



## The Effect of Non-Uniform Heat Source/Sink On Mixed Convection Flow Of Hybrid Nanofluid Over A Stretching Sheet

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### Abstract

This study considers the effect of non-uniform heat source/sink on mixed convection flow of hybrid nanofluid past a stretching sheet. The governing equations consist of coupled nonlinear partial differential equations transformed into ordinary differential equations by applying the appropriate similarity transformation. Then, the ordinary differential equations are solved numerically by using the Keller-Box method. Numerical results are presented graphically with the aid of MATLAB. It is observed from the results that the presence of a heat source raises the fluid's temperature, whereas the presence of a heat sink reduces the temperature of the fluid.

**Keywords:** Hybrid nanofluid; Mixed convection; Non uniform heat source/sink; Numerical solution

### Introduction

Conventional heat transfer fluids have limited heat transfer capabilities due to its low thermal conductivity. This limitation urged researchers to introduce a new class of heat transfer known as nanofluids. Nanofluids are solid-liquid composite materials consisting of nanoparticles spread in base fluid. Further studies on nanofluids prove that nanoparticles successfully improved the thermophysical properties of a base fluid, thus enhancing heat transfer rate. In recent studies, researchers started to consider the implementations of combining two different nanoparticles in base fluid. The advanced process, named hybrid nanofluids, displayed greater improvement in heat transfer properties of the base fluid [1-6]. Hybrid nanofluids present better properties than nanofluids due to higher surface area, higher dispersion stability and a reduction in pumping power. Due to that reason, hybrid nanofluids have been widely used in several applications such as coolant applications, heat exchangers, solar energy and transportation, micro fluidics, naval structures and medical [1].

Additionally, the presence of a heat source/sink in the heat transfer flow can significantly increase the thermal conductivity of the conventional heat transfer fluid. Therefore, the effect of non-uniform heat source/sink on convection flow is interesting to study, especially when involving hybrid nanofluids.

### Problem formulation

Consider the unsteady two-dimensional boundary flow of hybrid nanofluids over a stretching sheet in the presence of non-uniform heat source/sink. The research analysis was conducted by using water as base fluid while copper and alumina as chosen nanoparticles.

**Table 1:** Thermophysical properties of water and nanoparticles

Thermophysical Properties	Water (H <sub>2</sub> O)	Copper (Cu)	Alumina (Al <sub>2</sub> O <sub>3</sub> )
$\rho$ (kg/m <sup>3</sup> )	997.1	8933	3970
$C_p$ (J/kgK)	4179	385	765
$k$ (W/mK)	0.613	401	40
$\beta$ (K <sup>-1</sup> )	$21 \times 10^{-5}$	$1.67 \times 10^{-5}$	$0.85 \times 10^{-5}$

The governing equations can be expressed as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu_{hnf}}{\rho_{hnf}} \frac{\partial^2 u}{\partial y^2} + g \frac{(\rho\beta)_{hnf}}{\rho_{hnf}} (T - T_\infty), \tag{2}$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{hnf} \frac{\partial^2 T}{\partial y^2} + \frac{q'''}{(\rho C_p)_{hnf}}, \tag{3}$$

subjected to the appropriate initial and boundary conditions for the velocity components at the surface and temperature

$$u = U_w(x,t) = \frac{ax}{1-ct}, \quad v = v_w(t) = 0, \quad T = T_w(x,t) = T_\infty + \frac{bx}{(1-ct)^2}, \quad \text{at } y = 0, \tag{4}$$

$$u = 0, \quad T = T_\infty, \quad \text{as } y \rightarrow \infty, \tag{5}$$

where  $u$  and  $v$  are the velocity components along the  $x$  and  $y$  axes respectively while  $t$  is time. The dynamic viscosity of the hybrid nanofluids denotes by  $\mu_{hnf}$ , the density of the hybrid nanofluids represents by  $\rho_{hnf}$  while  $\alpha_{hnf}$  is the thermal diffusivity of the hybrid nanofluids. Meanwhile,  $(\rho\beta)_{hnf}$  is the thermal expansion coefficients of the hybrid nanofluids and  $(\rho C_p)_{hnf}$  is the specific heat capacitance of hybrid nanofluids and  $g$  is the acceleration due to gravity. The suffixes  $hnf$  specify the hybrid nanofluids.

