Proceedings of Science and Mathematics
Faculty of Science, Universiti Teknologi Malaysia

Vol. 7, 2022, Page 82 - 87

Caputo Fractional Casson Fluid Flow Over Oscillated and Accelerated Plates

Marjan Mohd Daud[a], Lim Yeou Jiann[a], Sharidan Shafie[a]*, Rahimah Mahat[b]

[a]Department of Mathematical Sciences, Faculty of Science,
Universiti Teknologi Malaysia, 81310, Johor Bahru, MALAYSIA.
[b]Universiti Kuala Lumpur Malaysian Institute of Industrial Technology,
Persiaran Sinaran Ilmu, Bandar Seri Alam, 81750 Johor Bahru, MALAYSIA.
*Corresponding author: sharidan@utm.my

Abstract

Unsteady mixed convection boundary layer flow of Casson fluid with the impact of Caputo fractional derivative is studied. The flow over oscillating and accelerating plates is considered. A system of partial differential equations together with appropriate initial and boundary conditions are utilized to model the problems. Using appropriate dimensionless variables, the dimensional governing equations are transformed into dimensionless governing equations. By employing Caputo fractional derivative, the derived dimensionless equations are converted into fractional form. The Laplace transform method is used to acquire the exact solutions. The results of velocity and temperature profiles are presented graphically and analyzed to explore their behavior along both geometries. It shows that the accelerated plate has a higher velocity compared to the oscillated plate. The temperature profile is depicted to be the same. The increment of fractional parameters resulted in a higher pattern for both profiles. This research is useful for understanding Casson fluid flows in fractional system

Keywords: Oscillated plate; Accelerated plate; Caputo fractional derivative; Laplace transform

Introduction

The concept of fractional derivatives in applied mathematics modelling is recently gaining attention from researchers due to its function and specialty. In accordance with [1], a fractional derivative has existed for a long time since the conversation between Leibnitz and L'Hospital. Biographically, L'Hospital had written a letter to Leibenitz to argue on fractional derivative which is then said that it is a paradox from which helpful conclusions will be reached in the future [2]. Fractional partial differential equations have a lot of unusual properties that make them good mathematical tools for describing the complicated behavior of boundary layer flow. For modelling physical processes, the Caputo fractional derivative is more useful. The Caputo fractional derivative has been studied extensively in the past [3,4].

Newtonian and non-Newtonian fluids are the two types of fluids. Many academics have recently become interested in non-Newtonian fluids due to its significant implications in mechanical and engineering fields. Casson fluid is one type of non-Newtonian fluid that defies Newton's vicious law, displaying infinite viscosity at zero shear stress and is described as a shear thinning liquid. Furthermore, this fluid behaved as a solid when the yield stress exerted on it was greater than the shear stress. The fluid begins to flow when the shear stress is greater than the applied yield stress. Casson fluid includes toothpaste, blood and jelly. According to [5], Casson fluid has a wide range of uses in pharmaceutical and cosmetics. [6,7] are some of the studies that have been done on Casson fluid.

Because of their importance in real-world concerns, many academics are currently interested in studying this fluid over a variety of geometries such as over a plate. The benefits of plate may be seen in the aerodynamics of automotive and supersonic aircraft. The behavior of the plate varies adherents to its application of real-life problems including static, rotating and others. Among research that has been done on plate can be seen in [8,9].

The present study aims to observe and compare the Casson fluid flow characteristics over oscillated and accelerated plates, respectively. In addition, together with the application of the Caputo fractional derivative is to form time-fractional governing equations. The obtained fractional equationsare solved to find analytical solutions of velocity and temperature profiles by using Laplace transform method.

Methodology

The flow of unidirectional and unsteady incompressible Casson fluid is considered. The momentum and energy equations of the problem are given as

$$\frac{\partial u(y,t)}{\partial t} = \upsilon\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u(y,t)}{\partial y^2} + g\beta_T\left(T-T_o\right). \tag{1}$$

$$\rho c_p \frac{\partial T(y,t)}{\partial t} = k\frac{\partial^2 T(y,t)}{\partial y^2}. \tag{2}$$

where $u(y,t),\ T(y,t),\ \upsilon,\ \beta,\ g,\ \beta_T,\ \rho,\ c_p,\ k$ are the fluid velocity, fluid temperature, kinematic viscosity, Casson fluid parameter, gravitational acceleration, thermal expansion coefficient, fluid density, specific heat at constant pressure and thermal conductivity, respectively. This problem is solved under two different conditions as follows.

