Proceedings of Science and Mathematics
Faculty of Science, Universiti Teknologi Malaysia
Vol. 5, 2021, page 39-47

Perfect Codes of Conjugacy Class Graphs of Dihedral Groups of Order at Most 18

[1]Nur Hazirah Azman and [2]Ibrahim Gambo

[1,2]Department of Mathematical Sciences
Faculty of Science, Universiti Teknologi Malaysia,
81310 Johor Bahru, Johor, Malaysia.

e-mail: [1]nur.hazirah.azman@graduate.utm.my, [2]igambo@utm.my

Abstract This research aimed in computing the perfect codes of conjugacy class graph of dihedral group of order at most 18. The conjugacy classes of dihedral groups were determined, and the conjugacy class graphs were then constructed. The features of the perfect codes obtained were studied in order to differentiate the pattern through the order of the dihedral groups. The results obtained were then divided into 3 cases where the number of n of the dihedral groups, D_n congruent to $1 \bmod 2, 2 \bmod 4$ and $0 \bmod 4$ where n is between 3 until 9.

Keywords Conjugacy classes; Conjugacy class graph; Perfect codes; Graph theory; Coding theory.

1 Introduction

This research involves three main areas which are the coding theory, the group theory as well as the graph theory in fulfilling the aims of this research. The group theory part concerned on the dihedral group, D_n where $3 \leq n \leq 9$ with order 6 until 18. For $n \geq 3$, the dihedral group is defined as the rigid motions taking a regular n-gon back to itself, with the operation being composition [1]. According to Conrad, rigid motions here is defined as a distance-preserving transformation such as reflections and rotations [1]. In this part, the elementary aspects of the dihedral groups such as its elements, relations between reflections and rotations, its center as well as its conjugacy classes were explored. In previous findings, Sehrawat and Pruthi [2] had found codes over the dihedral groups. Similarly, in this research, this group was chosen in order to differentiate the pattern of the perfect codes for different values of n between the elements in the dihedral group. The conjugacy class of each element in the dihedral group of order 6 until 18 were determined to picture the elements' conjugacy class graph later in the second part involving the graph theory.

Next, the graph theory concerned about the conjugacy class graph. The conjugacy class were extended from the group theory which is obtain from the theorems and definitions of the conjugacy class [3]. The study on a graph related to the conjugacy classes of groups was introduced by Bertram *et al.* [4] in 1990 where the term was denoted as Γ_G^{cl}. According to findings by Bertram *et al.* [4], if the greatest common divisor of the sizes between any two conjugacy classes is greater than one, then the vertices of this graph in which two distinct vertices are connected by an edge are non-central conjugacy classes of a group. In addition, the study on the application of a graph related to the conjugacy classes had been carried out by Bianchi *et al*. [5] while in 2005, You *et al*. [6] discovered a new graph related to the conjugacy classes of finite group which is the conjugacy class graph. Erfanian [7], later conducted a study on the triangle-free commuting conjugacy class graph while Sarmin *et al.* [8] studied on the generalised conjugacy class graph of some dihedral groups in order to show the relation between orbits and their cardinalities which results in finding some properties of conjugacy class graph. In this research, the properties of the conjugacy class graph which were obtained based on the elements in the dihedral groups were then compared between one another in order to prove some theorems and lemmas.

Lastly, the study on perfect codes in graphs had also been carried out by many researchers such as Biggs [9] who developed a general theory that leads to a simple criterion for the existence of a perfect code in a distance-transitive graph. Furthermore, Mollard [10] studied on the perfect codes in Cartesian product of graphs. The findings discovered that the partition of the perfect codes is easily obtained in the Cayley graphs which leads in exploration of the example of applications as well as its generalizations. Meanwhile, Feng *et al.* [11] had done a research on the perfect codes in circulant graphs. Their research obtained a necessary and sufficient condition for a circulant graph of order *n* and degree $p^l - 1$ to have a perfect code, where *p* is a prime and p^l is the largest power of *p* dividing *n* [11]. Similarly, in this research, the properties and concepts of the perfect codes were explained to find the possible codes as well as the perfect codes of the conjugacy class graph of some dihedral groups of order at most 18.

2 Preliminaries

This section provides some concepts and previous research results that are used in this study in what follows.

2.1 Dihedral Group

Dihedral group can be defined as stated in Definition 2.1 while the centre of a group is defined as stated in Definition 2.2.

Definition 2.1 [1] Dihedral group, D_n is the symmetry group of an *n*-sided regular polygon where $n > 1$.

Definition 2.2 [8] The center $Z(G)$ of a group G is the set of elements in G that commute with every element of G. In symbols, $Z(G) = \{a \in G | ax = xa \text{ for all } x \in G\}$.

