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Comparing Discrete Grey Model DGM(1,1) and Grey Forecasting Model GM(1,1) For Carbon Dioxide Forecasting

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Abstract
The purpose of this study is to determine the correlation between Grey forecasting model GM(1,1) and Discrete Grey model DGM(1,1) and identify the best forecasting performance between these two models. Grey forecasting model GM(1,1), the main system of grey theory was used to obtain the forecast value of carbon dioxide emission. This model focused on solving problems where small data samples and uncertain information are used. In addition, the use of DGM(1,1) was also proposed to compare the prediction accuracy between these models. This research study applied the GM(1,1) and DGM(1,1) to forecast carbon dioxide emission from 2008 to 2015 in several of ASEAN countries; Indonesia, Malaysia, Philippines, Singapore and Thailand. The results were separated into two parts which are simulative value (2008-2015) and forecast value (2016-2018) and compared between these two models. The results showed that the GM(1,1) produced accurate forecasts in 2 ASEAN countries with MAPE (Mean Average Percentage Error) value of 6.33%(Philippines) and 1.26%(Singapore). While DGM (1,1) produced accurate forecasts in 3 ASEAN countries with MAPE value of 3.55%(Indonesia), 9.22%(Thailand) and 10.92%(Malaysia). Comparing these two models, the results proved that DGM(1,1) is more accurate and showed the best prediction performance. From the result obtained, Singapore was highlighted due to the high carbon dioxide emission compared to other ASEAN countries.

Keywords: Forecasting; Grey theory; carbon dioxide

1. Introduction
The forecasting technique is commonly used in various sectors. Basically, forecasting is a technique of predicting the future consequence of the events by examining the past data. Through this technique, it can be said as decision-making tool that help any sector activities coping with the condition of future's uncertainty impact and analyze the patterns and trends of their stability condition.

However, the past literature has discovered that some of the conventional methods in forecasting face the problems due to difficult and complex calculations for production forecasting. For example, linear regression, Markov prediction and factor analysis have been recognized as not suitable for prediction of production purpose. In order to apply the suitable forecasting technique, the new model from Grey theory has been introduced which is Grey model. The grey model proven as an effective method for prediction especially when dealing with problems of uncertainty information and small sample of data [9].

The Association of Southeast Asian Nations (ASEAN) is a region that contributes more energy resources. The energy demand has grown and expected the increasing of CO2 emissions. Indonesia, Thailand and Malaysia collectively experienced increased CO2 emissions for almost 80% between 1971 and 2009 [11]. During this period, these three countries were likely promoted their economy by exporting the industrial products.

This research aims to apply the forecasting technique by using data collection of carbon dioxide (CO2) emissions. As we know, CO2 emissions have risen rapidly and significantly cause severe impacts and consequence for human and environment. The increasing of CO2 emissions requires planning and decision making for evaluation in order to obtain optimal result. However, obtaining optimal result is quite difficult because there is several uncertain information. For example, the weather condition and natural environment factors can be the factors of the uncertain information obtained.

2. Literature Review

2.1. Grey Model

Grey forecasting model is the main system of grey theory, which was formally named as grey model or abbreviated as GM (1,1) model. The GM type (1,1) is commonly used and proved the efficiency in terms of calculation rather than other grey theories. This type of model defined as Grey model First Order One Variable in the form of differential equation. This model focuses on solving problems where small data sample and uncertain information are used [9]. Another study determined that these characteristics produce an accurate forecasting model without using the statistical distribution of data [11].

Generally, GM (1,1) model is one the most widely used method in the grey system. This model proved that it has been successfully employed in various fields and determined better results in recent years. For example, the researchers proposed a grey forecasting method based on a data grouping approach to forecast quarterly hydropower production of China[10]. Other than that, on previous researches, grey forecasting model have been applied in electricity forecasting, fisheries production and agricultural sector.

