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Dispersion of Solute in Bingham Fluid Model of Blood Flow through a Stenosed Artery with the Effect of Varying Viscosity

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Abstract
The effects of varying viscosity on solute dispersion are investigated in blood flow through a stenosed artery. Blood is considered as Bingham fluid model. The Bingham fluid model in a circular straight pipe is formulated mathematically. The momentum and constitutive equations are solved to determine the velocity of the Bingham fluid model. To determine the concentration of solute, dispersion function and mean concentration, the Generalized Dispersion Model (GDM) is used to solve unsteady convective-diffusion. Bingham fluid with varying viscosity and yield stress affects blood velocity and diverts the solute dispersion process. The effect of varying viscosity on blood velocity, solute concentration, dispersion function and mean concentration have been graphically discussed. When the yield stress is increased without the presence of varying viscosity, the blood velocity decreases further. When the height of the stenosis rises, the velocity drops significantly, and when varying viscosity is present, the velocity drops only slightly. The effect of varying viscosity and stenosis height on blood flow were to increase the rate of blood flow in the artery. Increased in yield stress tends to raise the concentration of solute at the core region, which then increases the mean solute concentration. In terms of dispersion function, the solute of dispersion function increases when the stenosis height increases in the plug core region near the center of artery and lower in the outer region. As the time increases, the dispersion function decreases slightly. The presence of varying viscosity causes the mean concentration to be disturbed. This study can help to predict the transportation of the drug to the targeted stenosed region where an abnormal plaque has formed.

Keywords: Varying viscosity; Electric field; Bingham fluid; Stenosed artery; Solute dispersion

1. Introduction

Viscosity is a fluid property that refers to the internal friction between adjacent fluid layers as they slide past one another. The chemical nature of the fluid and whether it is homogeneous or heterogeneous in composition determine the interactions between fluid layers [1]. Blood is non-Newtonian, unlike water, because its viscosity increases at low flow velocities. The viscosity of plasma is increased further when formed elements (red cells, white cells, and platelets) are added. Increased viscosity increases blood flow resistance, increasing the heart's workload and impairing organ perfusion.

The presence of stenosis inside an arterial blood vessel changes its flow pattern and hemodynamics conditions, according to Cooper *et al*. [2] and its continuous growth inside the blood vessel increases the risk of heart failure. Many researchers have studied the stenosis effects on blood flow while taking into account different shapes and sizes of stenosis. Mekheimer and Kot [3] proposed a mathematical model for the geometry of stenosis by specifying the shape and tapering parameters and determined that impedance lowers as the stenosis length and shape parameter values increase, while it increases when the stenosis size grows. Caro *et al.* [4] stated that when blood flows through larger arteries at high shear rates, it is Newtonian (homogenous/inhomogeneous), and when it flows through smaller arteries at low shear rates, it is non-Newtonian. Blood viscosity varies in the radial direction in bigger arteries stated by Roy *et al.* [5]. This occurs because of the inhomogeneous condition of blood causing suspended blood particles to migrate in a radial direction stated by Lighthill [6]. Blood inhomogeneity, and hence nonuniform viscosity, can drastically alter the rheology of blood.

To author's knowledge, there is no attempt to observe the effect of varying viscosity in Bingham fluid for dispersion of solutes in blood flow through tapered stenosed artery. The aim of this study is to formulate a Bingham fluid model and mathematically investigate the dispersion of solutes in the bloodstream with the effect of varying viscosity through a tapered artery.

2 Mathematical Formulation

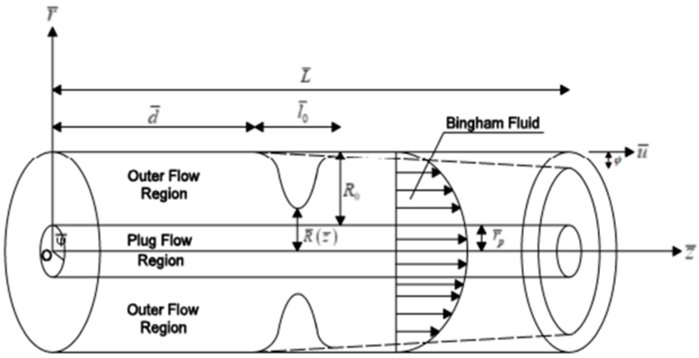

Figure 1: The geometry of pipe (artery) of Bingham fluid

The geometry of fluid flow with the effect of electric field is shown in Figure 1, where $\bar{L}$ is the length of artery, $\bar{l}_0$ is length of stenosis, $\bar{d}$ is semi-width of the cross-sectional artery, $\bar{R}_0$ is constant radius of artery, $\bar{R}(z)$ is stenosed radius and θ is angle of tapering.