The theorems listed below explained on the properties of the dihedral group.

Theorem 2.1 [1] D_n has an order of $2n$.

Theorem 2.2 [1] The *n* rotations in D_n are $\{e, a, a^2, \dots, a^{n-1}\}$.

Theorem 2.3 [1] The *n* reflections in D_n are $\{b, ab, a^2b, \dots, a^{n-1}b\}$.

Theorem 2.4 [1] The group D_n has $2n$ elements where $D_n = \{e, a, a^2, .., a^{n-1}, b, ab, \dots, a^{n-1}b\}$. All elements in D_n with order greater than 2 is a power of a.

Theorem 2.5 [1] When $n \geq 3$ is odd, the center of D_n is trivial. However, when $n \geq 3$ is even, the center of D_n is $\left\{e, a^{\frac{n}{2}}\right\}$.

According to Conrad, for a^j where j is the power of a, to commute with b, $a^j b = ba^j$ which is equivalent to $a^j b = a^{-j}b$, which then implies that $a^{2j} = e$ [1]. Since a has order n, $a^{2j} = e$ only if $n|2j$. For the odd n, this implies $n|j$, which gives j is a multiple of n and thus, $a^j = e$. Therefore, for odd n, the only rotation that could be in the center of D_n is e. Thus, the center of D_n is $\{e\}$ [1].

For the n even case, the condition where $n|2j$ is equivalent to $\frac{n}{2}|j$. For $0 \leq j \leq n-1$, the only possible choices for j are $j = 0$ and $j = \frac{n}{2}$. Then, $a^j = a^0 = e$ or $a^j = a^{\frac{n}{2}}$. Certainly e is in the center. As $a^{\frac{j}{2}}$ commutes with every rotation and reflection in D_n, then $a^{\frac{j}{2}}$ is also the center of D_n [1].

2.2 Conjugacy Class Graph

Next, the conjugate between two elements in a group G as well as the conjugacy class are defined as in Definition 2.3 and Definition 2.4, respectively.

Definition 2.3 [8] Let a and b be two elements in finite group G, then a and b are called conjugate if there exists an element g in G such that $gag^{-1} = b$.

Definition 2.4 [12] Let a and b be two elements in finite group G, then a and b are conjugate in G (and call b a conjugate of a) if $x^{-1}ax = b$ for some $x \in G$. The conjugacy class of a is the set $cl(a) = \{x^{-1}ax | x \in G\}$.

Then, Theorem 2.6 shows the concept of conjugacy classes in the dihedral group.

Theorem 2.6 [13] The conjugacy classes of D_n are as follows:

If n is odd,

(i) the identity element: $\{e\}$,

(ii) $(\frac{n-1}{2})$ conjugacy classes of size 2: $\{a^1\}, \{a^2\}, \dots, \{a^{(\frac{n-1}{2})}\}$,

(iii) all the reflections: $\{a^i b: 0 \leq i \leq n-1\}$.

If n is even,

(i) Two conjugacy classes of size 1: $\{e\}, \{a^{\frac{n}{2}}\}$,

(ii) $(\frac{n-1}{2})$ conjugacy classes of size 2: $\{a^1\}, \{a^2\}, \dots, \{a^{(\frac{n}{2}-1)}\}$,

(iii) the reflections fall into two conjugacy classes: $\{a^{2i}b: 0 \leq i \leq \frac{n}{2} - 1\}$ and $\{a^{2i+1}b: 0 \leq i \leq \frac{n}{2} - 1\}$.

The conjugacy class graph is defined as stated by Bertram *et al.* [8] in Definition 2.5.

Definition 2.5 [4] Suppose G is a finite group with $Z(G)$ as the center of G. The vertices of the conjugacy class graph of G, $V(\Gamma)$ where non-central conjugacy classes of G denoted as $V(\Gamma G)$, for which $V(\Gamma G) = K(G) - |Z(G)|$, where $K(G)$ is the number of conjugacy classes in G. Two vertices are adjacent if their cardinalities are not coprime, that is, the greatest common divisor of the number of vertices is not equal to one.

2.3 Perfect Codes

Codes and neighbourhood are defined as in Definition 2.6 and Definition 2.7, respectively.

Definition 2.6 [14] Let Γ be a graph with vertex set $V(\Gamma)$ and edge set $E(\Gamma)$, every subset of $V(\Gamma)$ can be considered as a code. Thus, $C \subseteq V(\Gamma)$, then C is called a code.