The non-uniform heat source/sink,  $q'''$ , is modeled as [7]

$$q''' = \frac{U_w(x)k_{hnf}}{x\nu_{hnf}} [A(T_w - T_\infty)f'(\eta) + B(T - T_\infty)] \tag{6}$$

where  $U_w$  is the surface velocity,  $k_{hnf}$  and  $\nu_{hnf}$  is the thermal conductivity and the kinematic viscosity of hybrid nanofluids, respectively while  $T_\infty$  is the ambient fluid temperature, and  $T_w$  is the temperature near the surface sheet. Given that  $A$  and  $B$  are the parameters of space-dependent and temperature-dependent heat source/sink, respectively. Note that  $A, B > 0$  represents the heat source while  $A, B < 0$  holds the heat sink.

Considering the following similarity transformation [8]

$$\eta = \sqrt{\frac{a}{v_f(1-ct)}}y, \quad \psi(x, y, t) = \sqrt{\frac{av_f}{1-ct}}xf(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \tag{7}$$

where  $\eta$  is the similarity variable and  $\psi(x, y, t)$  is the stream function that is defined by  $u = \frac{\partial\psi}{\partial y}$  and  $v = -\frac{\partial\psi}{\partial x}$ . In view of the suitable transformations given in equation (6), equations (2) and (3) are transformed to the following forms;

$$f''' + (1-\phi)^{2.5} \left[ D_1 \left[ ff'' - (f')^2 - S \left( f' + \frac{\eta}{2} f'' \right) \right] + D_2 \lambda \theta \right] = 0, \tag{8}$$

$$\theta'' + (1-\phi)^{2.5} D_1 [A f' + B \theta] + \frac{\text{Pr} D_3}{D_4} \left[ f \theta' - f' \theta - S \left( 2\theta + \frac{\eta}{2} \theta' \right) \right] = 0, \tag{9}$$

accordingly, the boundary conditions (4) become

$$\begin{aligned} f' = 1, & \quad f = 0, & \quad \theta = 1, & \quad \text{at } \eta = 0, \\ f' = 0, & & \quad \theta = 0, & \quad \text{at } \eta \rightarrow \infty, \end{aligned}$$

where  $D_1, D_2, D_3$  and  $D_4$  are constants given by

$$D_1 = (1-\phi) + \phi_{1p} \frac{\rho_{1p}}{\rho_f} + \phi_{2p} \frac{\rho_{2p}}{\rho_f}, \quad D_2 = (1-\phi) + \phi_{1p} \frac{(\rho\beta)_{1p}}{(\rho\beta)_f} + \phi_{2p} \frac{(\rho\beta)_{2p}}{(\rho\beta)_f}, \tag{10}$$

$$D_3 = (1-\phi) + \phi_{1p} \frac{(\rho C_p)_{1p}}{(\rho C_p)_f} + \phi_{2p} \frac{(\rho C_p)_{2p}}{(\rho C_p)_f}, \quad D_4 = \frac{k_{mf}}{k_f}. \tag{11}$$

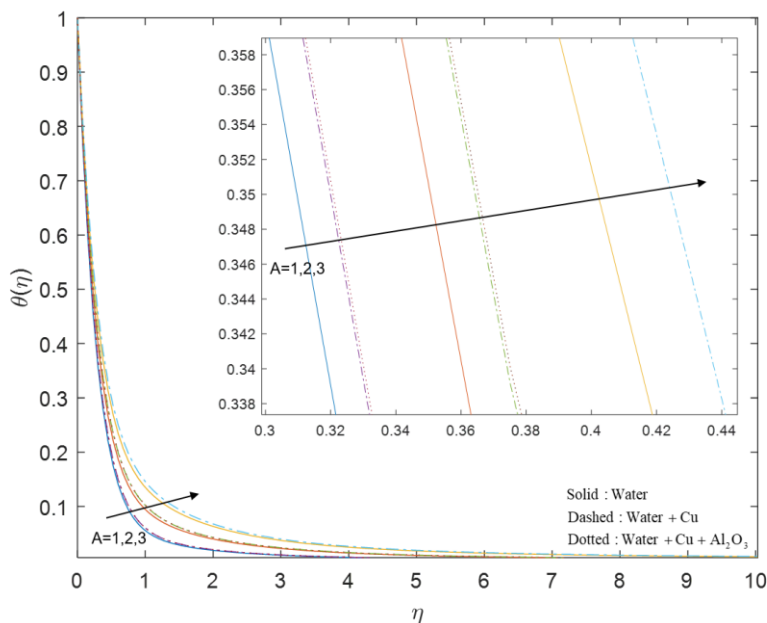
### Results and discussion

This section presented and discussed the numerical results through analysis on the parameters involved such as the space-dependent non-uniform heat source/sink parameter,  $A$  and the temperature-dependent non-uniform heat source/sink parameter,  $B$ , on the temperature profile. Table 2 presents the comparison of steady state solution,  $S = 0$  with those obtained by [8] and [9] which seems to be in close agreement with the present result.