Oscillating Plate

The plate oscillates at its axis,

$$u(y,0)=0, \qquad T(y,0)=T_\infty, \tag{3}$$

$$u(0,t)=f\mathrm{H}(t)\cos\omega t, \qquad T(0,t)=T_w, \tag{4}$$

$$u(\infty,t)=0, \qquad T(\infty,t)=T_\infty, \tag{5}$$

where $\mathrm{H}(t)$ is the unit step function, ω is the frequency of oscillations and f is the amplitude of oscillations.

Accelerating Plate

As time increases, the plate accelerated at *At*. Therefore, the equations (1) and (2) are subjected to

$$u(y,0)=0, \qquad T(y,0)=T_\infty, \tag{6}$$

$$u(0,t)=At, \qquad T(0,t)=T_w, \tag{7}$$

$$u(\infty,t)=0, \qquad T(\infty,t)=T_\infty, \tag{8}$$

where the constant A represents the plate's acceleration.

Solution of the Problems

Using Caputo fractional derivative defined as

$$\frac{\partial^\alpha f(t)}{\partial t^\alpha} = \begin{cases} \dfrac{1}{\Gamma(1-\alpha)}\displaystyle\int_0^t \frac{1}{(t-\tau)^\alpha}\frac{\partial f(\tau)}{\partial \tau}\partial\tau; 0<\alpha<1, \\ \dfrac{\partial f(t)}{\partial t}; \alpha=1. \end{cases} \tag{9}$$

where Γ is Euler Gamma function, τ is the convolution product variable, and α is a fractional parameter. The governing equations are transformed into fractional equations.

Solution of Oscillating Plate
Dimensionless variables are introduced as

$$v=\frac{u}{f},\quad \gamma=\frac{v}{f},\quad \xi=\frac{y}{\gamma},\quad \tau=\frac{t}{\lambda},\quad \theta=\frac{T-T_{\infty}}{T_w-T_{\infty}}. \tag{10}$$

Hence, by using (10) equations (1) and (2) are in dimensionless fractional equations form as

$$D_{\tau}^{\alpha}\mathrm{v}=\left(1+\frac{1}{\beta}\right)\frac{\partial^2 \mathrm{v}}{\partial\xi^2}+Gr\theta, \tag{11}$$

$$\Pr\frac{\partial\theta}{\partial\tau}=\frac{\partial^2\theta}{\partial\xi^2}. \tag{12}$$

Laplace transform is a method for solving unsteady differential equations that uses an integral transform. The partial differential equations are transformed into linear ordinary differential equations which are then solved by undetermined coefficient method and can be expressed as

$$\overline{\mathrm{v}}=\frac{q}{q^2+w^2}e^{-\xi\sqrt{\beta_o q^{\alpha}}}+\frac{Gr_o}{(\Pr q^{\alpha}-\beta_o q^{\alpha})q}e^{-\xi\sqrt{\beta_o q^{\alpha}}}-\frac{Gr_o}{(\Pr q^{\alpha}-\beta_o q^{\alpha})q}e^{-\xi\sqrt{\Pr q^{\alpha}}}, \tag{13}$$

$$\overline{\theta}=\frac{1}{q}e^{-\xi\sqrt{\Pr q^{\alpha}}}.$$

Then, the inverse Laplace transform method is applied to equations (11) and (12), yields to

$$\mathrm{v}=\cos\,\omega\tau\times\tau^{-1}\phi\left(0,-\frac{\alpha}{2},-\xi\sqrt{\frac{\beta_o}{\tau^{\alpha}}}\right)+\frac{Gr_o}{(\Pr q^{\alpha}-\beta_o q^{\alpha})}\begin{bmatrix}\phi\left(1,-\frac{\alpha}{2},-\xi\sqrt{\frac{\beta_o}{\tau^{\alpha}}}\right)\\ \phi\left(1,-\frac{\alpha}{2},-\xi\sqrt{\frac{\Pr}{\tau^{\alpha}}}\right)\end{bmatrix}, \tag{14}$$

$$\theta=\phi\left(1,-\frac{\alpha}{2},-\xi\sqrt{\frac{\Pr}{\tau^{\alpha}}}\right). \tag{15}$$

Solution of Accelerating Plate
The appropriate dimensionless variables are

$$v=\frac{u}{(\upsilon A)^{\frac{1}{3}}},\quad \tau=\frac{tA^{\frac{2}{3}}}{\upsilon^{\frac{1}{3}}},\quad \xi=\frac{yA^{\frac{1}{3}}}{\upsilon^{\frac{2}{3}}},\quad \theta=\frac{T-T_o}{T_w-T_o}. \tag{16}$$