2.1 Governing Equation

The constitutive equation of Bingham fluid model as follows:

$$f(\bar{\tau}) = \begin{cases} \dfrac{\bar{\tau} - \bar{\tau}_y}{\bar{\mu}_B}, & \text{if } \bar{\tau} \geq \bar{\tau}_y, \\ 0, & \text{if } \bar{\tau} < \bar{\tau}_y, \end{cases} \tag{1}$$

where $\bar{\mu}_B$ is the Bingham varying viscosity, $\bar{\tau}$ is the shear stress $f(\bar{\tau}) = d\bar{u} / d\bar{r}$ where $\bar{r}$ is the radius of the plug region and $\bar{u}$ is the velocity in radial direction. $\bar{\tau}_y$ is the yield stress. From Eq. (1), the constitutive equation for Bingham fluid for $\bar{\tau} \geq \bar{\tau}_y$ is shown as follows:

$$\bar{\tau} = \left(-\bar{\mu}_B \frac{d\bar{u}}{d\bar{r}} \right) + \bar{\tau}_y, \tag{2}$$

where $\bar{\mu}_B$ signify the radially varying viscosity in the core region, which is $\bar{\mu}_B(\bar{r}) = \bar{\mu}_0 \left\{ 1 + K - K\left(\frac{\bar{r}}{\bar{R}_0} \right)^m \right\}$ [7]. $\bar{\mu}_0$ where the viscosity is taken to be constant represents the coefficient of viscosity of plasma, $\bar{\tau}_y$ is the yield stress, *n* is the Bingham fluid parameter, m is the viscosity index appearing in the expression for variable viscosity and K is the constant viscosity relation.

The boundary condition of Eq. (2) for the expression for the shear stress is given by

$$\bar{\tau} \text{ is finite at } \bar{r} = 0. \tag{3}$$

Then, the momentum equation with electric field is defined as follows [2]

$$\frac{d\bar{p}}{d\bar{z}} = -\frac{1}{\bar{r}} \frac{d}{d\bar{r}} (\bar{r}\bar{\tau}) + \rho_c E_t, \tag{4}$$

where $d\bar{p} / d\bar{z}$ is the axial pressure gradient, $\bar{p}$ is the fluid pressure, $\bar{z}$ is the axial coordinate for a circular piper, ρ_c is the fluid density and E_t is the electric field in non-axial coordinate.

The boundary condition of Eq. (4) for the expression for the shear stress is given by:

$$\bar{u} = u_{\bar{s}} \text{ at } \bar{r} = \bar{R}(\bar{z}). \tag{5}$$

where $\bar{R}(\bar{z})$ is given by

$$\bar{R}(\bar{z}) = \begin{cases} R_0 - w(\bar{z} + \bar{d}) - \dfrac{h\cos\theta}{2}\left(1 + \cos\dfrac{\pi\bar{z}}{z_0}\right) & |\bar{z}| \le z_0, \\ R_0 - w(\bar{z} + \bar{d}) & |\bar{z}| \ge z_0, \end{cases} \tag{6}$$

where R_0 is the non-stenotic radius of the straight artery, θ is the tapering angel, $h\cos\theta$ is the height of the stenosis, $\bar{d}$ is the circle radius, z_0 is the half-length stenosis, $\bar{z}$ is the length of artery into stenosis and $w(= \tan\theta)$is the tapered vessel slope.
A two-dimensional unsteady convective-diffusion equation is as follows

$$\frac{\partial\bar{C}}{\partial\bar{t}} = -\bar{u}_z \frac{\partial\bar{C}}{\partial\bar{z}} + \bar{D}_m\left(\frac{1}{\bar{r}}\frac{\partial}{\partial\bar{r}}\left(\bar{r}\frac{\partial\bar{C}}{\partial\bar{r}}\right) + \frac{\partial^2\bar{C}}{\partial\bar{z}^2}\right), \tag{7}$$

where $\bar{t}$ is the time, $\bar{u}_{\bar{z}}$ is the velocity in the $\bar{z}$ direction, $\bar{z}$ is the axial coordinate, $\bar{r}$ is the radial coordinate of a circular pipe, and $\bar{D}_m$ is the molecular diffusivity.
The subscript $\bar{z}$ in $\bar{u}$ in Eq. (7) is taken out of the equation and becomes

$$\frac{\partial\bar{C}}{\partial\bar{t}} + \bar{u}\frac{\partial\bar{C}}{\partial\bar{z}} = \bar{D}_m\left(\bar{l}^2 + \frac{\partial^2}{\partial\bar{z}^2}\right)\bar{C}. \tag{8}$$

