Proceedings of Science and Mathematics
Faculty of Science, Universiti Teknologi Malaysia
Vol. 5, 2021, page 48-56

Hybrid Nanofluid Flow and Heat Transfer Past a Permeable Shrinking Sheet with Convective Boundary Condition

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Abstract: The hybrid nanofluid flow and heat transfer past a permeable shrinking surface with convective boundary condition is investigated. Water is chosen as base fluid and the nanoparticles involved are copper and aluminium oxide. The governing equations of the problem are reduced to similarity equations using similarity transformation and then they were solved numerically using MATLAB bvp4c solver. Numerical results were displayed graphically to illustrate the influence of the volume fraction of nanoparticles and shrinking parameter on the fluid velocity and temperature profiles. It is found that the velocity and temperature profiles increase as the values of volume fraction of copper increases. For the increasing values of shrinking parameter, the fluid velocity shows an increment while the opposite trend is observed for temperature profiles.

Keywords: Hybrid nanofluid; boundary layer flow; shrinking surface; convective boundary condition

1 Introduction

Analytical and experimental studies on nanofluid flow have been conducted over the last several decades in order to fully understand its thermophysical and heat transfer properties. Most engineering machinery uses heat transfer fluids such as water, ethylene glycol, and oil. However, the thermal conductivity of these base fluids is low, limiting the heat transfer efficiency. The majority of studies in this area have created to improve the convective thermal performance and discover the potential to improve convective heat transfer through nanotechnology using a mixture of nanosized particles and a conventional heat transfer fluid. Thus, nanofluids, the recent form of high-potential heat transfer fluids, are introduced and implemented in the industrial sector. Choi and Eastman [1] were the one that introduced nanofluids in which the nanoparticles dispersed in base fluids to enhance the thermal conductivity. However, Babu *et al.* [2] learned a new type of nanofluid in heat transfer applications which is the mixture of two different nanoparticles into the base fluid. It is known as hybrid nanofluid. Nabil *et al.* [3] predicted hybrid nanofluid will have higher thermal conductivity and gives better thermophysical properties than nanofluids.

Several researchers investigated the boundary layer flow and heat transfer of hybrid nanofluids. Elsaid and Abdel Wahed [4] investigated the impact of hybrid nanofluids coolant (H_2O–Cu/Al_2O_3) on the mechanical properties of a moving cylinder during the heat treatment process. It is found that the use of hybrid nanoparticles within the base fluid (water) raises the temperature of the boundary layer while decreasing its velocity. Akbar *et al*. [5] investigated the hybrid nanofluid preparation and thermophysical properties measurement. It is found that the hybrid nanofluid improve its thermophysical properties and Nusselt number compared to pure water. Jasim *et al*. [6] studied mixed convection flow of hybrid nanofluid through a vented enclosure with an inner rotating cylinder and found that the thermal transmission is enhanced for hybrid nanofluid more than the pure water. Hanif *et al.* [7] studied the hybrid model of radiative Cu–Fe_3O_4/water nanofluid over a cone with prescribed heat flux (PHF) and prescribed wall temperature (PWT). The temperature of the fluid increases for both nanofluid and hybrid nanofluid as the estimates of magnetic and radiation parameters become increasingly accurate. The PHF case exhibits higher heat transfer rates than the PWT cases. It also found that the fluid's velocity increases as a result of the permeability effects.

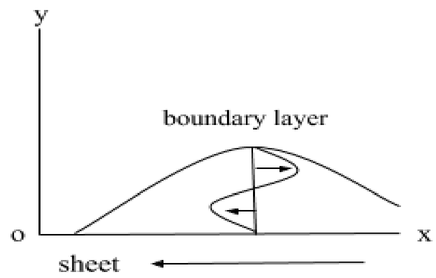
Fluid flow past a stretching sheet is important in engineering and industrial operations such as wire drawing, hot rolling, extrusion, and metal spinning. Crane [8] was the first to investigate steady flow over a stretching sheet and get at the similarity solution. Bachok *et al.* [9] conducted a study of the flow and heat transfer characteristics of a nanofluid over a stretching/shrinking sheet. It is found that the shrinking sheet is different from stretching sheet. The solutions for a shrinking sheet are discovered to be non-unique. The addition of nanoparticles into the base water fluid has resulted in an increase in skin friction and heat transfer coefficients, which increases noticeably as the nanoparticle volume fraction increases. Uddin and Bhattacharyya [10] studied the heat transfer in boundary layer stagnation-point flow towards a permeable shrinking sheet with variable sheet temperature and suction/injection. Mishra *et al.* [11] investigated the free convective micropolar fluid over a shrinking sheet in presence of heat source/sink. The flow created by a shrinking sheet exhibits physical phenomena which are different from the flow created by forward stretching.