Hence, by using (16) equations (1) and (2) are in dimensionless fractional equations form as

$$D_{\tau}^{\alpha}\mathrm{v}=\left(1+\frac{1}{\beta}\right)\frac{\partial^2 \mathrm{v}}{\partial\xi^2}+Gr\theta, \tag{17}$$

$$\Pr D_{\tau}^{\alpha}\theta=\frac{\partial^2\theta}{\partial\xi^2}. \tag{18}$$

Then, using Laplace transform method to solve (17) and (18) and linear ordinary differential equations are obtained. By using an undetermined coefficient method, the solution can be expressed as

$$\bar{v}=\frac{1}{q^2}e^{-\xi\sqrt{\beta_o q^\alpha}}+\frac{\beta_o Gr}{(\Pr q^\alpha-\beta_o q^\alpha)q}e^{-\xi\sqrt{\beta_o q^\alpha}}-\frac{Gr_o}{(\Pr q^\alpha-\beta_o q^\alpha)q}e^{-\xi\sqrt{\Pr q^\alpha}}, \quad (19)$$

$$\bar{\theta}=\frac{1}{q}e^{-\xi\sqrt{\Pr q^\alpha}}. \quad (20)$$

Then, inverse Laplace transform method is implemented to (19) and (20) and obtained the solutions as

$$v=\tau\left(2,-\frac{\alpha}{2},-\xi\sqrt{\frac{\beta_o}{\tau^\alpha}}\right)+\frac{Gr_o}{(\Pr q^\alpha-\beta_o q^\alpha)}\left[\begin{matrix}\phi\left(1,-\frac{\alpha}{2},-\xi\sqrt{\frac{\beta_o}{\tau^\alpha}}\right)\\ \phi\left(1,-\frac{\alpha}{2},-\xi\sqrt{\frac{\Pr}{\tau^\alpha}}\right)\end{matrix}\right], \quad (21)$$

$$\theta=\phi\left(1,-\frac{\alpha}{2},-\xi\sqrt{\frac{\Pr}{\tau^\alpha}}\right). \quad (22)$$

Results and Discussion

The results for the velocity and temperature profiles with the effect of fractional parameters are displayed in this section. Figure 1 depicts the impact of α on the fluid temperature. For both geometries, the same consistency pattern of graphs regardless of different geometry are obtained. Maximum curve of temperature profile is obtained at $\alpha=1$.

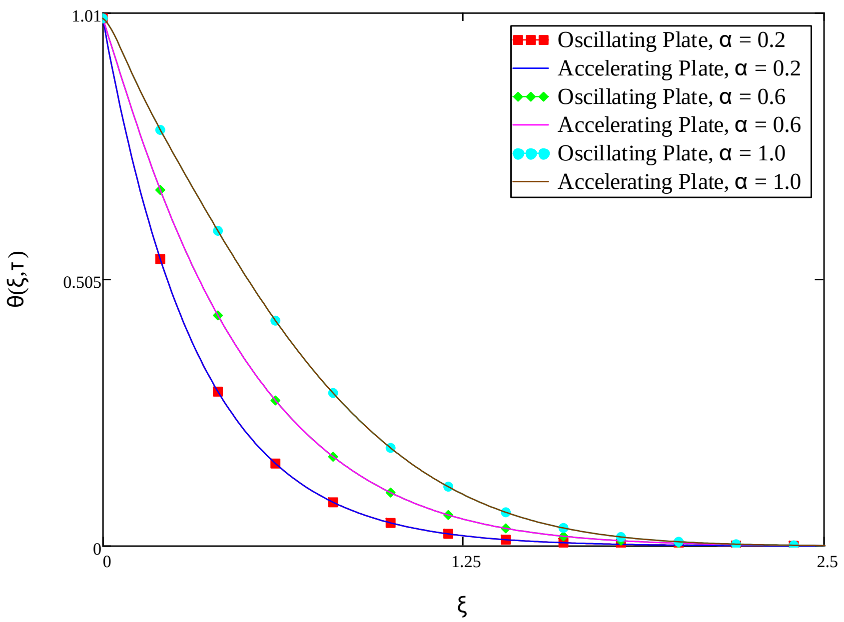

Figure 1 Temperature profile

Meanwhile, Figure 2 shows the result of the velocity profile for both geometries. It shows that an oscillating plate has a lesser velocity as compared to the accelerating plate. Due to the movement of the plate, it depicts that the accelerating plate has a higher velocity since its tendency is elevated. The velocity profile grows in lockstep with α.