The initial and boundary condition of solute concentration [8] are given by:

$$\bar{C}(\bar{r}, \bar{z}, 0) = \begin{cases} \bar{C}_0 \;\; \text{if } |\bar{z}| \le \dfrac{\bar{z}_s}{2}, \\ 0 \;\; \text{if } |\bar{z}| > \dfrac{\bar{z}_s}{2}, \end{cases} \tag{9}$$

$$\bar{C}(\bar{r}, \infty, \bar{t}), \tag{10}$$

$$\frac{\partial\bar{C}}{\partial\bar{r}}(0, \bar{z}, \bar{t}) = 0. \tag{11}$$

2.2 Non-Dimensional Variables

The following is the non-dimensional variables:

$$C = \frac{\bar{C}}{\bar{C}_0}, u = \frac{\bar{u}}{\bar{u}_0}, u_m = \frac{\bar{u}_m}{\bar{u}_0}, r = \frac{\bar{r}}{\bar{R}_0}, r_p = \frac{\bar{r}_p}{\bar{R}_0}, p = \frac{\bar{p}\bar{R}_0}{\bar{\mu}\bar{u}_0},$$
$$t = \frac{\bar{D}_m\bar{t}}{\bar{R}_0^2}, \tau_y = \frac{\bar{\tau}_y R_0}{\bar{\mu}\bar{u}_0}, z = \frac{\bar{z}}{\bar{R}_0}, z_s = \frac{\bar{D}_m\bar{z}_s}{\bar{R}_0^2\bar{u}_0}, E_z = \frac{\bar{E}_z}{\bar{\varepsilon}}, \tag{12}$$

where $\bar{u}_0$ is the fluid characteristic velocity, $C, u, u_m, r, z, z_s, t, \tau$ and τ_y are the concentration of the solute, velocity of the blood flow, mean velocity, radius artery, axial distance, length of solute, time, shear stress and yield stress respectively.

2.3 Method of Solution

Using non-dimensional variables Eq. (12) in Eq. (4), yields

$$\frac{dp}{dz} = -\frac{1}{r}\frac{d}{dr}(r\tau) + \rho_c E_t, \tag{13}$$

Integrating Eq. (13) with respect to r, the expression for the shear stress is given as follows:

$$\tau = \frac{r}{2}\left(\rho_c E_t - \frac{dp}{dz}\right). \tag{14}$$

Substituting Eq. (14) into Eq. (13) and integrate the resulting equation in term of r in the outer non-plug core region, the velocity expression is obtained as

$$u(r) = u_S - \frac{1}{4\mu}\left(\rho_c E_t \varepsilon - \frac{dp}{dz}\right)\left(r^2 - R^2(z) - 2r_p r + 2r_p R(z)\right). \tag{15}$$

Evaluating $r = r_p$ in the Eq. (15), the velocity of the blood in the plug flow region is obtained as follows:

$$u(r_p) = u_S - \frac{1}{4\mu}\left(\rho_c E_t \varepsilon - \frac{dp}{dz}\right)\left(-r_p^2 - R^2(z) + 2r_p R(z)\right). \tag{16}$$

The mean velocity of the Bingham fluid is defined as [3]

$$u_m = \int_0^{2\pi}\int_0^{R(z)} \frac{urdr\theta}{rdr\theta},$$
$$u_m = \frac{2}{R_0}\int_0^{R(z)} u(r)rdr. \tag{17}$$

Substituting Eq. (15) and Eq. (16) into Eq.(17), a velocity of Bingham fluid is formed as

$$u_m = \frac{R^2(z)}{8\mu}\left(\rho_c E_t \varepsilon - \frac{dp}{dz}\right)\left(1 - \frac{4r_p}{3R(z)} + \frac{r_p^4}{3R^4(z)}\right) + u_s. \tag{18}$$