In heat transfer analysis, temperature conditions such as constant surface temperature and heat flux are widely applied. In some cases, the transfer of heat to the surface is dependent on the temperature of the surface, as is common in heat exchangers. As a result, the convective boundary condition must be considered. Makinde and Aziz [12] investigated the influence of convective boundary condition on hydromagnetic mixed convection with heat and mass transfer past a vertical plate embedded in a porous medium. It is revealed that both the fluid velocity and temperature increase with an increase in the convective heat transfer parameter, Biot Number (Bi).

With motivation from previous studies, the behavior of hybrid nanofluid flow and heat transfer across a permeable shrinking surface due to convective boundary condition is investigated. In this research, Tiwari and Das [13] nanofluid model i used. The chosen nanoparticles which are Copper (Cu) and Alumina (Al_2O_3) then suspended in water to form a hybrid nanofluid.

2 Mathematical Formulations

A constant or steady flow and heat transfer is exhibited on a permeable shrinking surface of a hybrid nanofluid. In Figure 3.1 below, the *x*-axis is measured at the plate and the *y*-axis at the end of the plate, where there is a plate at $y = 0$.



$$u = \lambda u_w(x)$$

Figure 1: The geometry of the problem for shrinking sheet $(\lambda < 0)$

Assume $u_w = ax$ is a velocity to stretch or shrink the surface, where *a* is a constant and the wall mass flux velocity is v_0. In addition, the bottom surface of the sheet is heated through a convection of a hot fluid with a uniform temperature, T_f, which provides a heat transfer coefficient, h_f. The surface temperature T_w benefits from the heating phase, defined by hot fluid, and T_∞ is the ambient fluid temperature. As proposed by Waini *et al.* [14], the hybrid nanofluid nanoparticles size is assumed to be uniform and the nanoparticles agglomeration effects on the thermophysical properties is ignored. The governing equations for hybrid nanofluid flow over a permeable stretching and shrinking surface with convective boundary condition can be written as follows (Devi and Devi [15] and Tiwari and Das [13]):

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

Momentum equation:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu_{hnf}}{\rho_{hnf}}\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

Energy equation:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k_{nf}}{(\rho c_p)_{hnf}}\frac{\partial^2 T}{\partial y^2}. \tag{3}$$

The appropriate boundary conditions according to Waini *et al*. [14] are,

$$\begin{gathered} v(x,y) = v_0, \quad u(x,y) = \lambda u_w(x), \quad -k_{hnf}\left(\frac{\partial T}{\partial y}\right) = h_f(T_f - T) \quad \text{at } y = 0, \\ u(x,y) = 0, \quad T(x,y) = T_\infty \quad \text{as } y \to \infty, \end{gathered} \tag{4}$$

where (x, y) denotes the Cartesian coordinates along the sheet and normal to it, *u* and *v* are the velocity components of the hybrid nanofluid in the *x*- and *y*-directions, respectively, T is the temperature of the hybrid nanofluid, C_p is the specific heat at constant pressure, ρ_{hnf} is the effective density, μ_{hnf} is the effective dynamic viscosity, $(\rho C_p)_{hnf}$ is the heat capacitance, k_{hnf} is the thermal conductivity. Here, the subscript *hnf* represents the hybrid nanofluid. λ represents the stretching or shrinking sheet parameter with the corresponding stretching sheet, $\lambda > 0$ shrinking sheet, $\lambda < 0$ and static sheet, $\lambda = 0$ and heat transfer coefficient is denoted as h_f .

Following Devi and Devi [15], we are looking for a similarity solution of equations (1) - (4) by using following similarity variables:

$$\psi = x\sqrt{av_f}\, f(\eta), \quad \theta(n) = \frac{T - T_\infty}{T_f - T_\infty}, \quad \eta = y\sqrt{\frac{a}{v_f}} \tag{5}$$

where the stream function denoted by ψ with $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$ so that the continuity equation in (1) is satisfied identically.

