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Spliced Graphs of One Cutting Site in Graph Splicing System

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Abstract

A graph splicing is an operation that can be performed on graphs. A graph splicing system was originally introduced to illustrate the one-dimensional string of deoxyribonucleic acid (DNA) splicing in the form of graphs. The important components of DNA splicing which are the initial strands and splicing rules can be written in the form of graphs. The entire graph splicing system can be explained by one main component namely the graph splicing scheme in which at least one graph splicing rule and one initial graph are defined. By applying graph splicing rules on initial graphs, components of new graphs with the decomposed edges and vertices will be generated. In this research, the generated new graph is introduced as a spliced graph in which the graph splicing rule used consists of one cutting site only. The generated spliced graph constructed with four main components which are the vertices, semivertices, edges and semiedges.

Keywords: Spliced graphs, Graph splicing system, Graph theory, DNA

Introduction

The formulation of one-dimensional string of splicing system on a complex DNA is said to be inadequate since the structure of the DNA molecule is described in three dimension [1]. Hence, Freund [1] in 1995 introduced a notion of graph splicing system to illustrate the DNA splicing in the form of graphs. A graph splicing system is constructed with one main component which is a graph splicing scheme. A graph splicing scheme can describe the whole process of splicing (the cutting and recombining) of the graphs where at least one graph splicing rule is defined. The graph splicing rule is used to restrict the edges that are to be cut before the edges are recombined.

In [1], two types of graph splicing rules which are regular graph splicing rule and self-splicing rule are presented. A graph splicing rule is regular if any two edges to be cut are from two distinct initial graphs; meanwhile, a graph splicing rule is self-splicing if any two edges to be cut are from the same initial graph. Besides that, a graph splicing scheme can contain more than one splicing rule in which each splicing rule restricts different set of edges to be cut. If all splicing rules in a graph splicing scheme are regular, then the graph splicing scheme is called as a regular graph splicing scheme. These types of schemes are only applicable if the graph is linear or circular.

The study of splicing systems on graphs has expanded in other types of graphs such as semigraph [2]. A semigraph is a type of graphs, firstly introduced by Sampathkumar [3] in year 2000. A semigraph is a generalization of graphs where each vertex of a graph is drawn as a node in a semigraph, whereas each edge of a graph is drawn as a line in a semigraph. Hence, a semigraph is said to have a more straightforward generalization of a graph instead of hypergraph where the edges are illustrated in the form of sets. The idea of graph splicing system on semigraph is used by Jeyabharathi et. al. [2] to illustrate the cleavage pattern of DNA splicing. This type of splicing is called as n-cut splicing where n represents the length of the cleavage site of the splicing. Each splicing on the semigraph will generate two components of n-cut spliced semigraphs which represent sticky ends in DNA splicing. Next, some studies have been done on the n-cut spliced semigraphs.

For instance, the studies on bipartite structure and the characteristics of self-assembled semigraphs in DNA splicing through n-cut spliced semigraphs have been done by Jeyabharathi et. al. in [4] and [5], respectively. Besides that, Thiagarajan et. al [6] generates the language on n-cut spliced semigraphs by applying the folding techniques. Furthermore, Jeyabharathi et. al. [7] studied the norm of Parikh matrices in DNA splicing using the 1-cut spliced semigraphs and 2-cut spliced semigraphs, while Aisah et. al [8] used the idea of the previous author to study on the Parikh matrices involving 2-cut spliced semigraphs and 4-cut spliced semigraphs.

In n-cut splicing, two components of n-cut spliced semigraphs will be generated whenever a splicing is applied on a semigraph. Meanwhile, generally in graph splicing systems, other types of graphs can be considered as the initial graphs as well since there can be various form of graph splicing rules defined in the graph splicing schemes. This indirectly will generate other form of new graphs resulted from the decomposition of the edges of the initial graphs. By using the idea of n-cut spliced semigraphs, a new definition is stated in this research to generalize the new graphs generated by applying graph splicing rule with one cutting site on graphs.

The next section discusses on some preliminaries for this research.

Preliminaries

A graph is a mathematical structure that is usually used in representing problems. A graph G is constructed by two sets of components which are the nodes, also known as vertices and the lines connecting between nodes, also known as edges. The set of vertices and edges can be denoted as $V(G)$ and $E(G)$, respectively [9]. If the sets of vertices and edges of a graph h is in the sets of vertices and edges in G such that $V(h) \subseteq V(G)$ and $E(h) \subseteq E(G)$, then h is called a subgraph of G.