GM(1,1) model applied to forecast China’s energy production and consumption [9]. Other researcher predicted the flicker severity level linked with utilizing a large electric arc furnace load [5]. They obtained the high prediction accuracy in annual net income forecasting for local household in China [7]. Also, they analyzed the prediction of income of the mobile communication service industry [10]. It successfully obtains a high accuracy rate and able to reduce the effect of shock disturbance. These past studies shows that GM(1,1) is able to predict the real world problems for future strategic planning.

3. Methodology

3.1. Research Data

The data in this study is the amount of carbon dioxide emission released from 2008 to 2015 in several ASEAN countries: Indonesia, Malaysia, Philippines, Singapore and Thailand.

3.2. Grey Model, GM(1,1)

This type of model defined as Grey model First Order One Variable in the form of differential equation. The first order differential equation of GM(1,1) model is

$$\frac{dX^{(1)}}{dt} + aX^{(1)} = b. \tag{1}$$

Suppose

$$X^{(0)} = (\mathrm{P}^{(0)}(1), \mathrm{P}^{(0)}(2), \dots, \mathrm{P}^{(0)}(n)) \tag{2}$$

as a primitive sequence.

In grey system, the model constructed by applying one-order accumulated generating operation (AGO) to generate new sequence X from primitive sequence

Let sequence

$$X^{(1)} = (\mathrm{P}^{(1)}(1), \mathrm{P}^{(1)}(2), \ldots, \mathrm{P}^{(1)}(n)) \tag{3}$$

Where $X^{(1)} = (\sum_{t=1}^{1} \mathrm{P}^{(0)}(\mathrm{t}), \sum_{t=1}^{2} \mathrm{P}^{(0)}(\mathrm{t}), \ldots, \sum_{t=1}^{n} \mathrm{P}^{(0)}(\mathrm{t}))$

Furthermore, construct the accumulated matrix ***B*** and constant vector ***Y_N*** as given formula below,

$$B = \begin{bmatrix} -\frac{1}{2}\left[\mathrm{P}^{(1)}(1) + \mathrm{P}^{(1)}(2)\right] & 1 \\ -\frac{1}{2}\left[\mathrm{P}^{(1)}(2) + \mathrm{P}^{(1)}(3)\right] & 1 \\ \vdots & \vdots \\ -\frac{1}{2}\left[\mathrm{P}^{(1)}(n-1) + \mathrm{P}^{(1)}(n)\right] & 1 \end{bmatrix}, Y_N = \left[\mathrm{P}^{(0)}(2), \mathrm{P}^{(0)}(3), \ldots, \mathrm{P}^{(0)}(n)\right]^T$$

By using Cramer's method, the coefficient of a and b become

$$\begin{bmatrix} a \\ b \end{bmatrix} = (B^T.B)^{-1}B^T.Y_N \tag{4}$$

Besides, generate the $\hat{P}(t+1)$ by substituting the coefficient of *a* and *b* the equation

$$\hat{x}(k+1) = \left(P^{(0)}(1) - \frac{b}{a}\right)e^{-at} + \frac{b}{a}; t = 1,2, \ldots, n \tag{5}$$

is called the time response of GM(1,1).

To obtain the forecast value, let

$$\hat{X}^{(1)} = (\widehat{\mathrm{P}}^{(0)}(1), \widehat{\mathrm{P}}^{(0)}(2), \ldots, \widehat{\mathrm{P}}^{(0)}(n+1)) \tag{6}$$

where $\widehat{\mathrm{P}}^{(0)}(t+1) = \widehat{\mathrm{P}}^{(1)}(n+1) - \widehat{\mathrm{P}}^{(1)}(t); t = 1,2, \ldots, n$

3.3 Discrete Grey Model, DGM(1,1)

The definition of discrete grey model DGM(1,1) is described as given below:

The equation

$$x^{(1)}(k+1) = \beta_1 x^{(1)}(k) + \beta_2 \quad (7)$$

is called discrete grey model abbreviated as DGM(1,1)