Definition 2.7 [14] Let x be an element in F_q^n, q-nary code of length n where F_q is a field of size q and $r \geq 0$. Then, the neighbourhood of x with radius r is denoted as $S_r(x)$ and defined as the following :

$$S_r(x) = \{Y \in F_q^n \mid d(x, y) \leq r\}.$$

Next, the perfect code is defined as in Definition 2.8.

Definition 2.8 [15] Let $C \subseteq F_q^n$ be a q-nary code of length n where F_q is a field of size q. Then C is called a perfect code if it has the following conditions:

i. $S_1(x) \cap S_1(y) = \phi$ where $x, y \in C, x \neq y$.
ii. $F_q^n = \bigcup_{i=1}^{n} S_1(x_i)$ where $x \in C$.

In addition, Theorem 2.7 explained the properties of perfect codes when it evolves in graphs.

Theorem 2.7 [11] Let Γ be a graph and $C \subseteq V(\Gamma)$ is a code. Then C is a perfect code if and only if it satisfy the following two conditions:
i. C is an independent set,
ii. Every vertex in $V(\Gamma)$ is either in C or it is adjacent to exactly one element in C.

3 Methodology

In this research, the elements of dihedral group were first determined while the conjugacy class of each of the elements was obtained by using the definition of conjugacy class. Then, the graph of the conjugacy class was constructed based on the conjugacy classes obtained earlier. Next, as the codes for each of the graph were computed, the perfect codes were determined based on the definition stated. The features of the perfect codes of the conjugacy class graph for the dihedral groups were also considered in this research.

4 Results and Discussion

The results obtained from this research are presented in the sections that follows.

4.1 The Conjugacy Classes of Dihedral Groups of Order at Most 18

In this section, the conjugacy classes of each dihedral groups were determined based on Definition 2.4 where the number of conjugacy class was denoted as $K(G)$.

Proposition 4.1 Let G be a dihedral group of order 6, namely D_3 such that $G = \{e, a, a^2, b, ab, a^2b\}$. Then, the number of conjugacy classes of D_3, $K(D_3) = 3$.

Proof Given the elements of group D_3 were listed as $G = \{e, a, a^2, b, ab, a^2b\}$.

First, let $x = e$, then $cl(e) = \{e\}$.
Thus, it has order of 1.

Next, let $x = a$, where $cl(a) = g(a)g^{-1}, g \in D_3$.
If $g = e, g^{-1} = e$ then $(e)a(e) = a$.
If $g = a, g^{-1} = a^2$ then $(a)a(a^2) = (a^2)(a^2) = a$.
If $g = a^2, g^{-1} = a$ then $(a^2)a(a) = (a^2)(a^2) = a$.
If $g = b, g^{-1} = b$ then $(b)a(b) = a^2$.
If $g = ab, g^{-1} = ab$ then $(ab)a(ab) = (ab)(a^2b) = a^2$.
If $g = a^2b, g^{-1} = a^2b$ then $(a^2b)a(a^2b) = (a^2b)(b) = a^2$.
Thus, $cl(a) = \{a, a^2\}$ which means $cl(a) = cl(a^2)$.
Since both elements $cl(a)$ and $cl(a^2)$ were in the same conjugacy class, then they have the same order which is 2.

Next, let $x = b$, where $cl(b) = g(b)g^{-1}, g \in D_3$.
If $g = e, g^{-1} = e$ then $(e)b(e) = b$.
If $g = a, g^{-1} = a^2$ then $(a)b(a^2) = (ab)(a^2) = a^2b$.
If $g = a^2, g^{-1} = a$ then $(a^2)b(a) = (a^2b)(a) = ab$.
If $g = b, g^{-1} = b$ then $(b)b(b) = b$.
If $g = ab, g^{-1} = ab$ then $(ab)b(ab) = (a)(ab) = a^2b$.
If $g = a^2b, g^{-1} = a^2b$ then $(a^2b)b(a^2b) = (a^2)(a^2b) = ab$.
Thus, $cl(b) = \{b, ab, a^2b\}$ which means $cl(b) = cl(ab) = cl(a^2b)$.
Since the elements $cl(b)$, $cl(ab)$ and $cl(a^2b)$ were in the same conjugacy class, then they have the same order which is 3.

It follows that the conjugacy class of D_3 were listed as follows:
i. $cl(e) = \{e\}$.
ii. $cl(a) = \{a, a^2\} = cl(a^2)$.
iii $cl(b) = \{b, ab, a^2b\} = cl(ab) = cl(a^2b)$.

Thus, $K(D_3) = 3$.

Proposition 4.2 Let G be a dihedral group of order 8, namely D_4 such that $G = \{e, a, a^2, a^3, b, ab, a^2b, a^3b\}$. Then, the number of conjugacy classes of D_4, $K(D_4) = 5$.