2.4 Generalized Dispersion Model

The GDM for $C_m(z_1, t)$ is given by [8]

$$\frac{\partial C_m}{\partial t}(z_1, t) = \sum_{i=1}^{\infty} K_i(t) \frac{\partial^i C_m}{\partial z_1^i}(z_1, t), \tag{19}$$

where $K_i(t)$ is the transport coefficient given by

$$K_i(t) = \frac{\partial_{i2}}{\partial t} + 2\frac{\partial f_i}{\partial r}(1, t) - 2\int_0^{R(z)} f_{i-1}(r, t)u(r)rdr, \quad \text{i} = 1,2,3,... \tag{20}$$

with the Kronecker delta δ_{ij} given by

$$\delta_{ij} = \begin{cases} 0 \text{ if } i \neq j, \\ 1 \text{ if } i \neq j. \end{cases} \tag{21}$$

From Eq. (20), the longitudinal convection coefficient is $K_i(t)$ and the longitudinal diffusion coefficient is $K_2(t)$. The effective axial diffusivity is the coefficient expresses $K_2(t)$ in the entire dispersion process in a simple diffusion process.

2.5 Method of Solution for Dispersion Function

The dispersion function $f_1(r, t)$ is given by

$$f_1(r, t) = f_{1s}(r) + f_{1t}(r, t), \tag{22}$$

where $f_{1s}(r)$ is the dispersion function in the steady state and $f_{1t}(r, t)$ is the dispersion function in the unsteady state.
The differential equation of dispersion function at the steady state in plug flow region is obtained as [8]

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f_{1s}}{\partial r}\right) - (u(r_p) - u_m) = 0 \;\; \text{if } 0 \leq r \leq r_p \tag{23}$$

and in outer flow regions obtained as [8]

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f_{1s_+}}{\partial r}\right)-(u(r_p)-u_m)=0 \text{ if } r_p \le r \le R(z). \tag{24}$$

Eq. (23) is used to solve the differential equation of dispersion function at steady state in the plug flow region and it is obtained as follows:

$$f_{1s_-}=CI+\frac{1}{16}Ar^2r_p^2-\frac{Ar^2r_p^4}{96R^2(z)}-\frac{1}{12}Ar^2r_pR(z)+\frac{1}{32}Ar^2R^2(z) \tag{25}$$

and in outer flow region,

$$\begin{aligned} f_{1s_+}&=CI-\frac{Ar^4}{64}+\frac{1}{18}Ar^3r_p+\frac{13Ar_p^4}{576}-\frac{Ar^2r_p^4}{96R^2(z)}-\frac{1}{12}Ar^2r_pR(z)\\ &+\frac{1}{32}Ar^2R^2(z)+\frac{1}{48}Ar_p^4\log\left[r\right]-\frac{1}{48}Ar_p^4\log\left[r_p\right], \end{aligned} \tag{26}$$

where $A=\rho_c E_t \varepsilon-\dfrac{dp}{dz}$ and CI is obtained

$$\begin{aligned} CI&=-\frac{Ar_p^4}{144}-\frac{Ar_p^6}{480R^2(z)}+\frac{7}{360}Ar_pR^3(z)\\ &-\frac{AR^4(z)}{96}+\frac{1}{48}Ar_p^4\log\left[r_p\right]-\frac{1}{48}Ar_p^3\log\left[R(z)\right]. \end{aligned} \tag{27}$$

The unsteady dispersion function of $f_{1t}(r,t)$ is given as

$$f_{1t}(r,t)=\sum_{m=1}^{\infty}A_m e^{-\lambda_m^2 t}J_0(\lambda_m r). \tag{28}$$

2.6 Method of Solution for Mean Concentration

Since the value of $K_3(t)$ for Newtonian fluid is negligibly small from Eq. (19), $K_3(t\to\infty)=-1/23040$, the terms $K_3(t), K_4(t), K_5(t)$ and so on are ignored by ignoring the terms involving these coefficients [9]. As a result, Eq. (21) is reduced to

$$\frac{\partial C_m(r,t)}{\partial t}=K_2(t)\frac{\partial^2 C_m(z_1,t)}{\partial z_1^2}. \tag{29}$$