Suppose

$$X^{(0)} = (\mathrm{x}^{(0)}(1), \mathrm{x}^{(0)}(2), \dots, \mathrm{x}^{(0)}(n)) \quad (8)$$

is a nonnegative series and the first-order accumulative generating operator (1-AGO) series is

$$X^{(1)} = (\mathrm{x}^{(1)}(1), \mathrm{x}^{(1)}(2), \dots, \mathrm{x}^{(1)}(n)) \quad (9)$$

where $\mathrm{x}^{(1)}(k) = \sum_{t=1}^{1} x^{(0)}(t), k = 1,2,\dots,n$

From equation (1), the system parameters could be estimated by using Cramer's method, which are

$$(\beta_1, \beta_2)^T = (B^T.B)^{-1}B^T.Y_N \quad (10)$$

where $$B = \begin{bmatrix} \mathrm{x}^{(1)}(1) & 1 \\ \mathrm{x}^{(1)}(2) & 1 \\ \vdots & \vdots \\ \mathrm{x}^{(1)}(\mathrm{n}-1) & 1 \end{bmatrix}, Y_N = \left[\mathrm{x}^{(1)}(2), \mathrm{x}^{(1)}(3), \dots, \mathrm{x}^{(1)}(n)\right]^T \quad (11)$$

Then, the recursive function of (1) could be written as follows, given that the initial condition is

$$\hat{\mathrm{x}}^{(1)}(\mathrm{k}+1) = \beta_1^k \left(x^{(0)}(1) - \frac{\beta_2}{1-\beta_1}\right) + \frac{\beta_2}{1-\beta_1}, k = 1,2,\dots,n-1$$

To obtain the forecast value is

$$\hat{\mathrm{x}}^{(0)}(\mathrm{k}+1) = \alpha^{(1)}\hat{x}^{(1)}(k+1) = \hat{\mathrm{x}}^{(1)}(\mathrm{k}+1) - \hat{\mathrm{x}}^{(1)}(\mathrm{k}), k = 1,2,\dots,n$$

4. Results and discussion

4.1. Simulative values

Starting from 2008 to 2015, the data were used to obtain the simulative values by using GM(1,1) and DGM(1,1). From the result obtained, GM(1,1) performs better than DGM(1,1). The result of simulative values is shown as in tables below.

Table 1: The simulative value and MAPE value of GM(1,1) and DGM(1,1) from 2008 to 2015 in Indonesia

Year		Actual data	GM (1,1) model		DGM (1,1) model	
			Simulative value $\hat{x}^{(0)}(k)$	Error (%)	Simulative value $\hat{x}^{(0)}(k)$	Error (%)
2008	$x^{(0)}(1)$	1.602	1.602		1.602	
2009	$x^{(0)}(2)$	1.653	1.746	5.59	1.747	5.63
2010	$x^{(0)}(3)$	1.724	1.778	3.13	1.778	3.14

2011	$x^{(0)}(4)$	1.960	1.811	7.61	1.811	7.63
2012	$x^{(0)}(5)$	1.959	1.845	5.84	1.844	5.89
2013	$x^{(0)}(6)$	1.804	1.879	4.14	1.878	4.06
2014	$x^{(0)}(7)$	1.921	1.914	0.36	1.912	0.47
2015	$x^{(0)}(8)$	1.900	1.949	2.62	1.947	2.48
MAPE (%)				4.1833		4.1834

Table 2: The simulative value and MAPE value of GM(1,1) and DGM(1,1) from 2008 to 2015 in Malaysia

Year		Actual data	GM (1,1) model		DGM (1,1) model	
			Simulative value $\hat{x}^{(0)}(k)$	Error (%)	Simulative value $\hat{x}^{(0)}(k)$	Error (%)
2008	$x^{(0)}(1)$	7.385	7.385		7.385	
2009	$x^{(0)}(2)$	6.527	6.664	2.10	6.666	2.13
2010	$x^{(0)}(3)$	7.059	6.840	3.10	6.842	3.07
2011	$x^{(0)}(4)$	7.039	7.021	0.26	7.022	0.24
2012	$x^{(0)}(5)$	6.993	7.206	3.05	7.207	3.06
2013	$x^{(0)}(6)$	7.459	7.397	0.84	7.398	0.83
2014	$x^{(0)}(7)$	7.757	7.592	2.12	7.593	2.12
2015	$x^{(0)}(8)$	7.682	7.793	1.44	7.793	1.44
MAPE (%)				1.8438		1.8420