Proof Similar to the proof of Proposition 4.1.

Proposition 4.3 Let G be a dihedral group of order 10, namely D_5 , $G = \{e, a, a^2, a^3, a^4, b, ab, a^2b, a^3b, a^4b\}$. Then, the number of conjugacy class of D_5, $K(D_5) = 4$.

Proof Similar to the proof of Proposition 4.1.

Proposition 4.4 Let G be a dihedral group of order 12, namely D_6 , $G = \{e, a, a^2, a^3, a^4, a^5, b, ab, a^2b, a^3b, a^4b, a^5b\}$. Then, the number of conjugacy class of D_6 , $K(D_6) = 6$.

Proof Similar to the proof of Proposition 4.1.

Proposition 4.5 Let G be a dihedral group of order 14, namely D_7 , $G = \{e, a, a^2, a^3, a^4, a^5, a^6, b, ab, a^2b, a^3b, a^4b, a^5b, a^6b\}$. Then, the number of conjugacy class of D_7, $K(D_7) = 5$.

Proof Similar to the proof of Proposition 4.1.

Proposition 4.6 Let G be a dihedral group of order 16, namely D_8 , $G = \{e, a, a^2, a^3, a^4, a^5, a^6, a^7, b, ab, a^2b, a^3b, a^4b, a^5b, a^6b, a^7b\}$. Then, the number of conjugacy class of D_8, $K(D_8) = 7$.

Proof Similar to the proof of Proposition 4.1.

Proposition 4.7 Let G be a dihedral group of order 18, namely D_9 , $G = \{e, a, a^2, a^3, a^4, a^5, a^6, a^7, a^8, b, ab, a^2b, a^3b, a^4b, a^5b, a^6b, a^7b, a^8b\}$. Then, the number of conjugacy class of D_9, $K(D_9) = 6$.

Proof Similar to the proof of Proposition 4.1.

4.2 The Conjugacy Class Graph of Dihedral Groups of Order at Most 18

In this section, the conjugacy class graphs of each dihedral groups were determined based on Definition 2.5 where the conjugacy class graph was denoted as Γ_{D_n} .

Lemma 4.1 Let G be a dihedral group of order 6,D_3. Then, the conjugacy class graph of D_3, Γ_{D_3} is an empty graph.

Proof Based on Proposition 4.1, the number of conjugacy classes is three which are $cl(e), cl(a)$ and $cl(b)$. The non-central conjugacy classes are $cl(a)$ and $cl(b)$ with order 2 and 3 respectively. Since the greatest common divisor between the order of $cl(a)$ and $cl(b)$is 1, thus the vertices among these classes is not connected which results to an empty graph.

Lemma 4.2 Let G be a dihedral group of order 8,D_4. Then, the conjugacy class graph of D_4, Γ_{D_4} is K_3.

Proof Follows from Proposition 4.2 and similar to the proof of Lemma 4.1.

Lemma 4.3 Let G be a dihedral group of order 10,D_5. Then, the conjugacy class graph of D_5, Γ_{D_5} is $K_2 \cup K_1$.

Proof Follows from Proposition 4.3 and similar to the proof of Lemma 4.1.

Lemma 4.4 Let G be a dihedral group of order 12,D_6. Then, the conjugacy class graph of D_6, Γ_{D_6} is $K_2 \cup K_2$.

Proof Follows from Proposition 4.4 and similar to the proof of Lemma 4.1.

Lemma 4.5 Let G be a dihedral group of order 14,D_7. Then, the conjugacy class graph of D_7, Γ_{D_7} is $K_3 \cup K_1$.

Proof Follows from Proposition 4.5 and similar to the proof of Lemma 4.1.

Lemma 4.6 Let G be a dihedral group of order 16,D_8. Then, the conjugacy class graph of D_8, Γ_{D_8} is K_5.

Proof Follows from Proposition 4.6 and similar to the proof of Lemma 4.1..

Lemma 4.7 Let G be a dihedral group of order 18,D_9. Then, the conjugacy class graph of D_9, Γ_{D_9} is is $K_4 \cup K_1$.

Proof Follows from Proposition 4.7 and similar to the proof of Lemma 4.1.

4.3 The Perfect Codes of Conjugacy Classes of Dihedral Groups of Order at Most 18

In this section, the perfect codes of conjugacy class graphs of dihedral groups of order 6 until 18 were determined based on Definition 2.8. The results obtained are shown as the following theorems.

Theorem 4.1 Let $V(\Gamma_{D_3}) = \{a, b\}$. Then, the perfect codes of conjugacy class graph of order 6, D_3 is $\{a, b\}$.