Following Waini *et al*. [14], the velocities are expressed as:

$$u = ax\, f'(\eta) \text{ and } v = -\sqrt{av_f} \cdot f(\eta) \tag{6}$$

and that the wall mass transfer velocity becomes,

$$v_0 = -\sqrt{av_f}\, S, \tag{7}$$

where $S = f(0)$ is the mass flux parameter with $S > 0$ represents fluid suction while $S < 0$ represents fluid injection and v_f represents the base fluid kinematic viscosity.

Table 1: Expression of thermophysical characteristic of hybrid nanofluid

Properties	**Hybrid nanofluid**
Heat capacity	$(\rho C_p)_{hnf} = (1-\varphi_2)[(1-\varphi_1)(\rho C_p)_f + \varphi_1(\rho C_p)_{n1}] + \varphi_2(\rho Cp)_{n2}$
Density	$\rho_{hnf} = (1-\varphi_2)[(1-\varphi_1)\rho_f + \varphi_1\rho_{n1}] + \varphi_2\rho_{n2}$
Dynamic viscosity	$\mu_{hnf} = \frac{\mu_f}{(1-\varphi_1)^{2.5}(1-\varphi_2)^{2.5}}$
Thermal conductivity	$k_{hnf} = \frac{k_{n2} + 2k_{nf} - 2\varphi_2(k_{nf} - k_{n2})}{k_{n2} + 2k_{nf} - \varphi_2(k_{knf} - k_{n2})} \times (k_{nf})$ where, $k_{nf} = \frac{k_{n1} + 2k_f - 2\varphi_1(k_f - k_{n1})}{k_{n1} + 2k_f - \varphi_1(k_f - k_{n1})} \times (k_f)$

Following Devi and Devi [15] and Oztop and Abu-Nada [16], the expression of the effective thermophysical properties of nanofluid and hybrid nanofluid are given in Table 1. Noted that the subscripts *f*, *nf*, *n1* and *n2* represent the fluid, nanofluid, the nanoparticles of Al_2O_3 and Cu, respectively. The thermophysical properties of base fluid and nanoparticles of Al_2O_3 and Cu are provided in Table 2 as in Oztop and Abu-Nada [16].

Table 2: Nanoparticles and fluid thermophysical properties

Properties	**Al_2O_3 (n1)**	**Cu (n2)**	**Water**
k (W/mK)	40	400	0.613
C_p *(J/kgK)*	765	385	4179
ρ	3970	8933	997.1

Substituting (5) into equations (2) and (3), the following ordinary differential equations are obtained:

$$\mathrm{A}f''' + ff'' - f'^2 = 0\,, \tag{8}$$

$$\frac{1}{\mathrm{Pr}}\cdot \mathrm{B}\cdot\theta'' + f\theta' = 0. \tag{9}$$

Let A and B be the constants, where $\mathrm{A} = \left(\dfrac{\mu_{hnf}/\mu_f}{\rho_{hnf}/\rho_f}\right)$ and $\mathrm{B} = \dfrac{k_{hnf}/k_f}{(\rho C_p)_{hnf}/(\rho C_p)_f}$.

Equations (8) and (9) are subjected to:

$$\begin{gathered} f(0) = S,\quad f'(0) = \lambda,\quad \theta'(0) = -\frac{k_f}{k_{hnf}}\mathrm{Bi}\left(1-\theta(0)\right) \\ f'(\eta)\to 0,\quad \theta(\eta)\to 0 \ \text{ as } \ \eta\to\infty \end{gathered} \tag{10}$$

where prime denotes differentiation with respect to η, $\mathrm{Pr} = v_f/a_f$ represents the Prandtl number, $\mathrm{Bi} = (h_f/k_f)\cdot\sqrt{v_f/a}$ represents the Biot number and $\lambda < 0$ represents a shrinking. Note that, $\mathrm{Bi}\to\infty$ refers to the constant wall temperature condition, $\theta(0) = 1$.