Table 3: The simulative value and MAPE value of GM(1,1) and DGM(1,1) from 2008 to 2015 in Philippines

Year		Actual data	GM (1,1) model		DGM (1,1) model	
			Simulative value $\hat{x}^{(0)}(k)$	Error (%)	Simulative value $\hat{x}^{(0)}(k)$	Error (%)
2008	$x^{(0)}(1)$	0.844	0.844		0.844	
2009	$x^{(0)}(2)$	0.839	0.822	1.95	0.823	1.90
2010	$x^{(0)}(3)$	0.889	0.862	3.08	0.862	3.04
2011	$x^{(0)}(4)$	0.884	0.903	2.21	0.904	2.24

2012	$x^{(0)}(5)$	0.906	0.947	4.52	0.947	4.54
2013	$x^{(0)}(6)$	0.989	0.992	0.31	0.992	0.33
2014	$x^{(0)}(7)$	1.039	1.040	0.11	1.040	0.12
2015	$x^{(0)}(8)$	1.113	1.090	2.10	1.090	2.09
MAPE (%)				2.0379		2.0388

Table 4: The simulative value and MAPE value of GM(1,1) and DGM(1,1) from 2008 to 2015 in Singapore

Year		Actual data	GM (1,1) model		DGM (1,1) model	
			Simulative value $\hat{x}^{(0)}(k)$	Error (%)	Simulative value $\hat{x}^{(0)}(k)$	Error (%)
2008	$x^{(0)}(1)$	7.939	7.939		7.939	
2009	$x^{(0)}(2)$	7.785	8.175	5.00	8.175	5.00
2010	$x^{(0)}(3)$	8.354	8.187	2.00	8.186	2.00
2011	$x^{(0)}(4)$	8.637	8.199	5.06	8.198	5.08
2012	$x^{(0)}(5)$	8.224	8.212	0.15	8.209	0.18
2013	$x^{(0)}(6)$	8.135	8.224	1.10	8.221	1.06
2014	$x^{(0)}(7)$	8.128	8.237	1.33	8.233	1.28
2015	$x^{(0)}(8)$	8.220	8.249	0.35	8.244	0.29
MAPE (%)				2.1285		2.1419

Table 5: The simulative value and MAPE value of GM(1,1) and DGM(1,1) from 2008 to 2015 in Thailand

Year		Actual data	GM (1,1) model		DGM (1,1) model	
			Simulative value $\hat{x}^{(0)}(k)$	Error (%)	Simulative value $\hat{x}^{(0)}(k)$	Error (%)
2008	$x^{(0)}(1)$	3.432	3.432		3.432	
2009	$x^{(0)}(2)$	3.310	3.380	2.14	3.381	2.16
2010	$x^{(0)}(3)$	3.505	3.463	1.21	3.463	1.19
2011	$x^{(0)}(4)$	3.475	3.547	2.08	3.548	2.09
2012	$x^{(0)}(5)$	3.716	3.634	2.22	3.634	2.21
2013	$x^{(0)}(6)$	3.852	3.722	3.36	3.722	3.36

2014	$x^{(0)}(7)$	3.779	3.813	0.88	3.813	0.88
2015	$x^{(0)}(8)$	3.829	3.906	2.02	3.906	2.01
MAPE (%)				1.98541		1.98545

4.2. Forecast values

Starting from 2016 to 2018, the data were used to obtain forecast values by using GM(1,1) and DGM(1,1). From the result obtained, DGM(1,1) performs better than GM(1,1). The result of forecast values is shown as in tables below.

Table 6. The forecast values and MAPE value of GM(1,1) and DGM(1,1) from 2016 to 2018 in Indonesia.

