. $S_1(a) \cap S_1(b) \cap S_1(a^2b) = \{a, b, ab, a^2b\} \neq \emptyset$
vi. $S_1(a) \cap S_1(ab) \cap S_1(a^2b) = \{a, b, ab, a^2b\} \neq \emptyset$
vii. $S_1(a^2) \cap S_1(b) \cap S_1(ab) = \{a, b, ab, a^2b\} \neq \emptyset$
viii. $S_1(a^2) \cap S_1(b) \cap S_1(a^2b) = \{a, b, ab, a^2b\} \neq \emptyset$
ix. $S_1(a^2) \cap S_1(b) \cap S_1(a^2b) = \{a, b, ab, a^2b\} \neq \emptyset$
x. $S_1(b) \cap S_1(ab) \cap S_1(a^2b) = \{a, a^2, b, ab, a^2b\} \neq \emptyset$

Next, for $C = \{a, a^2, b, ab\}, \{a, a^2, b, a^2b\}, \{a, a^2, ab, a^2b\}, \{a, b, ab, a^2b\}$ and $\{a^2, b, ab, a^2b\}$, they are not a perfect code because it does not satisfy the first condition in Definition 2.3 such that,

i. $S_1(a) \cap S_1(a^2) \cap S_1(b) \cap S_1(ab) = \{a, b, ab, a^2b\} \neq \emptyset$
ii. $S_1(a) \cap S_1(a^2) \cap S_1(b) \cap S_1(a^2b) = \{a, b, ab, a^2b\} \neq \emptyset$
iii. $S_1(a) \cap S_1(a^2) \cap S_1(ab) \cap S_1(a^2b) = \{a, b, ab, a^2b\} \neq \emptyset$
iv. $S_1(a) \cap S_1(b) \cap S_1(ab) \cap S_1(a^2b) = \{a, b, ab, a^2b\} \neq \emptyset$
v. $S_1(a^2) \cap S_1(b) \cap S_1(ab) \cap S_1(a^2b) = \{a, b, ab, a^2b\} \neq \emptyset$

Next, for $C = \{a, a^2, b, ab, a^2b\}$, it is not a perfect code because it does not satisfy the first condition in Definition 2.3 such that,

i. $S_1(a) \cap S_1(a^2) \cap S_1(b) \cap S_1(ab) \cap S_1(a^2b) = \{b, ab, a^2b\} \neq \emptyset$

Therefore, a perfect code of dihedral group of order 6 is $\{b\}, \{ab\}$ and $\{a^2b\}$.

Theorem 4.2 Let $V(\Gamma_{D_4}) = \{a, a^3, b, ab, a^2b, a^3b\}$. Then, there are no codes that are perfect for non-commuting graph of dihedral group of order 8, D_4.

Theorem 4.3 Let $V(\Gamma_{D_5}) = \{a, a^2, a^3, a^4, b, ab, a^2b, a^3b, a^4b\}$. Then, a perfect codes of non-commutating graph of dihedral group of order 10, D_5 are $\{b\}, \{ab\}, \{a^2b\}, \{a^3b\}$ and $\{a^4b\}$.

Theorem 4.4 Let $V(\Gamma_{D_6}) = \{a, a^2, a^4, a^5, b, ab, a^2b, a^3b, a^4b, a^5b\}$. Then, there are no codes that are perfect for non-commuting graph of dihedral group of order 12, D_6.

Theorem 4.5 Let $V(\Gamma_{D_7}) = \{a, a^2, a^3, a^4, a^5, a^6, b, ab, a^2b, a^3b, a^4b, a^5b, a^6b\}$. Then, the perfect code of non-commutating graph of dihedral group of order 14, D_7 are $\{b\}, \{ab\}, \{a^2b\}, \{a^3b\}, \{a^4b\}, \{a^5b\}$ and $\{a^6b\}$.

Theorem 4.6 Let $V(\Gamma_{D_8}) = \{a, a^2, a^3, a^5, a^6, a^7, b, ab, a^2b, a^3b, a^4b, a^5b, a^6b, a^7b\}$. Then, there are no perfect codes for non-commuting graph of dihedral group of order 16, D_8.

Theorem 4.7 Let $V(\Gamma_{D_9}) = \{a, a^2, a^3, a^4, a^5, a^6, a^7, a^8, b, ab, a^2b, a^3b, a^4b, a^5b, a^6b, a^7b, a^8b\}$. Then, a perfect code of non-commutating graph of dihedral group of order 18, D_9 are $\{b\}, \{ab\}, \{a^2b\}, \{a^3b\}, \{a^4b\}, \{a^5b\}, \{a^6b\}, \{a^7b\}$ and $\{a^8b\}$.

5 Conclusion

In a nutshell, the results can be concluded based on the odd and even number of n in dihedral groups. The pattern of the perfect codes of dihedral groups for n odd is n where $n = 3, 5$ and 9 and can be generalize that all the perfect code of the dihedral group for n is odd are the n reflections in D_n such that $b, ab, a^2b, \ldots, a^{n-1}b$. Meanwhile, there are no perfect codes of dihedral group for n even that is $n = 4$,6 and 8. This happened because of the size of codes. If the size of codes is one (1) that is $C = \{x\}, \forall x \in V(\Gamma_{D_n})$ for $n = 4$, 6 and 8, then the intersection of neighbourhood of codes is not equal to the vertex set, $V(\Gamma_{D_n})$. Next, if the size of codes is two (2) such that $C = \{x, y\}, \forall x, y \in V(\Gamma_{D_n})$ for $n = 4$, 6 and 8, then the intersection of neighbourhood of codes is not equal to empty set, $\{\emptyset\}$. Similarly, if the size of codes is greater than two (2), the intersection of neighbourhood of codes is not equal to empty set, $\{\emptyset\}$. These three cases do not satisfy both conditions of perfect code. Hence, there are no perfect codes of non-commuting graph of dihedral groups for n even.

6 References

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