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Multidigraph of Fuzzy Autocatalytic Set for Ranking Purposes

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Abstract
Multidigraph is one of well-established tools to model a given complex system. A multidigraph allows parallel directed edges between any pair of its vertices. Thus, two vertices may be connected by more than one directed edge. This research will introduce a new concept in Fuzzy Autocatalytic Set (FACS), namely Multidigraph Fuzzy Autocatalytic Set (MFACS) and its mathematical properties. The mathematical structures of MFACS are investigated. The MFACS will be implemented on several multicriteria decision making (MCDM) problems such as in ranking the multiple intelligence parameters for the purpose to maximize the employment probability of People with Epilepsy (PWE), Covid-19 mitigation strategy and selection of project manager for a construction company.
Keywords: Graph Theory, Multidigraph, Autocatalytic Set, Complex Adjacency Matrix.

Introduction

The concept of fuzzy autocatalytic set (FACS) was defined by Ahmad et al. [1]. The researchers investigated and explored fuzzy graphs in its relation to autocatalytic sets. In general, an autocatalytic set is defined as a set of entities which have catalytic interaction among them. The word entities can be anything such as people, molecules, or objects. However, Jain and Krishna [2] first defined an autocatalytic set as a form of a graph. An autocatalytic set (ACS) is a subgraph, each of whose nodes has at least one incoming link from a node belonging to the same subgraph. However, FACS is an autocatalytic set with membership value in the interval (0,1]. Since catalytic interaction may involve multi-structures and inter-relation between its components, this research will study multidigraphs involving catalytic networks for the first time. Most of the previous research on ACS was without multiple directed edges. A multidigraph allows parallel directed edges between any pair of its vertices. In other words, two vertices may be connected by more than one directed edge. Multidigraph is one of well- established tools to model a given complex system such as multi-agent system, disease, and chemical reaction networks [2]–[4]. By the introduction of multidigraph into fuzzy autocatalytic sets, it has led into a new branch of graph theory, namely Multidigraph of Fuzzy Autocatalytic Set (MFACS).

Multiple directed graphs or multidigraphs are widely used to find patterns or clusters in data. In this section, some basic definitions of multidigraph and its several classes are reviewed. A multidigraph G is an ordered pair $G = (V, A)$ contains a set of vertices, V and a multiset of ordered pairs of vertices called directed edges or arcs, A such that for vertices $a, b \in V$ and the multiple directed edges, $A = \{E_1, E_2, \ldots, E_n\}$ [5]. Example of multidigraph is shown in Figure 1.

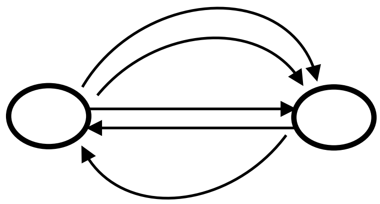

Figure 1 A multidigraph $\boldsymbol{G}$ with vertices, $\boldsymbol{a}, \boldsymbol{b}$ and multiple directed edges, $\boldsymbol{A}$.

This study constructs a new theoretical concept of multidigraph in fuzzy autocatalytic sets called Multidigraph Fuzzy Autocatalytic Set (MFACS). Besides, we write the Multidigraph Fuzzy Autocatalytic Set's algorithm and code it by using MATLAB software. Moreover, we verify the technique on multicriteria decision making problems such as in ranking the multiple intelligence parameters to maximize the employment probability of PWE, Covid-19 mitigation strategy and selection of project manager for a construction company.

Materials And Methodology

This study will develop a novel mathematical technique, namely, Multidigraph of Fuzzy Autocatalytic Set that consist of

- A new mathematical structure, namely, multidigraph fuzzy autocatalytic set (MFACS) will be presented.
- The procedure to construct the adjacency matrix for MFACS as shown in Figure 2.

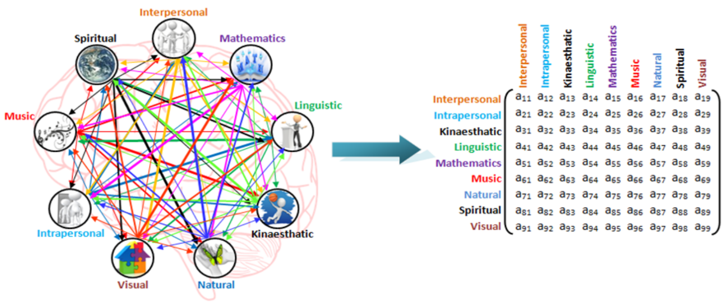

Figure 2 Complex Adjacency Matrix for MFACS

- Establish some mathematical properties for MFACS. Besides, we provide the MFACS algorithm and its coding using MATLAB software.
- Verify the effectiveness of MFACS in ranking multicriteria decision making for three complex systems. They are to rank nine intelligence skills based on results from ATIE© in order to maximize the employment probability of PWE, to analyze Coronavirus disease 2019 (Covid-19) in Malaysia and to select the most qualified project manager for a construction company.