The mean concentration of solute in Fourier transform is given by

$$C_m(\phi,t)=c_1(\phi). \tag{30}$$

The boundary conditions of Fourier transform for Eq. (30) is given by [8]

$$c_1(\phi)=\begin{cases}1 \text{ if } |\phi|\le z_s/2,\\ 0 \text{ if } |\phi|> z_s/2\end{cases} \tag{31}$$

and

$$\xi(t)=\int_0^t K_2(t)dt. \tag{32}$$

Using Inverse Fourier Transform (IFT) in Eq. (30), the mean concentration of solute $C_m(z_1,t)$ is obtained as follows:

$$C_m(z_1,t)=\frac{1}{2}\left[\operatorname{erf}\left(\frac{z_s/2-z_1}{2\sqrt{\xi}}\right)+\operatorname{erf}\left(\frac{z_s/2+z_1}{2\sqrt{\xi}}\right)\right], \tag{33}$$

where erf is the error function.

3 Results and Discussions

3.1 The Effect of Varying Viscosity

Figure 2 shows the variation of velocity *u* for different values of yield stress, τ_y in the blood flow through non tapered arteries without the effect of varying viscosity, μ_B with $E_t = 1, F = 1, u_s = 0, z = 1, \mu = 1,$ and $R(z) = 0.98.$ The velocity in Figure 2 indicates that the velocity reduces when the yield stress value increases from 0.02 to 0.10 due to the presence of stenosis that inhibits blood flow in the artery. The velocity is higher because constant viscosity enhanced the velocity. Yield stress acts as the way of the way of the flow of human blood and important in viscoelasticity change.

Figure 3 shows the variation of velocity *u* for different values of yield stress, τ_y in the blood flow through non tapered arteries with the effect of varying viscosity, μ_B with $E_t = 1, F = 1, u_s = 0, z = 1, K = 0.2, n = 2$ and $R(z) = 0.98.$ Figure 3 exhibits that the yield stress in the blood flow increases, the velocity also decreases. The radius of the artery in Figure 3 becomes smaller than the radius of the artery in Figure 2. This shows that velocity of the blood flow in Figure 3 is enhanced by varying viscosity.

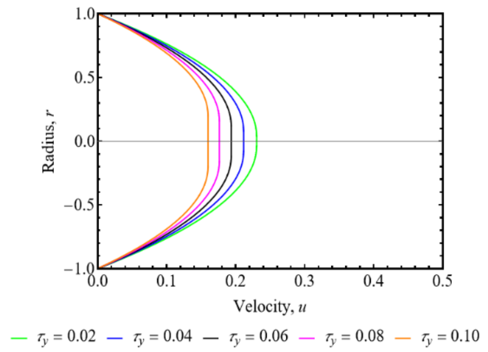

Figure 2: Variation of velocity for different values of yield stress, τ_y without the effect of varying viscosity, μ_B $(E_t = 1, F = 1, u_s = 0, z = 1, \mu = 1, R(z) = 0.98)$

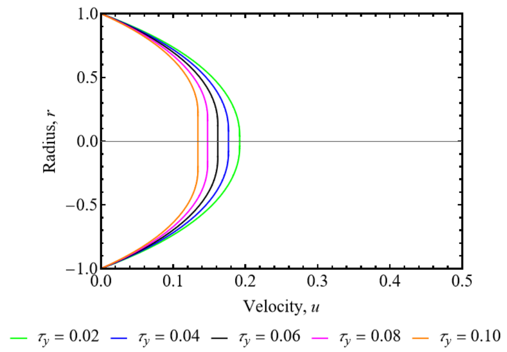

Figure 3: Variation of velocity for different values of yield stress, τ_y with the effect of varying viscosity, μ_B $(E_t = 1, F = 1, u_s = 0, z = 1, K = 0.2, n = 2, R(z) = 0.98)$

3.2 Steady Dispersion of Solute, f_{1s}

Figure 4 shows the variation of steady dispersion function, f_{1s} for different values of yield stress, τ_y in the blood flow through non tapered arteries without the presence of varying viscosity, μ_B with $F = 1, u_s = 0, z = 1, \mu = 1,$ and $R(z) = 0.98.$The dispersion function moves further than in Figure 4 because the viscosity is assumed to be constant $(\mu = 1).$ Bingham fluid is a non-Newtonian fluid with a yield stress, making it suitable for narrow arteries. Yield stress increases, the steady dispersion function decreases near the wall and opposite behaviour at the center of artery.