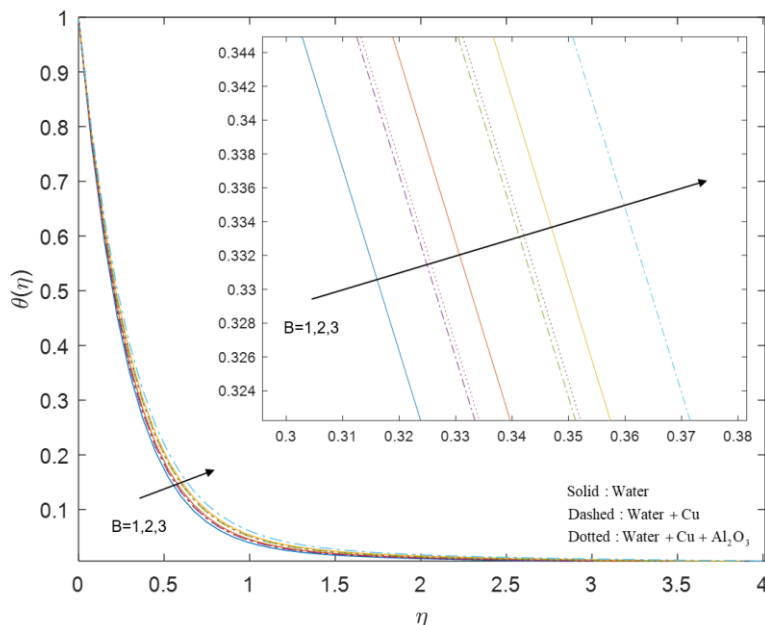
**Table 2:** Comparison of  $-\theta'(0)$  when  $\phi = S = \lambda = A = B = 0$  and various value of Pr

Pr	[9]	[8]	Present results
0.72	0.8086	0.80863	0.8088
1	1.0000	1.00001	1.0000
3	1.9237	1.92368	1.9237
7	3.0723	3.07224	3.0724

Throughout the study, the parameters are set as constant of  $S = \lambda = A = B = 0.5$  and  $\text{Pr} = 6.2$  except for those stated in the graphical results.



**Figure 1** The effect of space-dependent non-uniform heat source/sink on temperature profile.



**Figure 2** The effect of temperature-dependent non-uniform heat source/sink on temperature profile.

The behaviour of space-dependent non-uniform heat source/sink parameter,  $A$ , for the distribution of temperature are illustrated in Figure 1. The result yields to the increase in the fluid temperature due to the higher value of  $A$ . Note that the improvement in the thermal boundary layer leads to the non-uniform heat source/sink to act as a heat-generating agent. The nature of heat source measures that high strength of heat source leads to higher fluid temperature. Inversely, temperature decreases due to the increasing strength of the heat sink. It is recognised that the temperature levels of hybrid nanofluids are higher than in the nanofluids and base water.

Figure 2 observed the impact of temperature-dependent non-uniform heat source/sink parameter,  $B$ . The numerical results revealed that the involvement of  $B$  has a significant impact on the fluid temperature. It shows that the augmented value of  $B$  successfully accelerates the fluid temperature. Heat energy is generated by boosting the value of  $B$  and hence increasing the fluid temperature. Also, it is fascinating to note that hybrid nanofluids generally have a higher temperature than the nanofluids and base fluid. It is concluded that the presence of non-uniform heat source/sink continues to demonstrate that hybrid nanofluids have a higher temperature than that of nanofluids and base fluid. It can be seen in every figure that the temperature flow is significantly higher for hybrid nanofluids. It shows that hybrid nanofluids have the best performance in thermal conductivity.

### Conclusion

From the numerical results of the study on mixed convection flow of hybrid nanofluids in the presence of non-uniform heat source/sink, we draw the following conclusion:

- Increasing the space-dependent and temperature-dependent of non-uniform heat source/sink parameters resulted in the improvement of the temperature profiles.
- The enhancement of non-uniform heat source/sink parameters has the tendency to escalate the fluid temperature.
- The heat source increases the temperature of the fluid whilst the heat sink reduces the temperature of the fluid.
- Hybrid nanofluids have higher heat transfer performance when contrasted with nanofluids and base fluid.

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