Year		Forecast value of GM(1,1) model $\hat{x}^{(0)}(k)$	Error (%)	Forecast value of DGM(1,1) model $\hat{x}^{(0)}(k)$	Error(%)
2016	$x^{(0)}(9)$	1.986	4.92	1.982	4.75
2017	$x^{(0)}(10)$	2.022	0.44	2.019	0.25
2018	$x^{(0)}(11)$	2.060	5.44	2.056	5.64
MAPE (%)			3.60		3.55

Table 7. The forecast values and MAPE value of GM(1,1) and DGM(1,1) from 2016 to 2018 in Malaysia.

Year		Forecast value of GM(1,1) model $\hat{x}^{(0)}(k)$	Error (%)	Forecast value of DGM(1,1) model $\hat{x}^{(0)}(k)$	Error(%)
2016	$x^{(0)}(9)$	7.999	7.35	7.998	7.34
2017	$x^{(0)}(10)$	8.210	14.58	8.209	14.56
2018	$x^{(0)}(11)$	8.427	10.88	8.426	10.86
MAPE (%)			10.94		10.92

Table 8. The forecast values and MAPE value of GM(1,1) and DGM(1,1) from 2016 to 2018 in Philippines.

Year		Forecast value of GM(1,1) model $\hat{x}^{(0)}(k)$	Error (%)	Forecast value of DGM(1,1) model $\hat{x}^{(0)}(k)$	Error(%)
2016	$x^{(0)}(9)$	1.142	5.22	1.984	5.22
2017	$x^{(0)}(10)$	1.197	7.83	2.020	7.84
2018	$x^{(0)}(11)$	1.254	5.94	2.057	5.95
MAPE (%)			6.33		6.34

Table 9. The forecast values and MAPE value of GM(1,1) and DGM(1,1) from 2016 to 2018 in Singapore.

Year		Forecast value of GM(1,1) model $\hat{x}^{(0)}(k)$	Error (%)	Forecast value of DGM(1,1) model $\hat{x}^{(0)}(k)$	Error(%)
2016	$x^{(0)}(9)$	8.262	0.36	8.256	0.29
2017	$x^{(0)}(10)$	8.274	2.09	8.267	2.17
2018	$x^{(0)}(11)$	8.287	1.34	8.279	1.43
MAPE (%)			1.26		1.30

Table 10. The forecast values and MAPE value of GM(1,1) and DGM(1,1) from 2016 to 2018 in Thailand.

Year		Forecast value of GM(1,1) model $\hat{x}^{(0)}(k)$	Error (%)	Forecast value of DGM(1,1) model $\hat{x}^{(0)}(k)$	Error(%)
2016	$x^{(0)}(9)$	4.001	5.84	4.001	5.83
2017	$x^{(0)}(10)$	4.099	8.82	4.098	8.80
2018	$x^{(0)}(11)$	4.198	13.04	4.197	13.02
MAPE (%)			9.24		9.22

Based on the tables above, GM(1,1) produced accurate forecasts in 2 ASEAN countries with MAPE (Mean Average Percentage Error) value of 6.33%(Philippines) and 1.26%(Singapore). While DGM (1,1) produced accurate forecasts in 3 ASEAN countries with MAPE value of 3.55%(Indonesia), 9.22%(Thailand) and 10.92%(Malaysia).

4.3 Graph for GM(1,1) and DGM(1,1)

Figure 1. The output of simulative values and forecast values of GM(1,1) and DGM(1,1) in Indonesia.

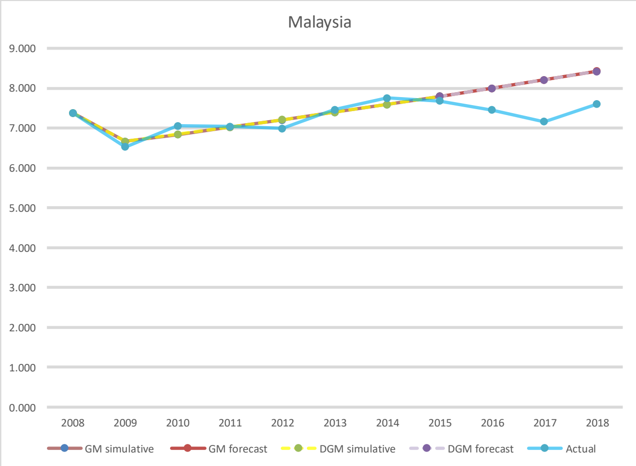

Figure 2. The output of simulative values and forecast values of GM(1,1) and DGM(1,1) in Malaysia