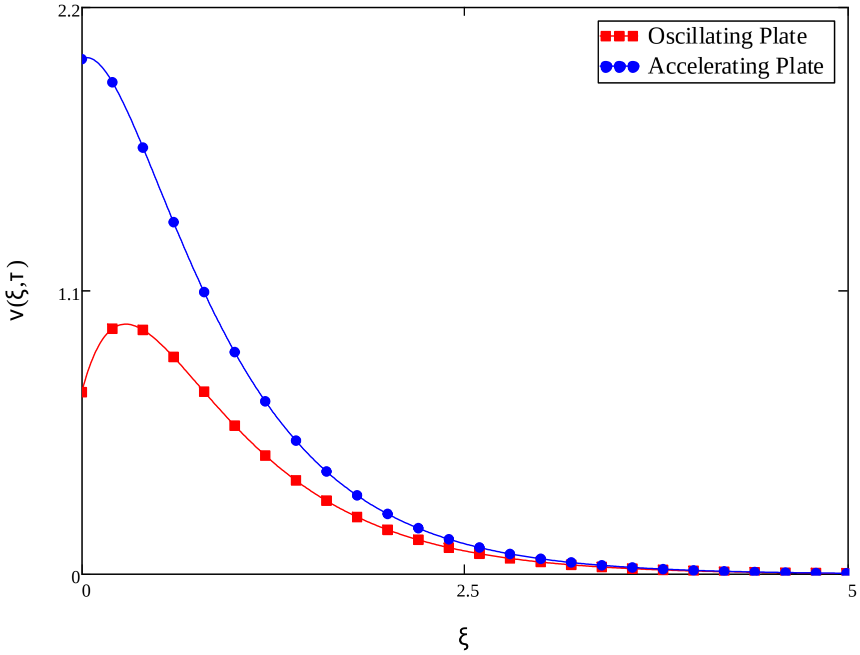

Figure 2 Velocity profile

Conclusion

In this paper, the Caputo fractional derivative is applied to the convective flow of Casson fluid over an oscillating and accelerating plate. The Laplace transform method was used to find the exact solutions which has then been developed for velocity and temperature profiles. The effect of fractional parameters is depicted. It shows that as fractional parameters increase, the velocity and temperature increase. The accelerated plate's velocity profile is higher than the oscillated plate. The temperature profile remains the same for both geometries.

Acknowledgement

The authors would like to acknowledge the Ministry of Higher Education Malaysia (MOHE) and Research Management Centre-UTM, Universiti Teknologi Malaysia (UTM) for financial support through vote numbers FRGS/1/2019/STG06/UTM/02/22 and 08G33.

References

[1] Khalil, R., Horani, M. Al, Yousef, A., and Sababheh, M. (2014) A new definition of fractional derivative. Journal of Computational and Applied Mathematics, 264: 65–70.

[2] Leibniz, G. (1849). Letter from Hanover, Germany, to GFA L'Hopital, September 30; 1695. Mathematische Schriften. 2: 301-302.

[3] Ali, F., Sheikh, N. A., Khan, I., and Saqib, M. (2017). Magnetic field effect on blood flow of Casson fluid in axisymmetric cylindrical tube: A fractional model. Journal of Magnetism and Magnetic Materials, 423: 327–336.

[4] Ahmad, M., Asjad, M. I., Akgül, A., and Baleanu, D. (2021). Analytical solutions for free convection flow of Casson nanofluid over an infinite vertical plate. AIMS Mathematics, 6(3): 2344–2358.

[5] Khalid, A., Khan, I., Khan, A., and Shafie, S. (2015). Unsteady MHD free convection flow of Casson fluid past over an oscillating vertical plate embedded in a porous medium. Engineering Science and Technology, an International Journal, 18(3): 309–317.

[6] Saqib, M., Ali, F., Khan, I., & Sheikh, N. A. (2018). Heat and mass transfer phenomena in the flow of Casson fluid over an infinite oscillating plate in the presence of first-order chemical reaction and slip effect. Neural Computing and Applications, 30(7): 2159–2172.

[7] Mahanthesh, B., Brizlyn, T., Shehzad, S. A., and B.J, G. (2019). Nonlinear thermo-solutal convective flow of Casson fluid over an oscillating plate due to non-coaxial rotation with quadratic density fluctuation: Exact solutions. Multidiscipline Modeling in Materials and Structures, 15(4): 818–842.

[8] Abdullah, M., Butt, A. R., and Raza, N. (2019). Heat transfer analysis of Walters'-B fluid with Newtonian heating through an oscillating vertical plate by using fractional Caputo–Fabrizio derivatives. Mechanics of Time-Dependent Materials, 23(2): 133–151.

[9] Ali, F., Sheikh, N. A., Khan, I., and Saqib, M. (2017). Solutions with Wright Function for Time Fractional Free Convection Flow of Casson Fluid. Arabian Journal for Science and Engineering, 42(6): 2565–2572.