In a graph splicing system, the process of cutting and recombining the graphs is described by a graph splicing scheme [1]. A graph splicing scheme is a pair that is denoted as $\sigma = (A,P)$ where A is a set of finite alphabets (in this research, A is considered empty unless otherwise stated) and P is a set of finite splicing rules. The finite set of P consisting k number of graph splicing rules is written in the form $((h[1],E_c[1]),(h[2],E_c[2]),\ldots,(h[k],E_c[k]);R)$ such that $k \geq 1$, and $k \in \mathbb{N}$ for all i with $1 \leq i \leq k$, where,

- $h[i] = (V(h)[i],E(h)[i])$ is a connected graph, where $V(h)[i]$ and $E(h)[i]$ are the edges of the i^{th} graph splicing rule; also, $(h)[i]$ is a subgraph of the initial graph(s) used in the graph splicing system,
- $E_c[i] \subseteq E(h)[i]$ is the set of edges that will be cut (the cutting pattern of ith graph splicing rules),
- the vertices $V(h)[i]$ are mutually disjoint,
- R is considered as the recombination rule.

The focus of this research is on generating the new graphs after the decomposition of edges before the recombination phase. Instead of being in the form of a set, the graph splicing rules can also be presented in the form of graphs.

Then, a graph splicing system is a set $S = (\sigma, I)$ where σ is the graph splicing scheme and I is the set of all initial graphs [1].

In the next section, the generated graph after decomposition of edges is defined by applying splicing rules on a graph. Note that the graph splicing rule only contains one cutting site.

Results and discussion

In this section, we introduce a new definition of a spliced graph.

Let G be an initial graph. In this section, a graph splicing rule is denoted as $p = ((h, E_c); R)$. By applying the graph splicing rule p on G, a new graph after the decomposition of the edges is obtained and called as a spliced graph, given in Definition 1.

Definition 1: Spliced Graph

Let $p = ((h, E_c); R)$ be a graph splicing rule with one cutting site, (h, E_c), and let G be an initial graph. By applying p on G, the spliced graph will be denoted as a 4-tuple,

$$SG(G,(h,E_c)) = (V, E, V', E')$$

where V, E, V' and E' are the set of vertices, set of edges, set of semivertices and set of semiedges, respectively, where V' and E' are the 'overhang' vertices and edges formed by the decomposition of edges of G, and whenever an edge $(v_i, v_j) \in E(G)$ for $v_i, v_j \in V(G)$ is cut,

i) V' consists at least two elements,
ii) E' must at least consists of a pair in the form of $(v_i, a'], [b', v_j)$ for $a', b' \in V'$.

The following example is given to illustrate a spliced graph of a graph splicing.

Example 1:

Consider a graph G represented in Figure 1.

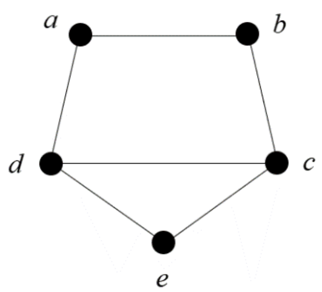

Figure 1 The graph G.

Then, let a graph splicing rule $p = ((h, E_c); R)$ be represented in Figure 2.

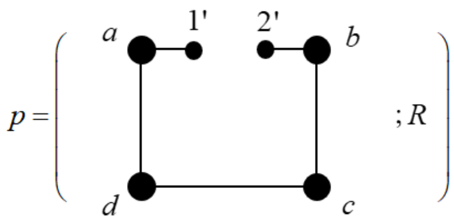

Figure 2 The graph splicing rule p.

By applying the graph splicing rule on G, a spliced graph is generated and can be represented as Figure 3 where $V = \{a, b, c, d, e\}$, $E = E(G) \setminus (a, b)$, $V' = \{1', 2'\}$ and $E' = \{(a, 1'], [2', b)\}$.

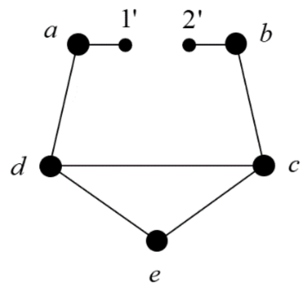

Figure 3 The spliced graph of G by applying the graph splicing rule, p.

The above example shows a spliced graph generated by applying the graph splicing rule p on a graph G.

Conclusion

Various real problems can be depicted and solved in the form of graphs. Various studies have been done involving graphs and one of them is graph splicing system. In this research, the idea of graph splicing system has been applied, focusing on the behavior of the cuts on edges of graph in generating various other types of graphs. The graph splicing rule used in this research is restricted to only contain one cutting site. This new generated graph is called as a spliced graph which consists of four components which are the vertices, edges, semivertices and semiedges. A semivertex and a semiedge are obtained by the decomposition of the edges of the initial graphs in which each semivertex and semiedge has its pair. Since a graph can hold many properties and characteristics, various studies can be done on the spliced graph in the future.

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