Expected Results

The FACS acts as a tool for introducing fuzziness to a graph's connectedness. The introduction of FACS has resulted in better analysis in real world applications. In the previous research, FACS has managed to represent a catalytic interaction between a set of entities using single directed edges. On the other hand, many complex systems require the representation of multiple directed edges between a pair of vertices. This complex system involves multi structures and inter-relation between its components.

Lin et al. [6] stated that graphs have been widely used in the analysis of various networks, but most of the existing embedding methods omit the network dynamics and the multiplicity of edges, hence it is difficult to accurately describe the detailed characteristics of the networks. As the solution, this study will develop a novel mathematical technique, namely, Multidigraphs of Fuzzy Autocatalytic Set (MFACS). The implementation MFACS can capture the dynamicity networks. The theory combines three mathematical structures which are multidigraph, ACS and FACS.

A new definition of MFACS and their complex adjacency representation of MFACS are revealed as shown in Figure 3. This theory gives a new insight in the development of multicriteria decision making (MCDM) tools involving multiple directed edges of its graph for complex systems. These systems are perceived intelligence parameters of people with epilepsy[7], Covid-19 mitigation strategy[8] and selection of project manager for a construction company by Mamat et al. [9].

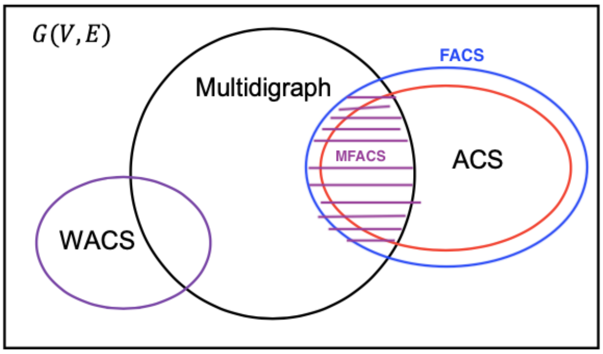

Figure 3 Multidigraph of Fuzzy Autocatalytic Set (MFACS)

This research will implement MFACS as a multi-criteria decision-making (MCDM) tool on three complex systems as follows.

First, the classification of multiple intelligences of people with epilepsy. One of the most challenging problems faced by people with epilepsy (PWE) is employment [7]. In order to suggest an ideal combination of the eight intelligence skills that the PWE should have to improve the probability of being employed. This theory will help to rank the intelligence skills based on results from the Ability Test in Epilepsy (ATIE©) to maximize the employment probability of PWE. Based on ATIE©, an inverse Ability Test in Epilepsy (i-ATIE) system was designed. This system was developed based on the Fuzzy Inverse ATIE (FIA) algorithm. Then, the algorithm was incorporated into a crisp Logistic Regression model in order to determine the best intelligence elements that are needed to maximize the employment chances of PWE.

Second, implementation of MFACS for analysis of Coronavirus disease 2019 (Covid-19), particularly, the pandemic outbreak in Malaysia. Hassan et al. [8] analysed the daily Covid-19 cases in states in Malaysia using c-FACS, to observe the trend and severity of the disease in Malaysia. The MFACS will be applied on the same case using multidigraph to enhance the dynamicity of the system.

Third, the project manager selection for Polband Construction company case study by Mamat et al. [10] is reexamined. The researchers used AHP to determine the weights and ranked the applicants using Fuzzy Technique for Order of Preference by Similarity to Ideal Solution (FTOPSIS). It consists of four levels. On the first level, the goal is to select a project manager. The second level contains four main criteria. There are 23 sub-criteria levels in the third level. The fourth level consists of 10 applicants.

Conclusion

This study investigates the concept of ACS associated with multidigraph for complex systems, particularly the problems involving multi-parallel edges due to inter-relation between related components.

However, it is limited to explore multidigraph without self-loop and have an incoming link at each vertex. Most of the previous research on ACS was without multiple directed edges. A multidigraph allows two vertices to be connected by more than one directed edge. Multidigraph is one of well- established tools to model a given complex system such as multi-agent system, disease, and chemical reaction networks [2,3,4].

In order to verify the effectiveness of MFACS, this research will apply MFACS in ranking multicriteria decision making systems. The secondary data from the Inverse Ability Test in Epilepsy (i-ATIE) system is used to rank the most suitable parameters of the eight intelligence skills of people with epilepsy, thereby increasing their chances of securing suitable employment. This system was developed based on the Fuzzy Inverse ATIE (FIA) algorithm.

For comparison purpose we also implement MFACS on published data, namely, from Covid-19 mitigation strategy by Hassan et al. [8] and data from project manager selection of Polband Construction Company by Mamat et al. [9]. By the introduction MFACS, it will give significant impact to multicriteria decision making system whereby we can consider all the parameters which involves multi structures and inter-relation between its components.

This theory gives a new insight in the development of multicriteria decision making (MCDM) tools involving multiple directed edges for complex systems. The advantages of MFACS can capture the dynamicity networks and the theory combines three mathematical structures which are multidigraph, ACS and FACS.

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