Figure 5 shows the variation of steady dispersion function, f_{1s} for different values of yield stress, τ_y in the blood flow through non tapered arteries with the presence of varying viscosity, μ_B with $F = 1, u_s = 0, z = 1, K = 0.2, n = 2,$ and $R(z) = 0.98.$The steady dispersion function decreases when the value of varying viscosity is present and the dispersion function in the whole region of the artery decreases slowly.

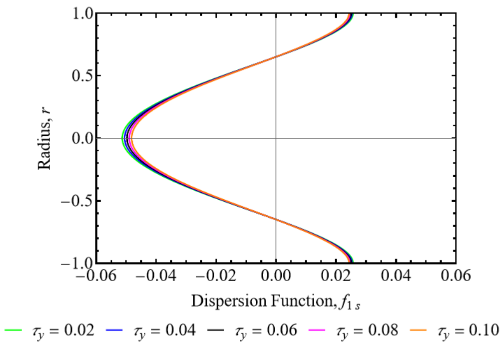

Figure 4: Variation of steady dispersion, f_{1s} for different values of yield stress, τ_y in the blood flow without the presence of varying viscosity, μ_B $(F = 1, u_s = 0, z = 1, \mu = 1, R(z) = 0.98)$

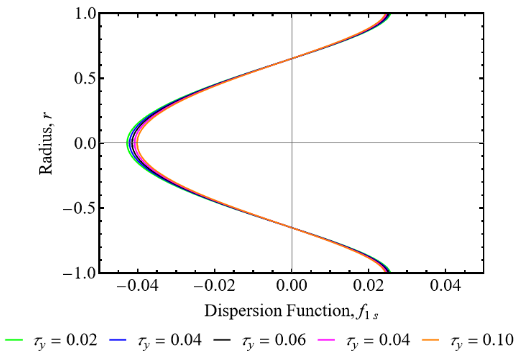

Figure 5: Variation of steady dispersion, f_{1s} for different values of yield stress, τ_y in the blood flow with the presence of varying viscosity, μ_B $(F = 1, u_s = 0, z = 1, K = 0.2, n = 2, R(z) = 0.98)$

3.3 Unsteady Dispersion of Solute, f_{1t}

Figure 6 represents the variation of unsteady dispersion function, f_{1t} for different values of time, *t* in the blood flow through non tapered arteries without the presence of varying viscosity, μ_B with $F = 1, u_s = 0, z = 1, \mu = 1,$ and $R(z) = 0.98$. The dispersion function decreases when the time increases. The unsteady dispersion function in Figure 6 is smaller than in Figure 7 because of the presence of the varying viscosity.

Figure 7 represents the variation of unsteady dispersion function, f_{1t} for different values of time, *t* in the blood flow through non tapered arteries with the presence of varying viscosity, μ_B with $F = 1, u_s = 0, z = 1, K = 0.2, n = 2,$ and $R(z) = 1$. In time $t = 0$, the unsteady dispersion function shows the maximum result and as time increases, the unsteady dispersion function decreases near to zero for both figure. The unsteady dispersion in Figure 7 is higher than Figure 6 because of the presence of varying viscosity in the blood flow.

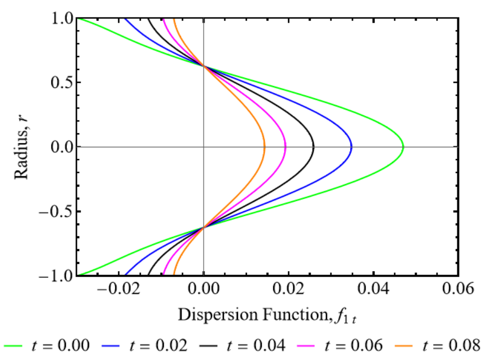

Figure 6: Variation of unsteady dispersion, f_{1t} for different values of time, *t* in the blood flow without the presence of varying viscosity, μ_B $(F = 1, u_s = 0, z = 1, \mu = 1, R(z) = 0.98)$