The parameters of physical interest are the skin friction coefficient, C_f and the local Nusselt number, Nu_x, which are defined as:

$$C_f = \frac{\tau_w}{\rho_f {u_w}^2},\quad Nu_x = \frac{xq_w}{k_f\left(T_f - T_\infty\right)} \tag{11}$$

where τ_w is the surface shear stress and q_w is the heat flux from the stretching or shrinking surface, which are defined as:

$$\tau_w = \mu_{hnf}\left(\frac{\partial u}{\partial y}\right)_{y=0}, \; q_w = -k_{hnf}\left(\frac{\partial T}{\partial y}\right)_{y=0}. \tag{12}$$

Using Equations (3.6), (3.11) and (3.12) one gets:

$$\mathrm{Re}_x^{1/2} C_f = \frac{\mu_{hnf}}{\mu_f} f''(0), \;\; \mathrm{Re}_x^{-1/2} Nu_x = -\frac{k_{hnf}}{k_f}\theta'(0) \tag{13}$$

where the local Reynolds number is $\mathrm{Re}_x = \dfrac{u_w(x)x}{v_f}$.

3 Results and Discussions

The boundary value problem solver which is available in MATLAB software called 'bvp4c' is employed to solve the equations (8) – (10) numerically. The bvp4c solver employs the finite difference scheme and the 3-stage Labatto IIIa formula, where the initial guess and changes step size is supplied to obtain the required accuracy of the solution. Also, the appropriate thickness of the boundary layer η_∞ must be selected relying on the parameters used to obtain the accurate solutions. The detail of this particular solver is clearly discussed in Shampine *et al.* [17]. To conduct this study, a solid volume fraction of alumina, Al_2O_3 is fixed at 0.1 (i.e $\varphi_1 = 0.1$) and it is added to the base fluid as suggested by Devi and Devi [15]. Consequently, copper nanoparticles, Cu are added into the mixture to form Cu-Al_2O_3/water hybrid nanofluid. To investigate velocity and temperature profiles, the values of volume fraction of Cu (φ_2) and shrinking sheet $(\lambda < 0)$ is evaluated at different values. As proposed by Oztop and Abu-Nada [16], the Prandtl number, Pr = 6.2 which represents water as the base fluid will be fixed for computed numerical results.

Table 3 present results for the heat transfer coefficient, $-\theta'(0)$ with various values of Prandtl number, Pr are compared with those in Waini *et al.* [14], Devi and Devi [15] and Wang [18] by setting the parameter $\varphi_1 = \varphi_2 = 0, \; \lambda = 1, \; S = 0, \; \mathrm{Bi} \to \infty$. The comparison is found to be in an excellent agreement.

Table 3: Comparison of $-\theta'(0)$ for different values of Prandtl number when $\lambda = 1, \; S = 0, \; \mathrm{Bi} \to \infty$.

Pr	**Wang**	**Devi and Devi**	**Waini *et al.***	**Present Result**
2	0.9114	0.91135	0.911357	0.911356
6.13	-	1.75968	1.759682	1.759688
7	1.8954	1.89540	1.895400	1.895409
20	3.3539	3.35390	3.353893	3.353909

The velocity, $f'(\eta)$ and temperature, $\theta(\eta)$ profiles for different values of copper nanoparticles volume fraction, φ_2 are presented in Figures 2 and 3. Other parameters such as $\varphi_1 = 0.1$, $\lambda = -1.5$, $S = 2.2$ and Bi=1 are fixed. Both velocity and temperature profiles increase when the values of φ_2 increases. The viscosity of hybrid nanofluid (Al_2O_3-Cu/H_2O) rises when φ_2 increases, which eventually improves the fluid velocity, as shown in Figure 2. Figure 3 illustrates that as the boundary layer thickness increases, the temperature profiles drop and approaching zero until it becomes constant.