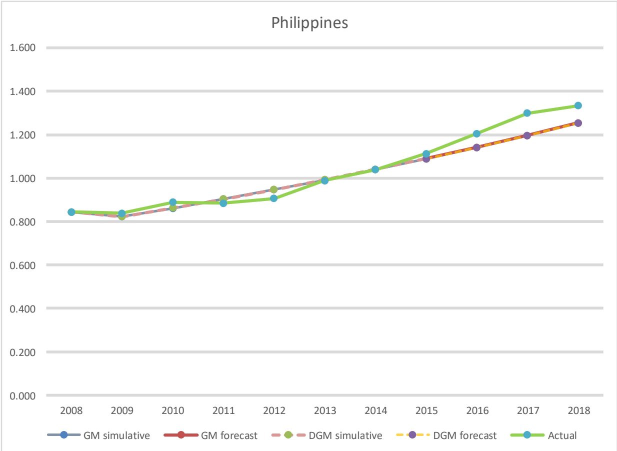

Figure 3. The output of simulative values and forecast values of GM(1,1) and DGM(1,1) in Philippines

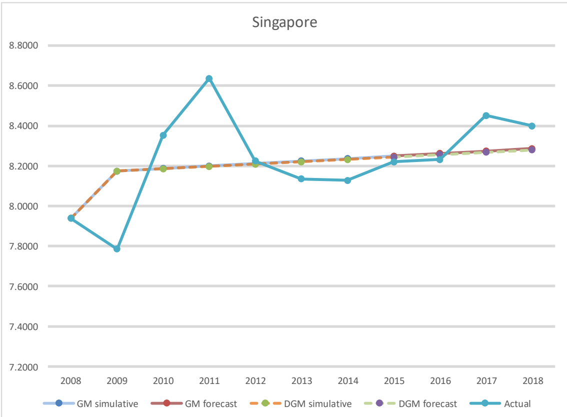

Figure 4. The output of simulative values and forecast values of GM(1,1) and DGM(1,1) in Singapore

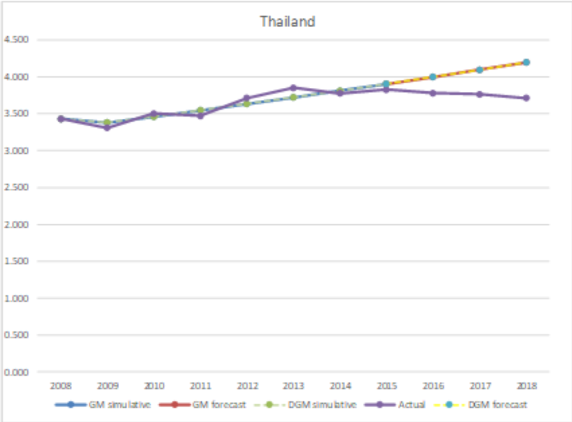

Figure 5. The output of simulative values and forecast values of GM(1,1) and DGM(1,1) in Thailand

Conclusion

The GM(1,1) model and DGM(1,1) model has been applied to forecast CO2 emission in Indonesia, Malaysia, Philippines, Singapore and Thailand for 2008 to 2018. The result shows that:

(1) By applying GM(1,1) model, it proves as the best forecasting performance in four countries: Indonesia, Philippines, Singapore and Thailand.

(2) Most of the results shows the small difference of residual error between GM(1,1) and DGM(1,1).

Since the grey forecasting model has been successfully applied in many fields, it also shows unsatisfied result in certain times. However, in this study has proposed a new grey model, DGM(1,1) model to enhance the prediction accuracy. The relationship of GM(1,1) model and DGM(1,1) has been analyzed. From the methodology, it shows that there exists a clear relationship between these models. The method of estimating the parameters is same but the pure index sequence is different.

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