Proof Based on Lemma 4.1, the vertex set of D_3, $V(\Gamma_{D_3}) = \{a, b\}$. Based on Definition 2.7, the neighbourhood of $V(\Gamma_{D_3})$ are :

$S_1(a) = \{a\}$.
$S_1(b) = \{b\}$.

Firstly, let $C = \{a\}$ and $\{b\}$ be a code.
Based on Definition 2.8, the code C did not satisfy the second condition of perfect code such that:
i. $S_1(a) \cup S_1(\phi) = \{a\} \cup \{\phi\} = \{a\} \neq \Gamma_{D_3}$.
ii. $S_1(b) \cup S_1(\phi) = \{b\} \cup \{\phi\} = \{b\} \neq \Gamma_{D_3}$.
Hence, $\{a\}$ and $\{b\}$ are not perfect codes.

Next, let $C = \{a, b\}$ be a code. Based on Definition 2.8, the code, C satisfies the two conditions of perfect codes such that:

i. $S_1(a) \cap S_1(b) = \{a\} \cap \{b\} = \phi$.

ii. $S_1(a) \cup S_1(b) = \{a\} \cup \{b\} = \Gamma_{D_3}$.

Hence, $\{a, b\}$ is a perfect code.

Thus, the perfect codes of Γ_{D_3} is $\{a, b\}$.

Theorem 4.2 Let $V(\Gamma_{D_4}) = \{a, b, c\}$. Then, the perfect codes of conjugacy class graph of order 8, D_4 are $\{a\}, \{b\}$ and $\{c\}$.

Proof Follows from Lemma 4.2 and similar to the proof of Theorem 4.1.

Theorem 4.3 Let $V(\Gamma_{D_5}) = \{a, b, c\}$. Then, the perfect codes of conjugacy class graph of order 10, D_5 are $\{a, c\}$ and $\{b, c\}$.

Proof Follows from Lemma 4.3 and similar to the proof of Theorem 4.1.

Theorem 4.4 Let $V(\Gamma_{D_6}) = \{a, b, c, d\}$. Then, the perfect codes of conjugacy class graph of order 12, D_6 are $\{a, c\}, \{b, c\}, \{a, d\}$ and $\{b, d\}$.

Proof Follows from Lemma 4.4 and similar to the proof of Theorem 4.1.

Theorem 4.5 Let $V(\Gamma_{D_7}) = \{a, b, c, d\}$. Then, the perfect codes of conjugacy class graph of order 14, D_7 are $\{a, d\}, \{b, d\}$ and $\{c, d\}$.

Proof Follows from Lemma 4.5 and similar to the proof of Theorem 4.1.

Theorem 4.6 Let $V(\Gamma_{D_8}) = \{a, b, c, d, e\}$. Then, the perfect codes of conjugacy class graph of order 16, D_8 are $\{a\}, \{b\}, \{c\}, \{d\}$ and $\{e\}$.

Proof Follows from Lemma 4.6 and similar to the proof of Theorem 4.1.

Theorem 4.7 Let $V(\Gamma_{D_9}) = \{a, b, c, d, e\}$. Then, the perfect codes of conjugacy class graph of order 18, D_9 are $\{a, e\}, \{b, e\}, \{c, e\}$ and $\{d, e\}$.

Proof Follows from Lemma 4.7 and similar to the proof of Theorem 4.1.

5 Conclusion

As a conclusion, the result could be summarized into the odd number of n in the dihedral group, D_n and for even number of n in the dihedral group, D_n such that $n \equiv 0 \bmod 4$ and $n \equiv 2 \bmod 4$. First, for the odd number of n in the dihedral group, D_n, it is observed that the graphs is a graph which consists of rotation elements with an isolated vertex of one reflection element and could be generalized as $\Gamma(D_n) = K_{(\frac{n-1}{2})} \cup K_1$ while the perfect codes would be a code of size 2 which consists of a pair of rotation and reflection elements of dihedral group in its conjugacy class graphs. Next, for the dihedral group, D_n of $n \equiv 0 \bmod 4$, the conjugacy class graphs for this case can be generalised as $\Gamma(D_n) = K_{(\frac{n}{2}+1)}$ while the perfect codes are the size 1 code of each element of the conjugacy class graph of dihedral groups. Lastly, for the dihedral group of $n \equiv 2 \bmod 4$, the general notation of the conjugacy class graph and the perfect codes could not be summarised as only one case were analysed in this research such that the dihedral group of order 6 with the perfect code is a code of size 2 which consists of a pair of rotation and reflection elements in its conjugacy class graph.

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