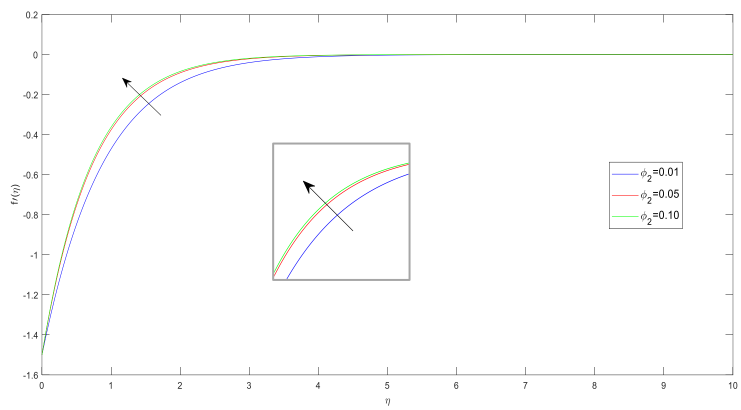


Figure 2: Effects of φ_2 on $f'(\eta)$

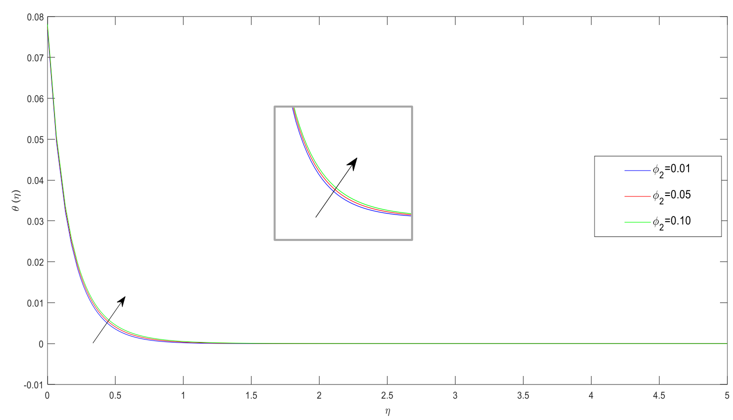


Figure 3: Effects of φ_2 on $\theta(\eta)$

Figures 4 and 5 illustrate the velocity, $f'(\eta)$ and temperature, $\theta(\eta)$ profiles for different values of λ. Other parameters are fixed at $\varphi_1 = 0.10$, $\varphi_2 = 0.10$, S=2.2 and Bi=1. The increasing in value of λ led to the increment in velocity and decrement in temperature profiles.

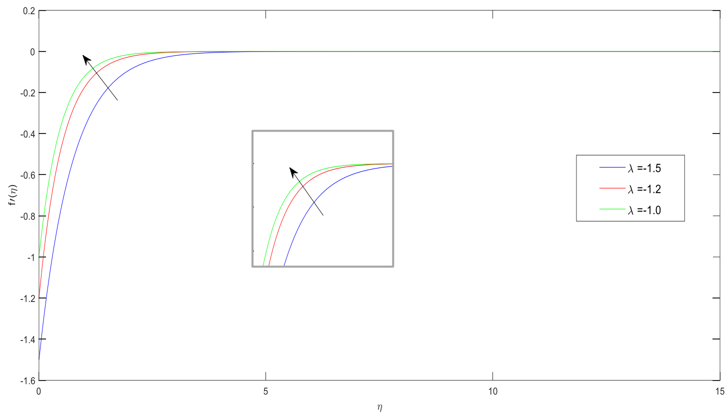


Figure 4: Effects of λ on $f'(\eta)$

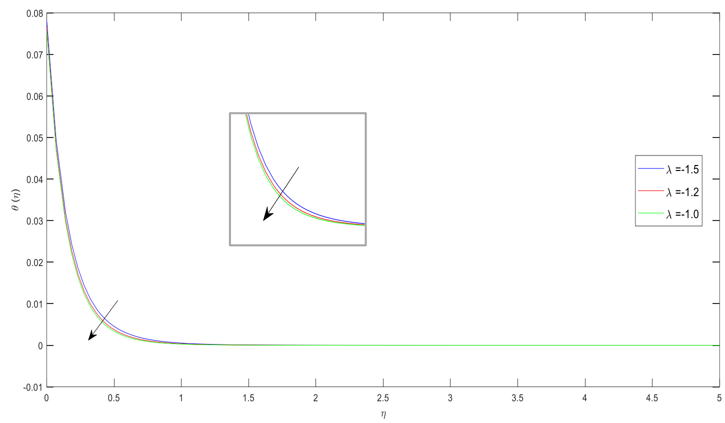


Figure 5: Effects of λ on $\theta(\eta)$

4 Conclusion

In this paper, the hybrid nanofluid flow and heat transfer past a permeable shrinking surface subject to convective boundary condition is investigated. Results showed that in hybrid nanofluid, the heat transfer increase as the Prandtl number increase. Next, the values for both velocity and temperature profiles increase as the values of the volume fraction of Copper (Cu) increase. The velocity profiles show an increment when the values of shrinking parameter $(\lambda < 0)$ increase. However, the temperature profiles show an opposite trend when the values of λ increase.

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