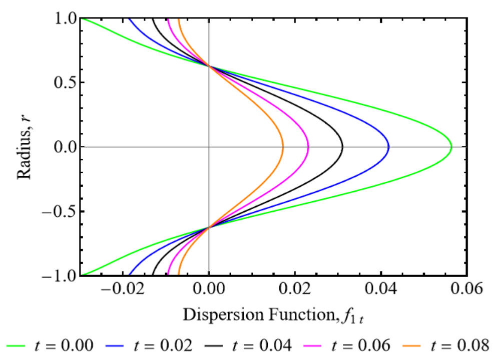

Figure 7: Variation of unsteady dispersion, f_{1t} for different values of time, *t* in the blood flow with the presence of varying viscosity, μ_B $(F = 1, u_s = 0, z = 1, K = 0.2, n = 2, R(z) = 0.98)$

3.4 Dispersion Function, f_1

Figure 8 shows the variation of dispersion function, f_1 for different values of yield stress, τ_y in the blood flow through non tapered arteries without the presence of varying viscosity, μ_B with $E_t = 1, F = 2, z = 1, z_0 = 1, \mu = 1, t = 0.05$ and $d = 1$. The dispersion function increase when the yield stress increase in the blood flow. Figure 9 shows the variation of dispersion function, f_1 for different values of yield stress, τ_y in the blood flow through non tapered arteries with the presence of varying viscosity, μ_B with $E_t = 1, F = 2, z = 1, z_0 = 1, K = 0.2, n = 2, t = 0.05$ and $d = 1$. In Figure 9, the dispersion function is smaller than in Figure 8 due to the presence of varying viscosity. The dispersion function increase when the yield stress increase and the presence of varying viscosity can distract the dispersion function of the blood flow. It shows that without the presence of varying viscosity, the medicine in the body flows faster through stenosed artery than with the presence of varying viscosity.

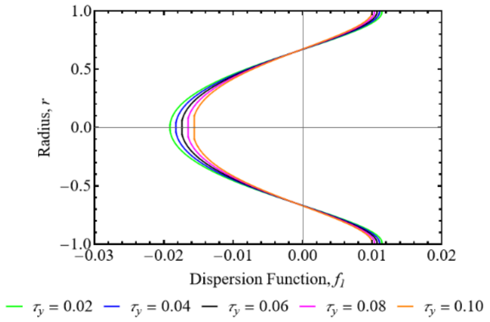

Figure 8: Variation of steady dispersion, f_1 for different values of yield stress, τ_y in the blood flow without the presence of varying viscosity, μ_B $(E_t = 1, F = 2, z = 1, z_0 = 1, \mu = 1, t = 0.05, d = 1)$

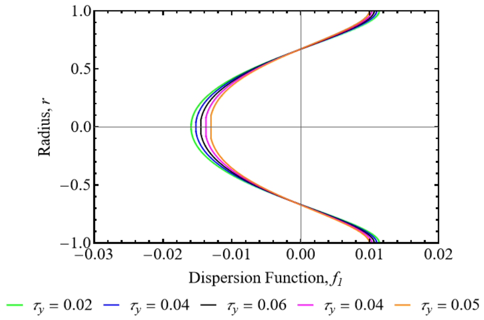

Figure 9: Variation of steady dispersion, f_1 for different values of yield stress, τ_y in the blood flow with the presence of varying viscosity, μ_B $(E_t = 1, F = 2, z = 1, z_0 = 1, K = 0.2, n = 2, t = 0.05, d = 1)$

3.5 Mean Concentration of the Solute, C_m

Figure 10 shows variation of mean concentration, C_m with different values of yield stress, τ_y with $E_t = 3, F = 1, Pe = 1, Z = 4, z_s = 1.4,$ and $R(z) = 0.98$. Increases in yield stress heighten the solute concentration at the core, which increases the mean solute concentration. It shows that the mean concentration is higher when the diameter of the artery in the circular pipe is small. The mean concentration increases rapidly around $0.15 \le t \le 0.4$ because the drug enters the body at a faster rate than it is removed. When drugs removed from the body, the mean concentration goes down slowly at $0.25 \le t \le 0.60$. It shows that when yield stress increases, the mean concentration decreases.

Figure 11 shows variation mean concentration, C_m with different value of yield stress, τ_y with the presence of varying viscosity, μ_B with $E_t = 3, F = 1, Z = 4, K = 0.2, n = 2, z_s = 1.4, R(z) = 0.98$. The presence of varying viscosity increases in the body, the concentration of blood flow through the artery decreases. As the solute concentrations increases, the heart pumps blood at a high rate. With a higher mean concentration of solute at the highest-pressure gradient, solute dispersion, such as medication, reaches its maximum efficacy.

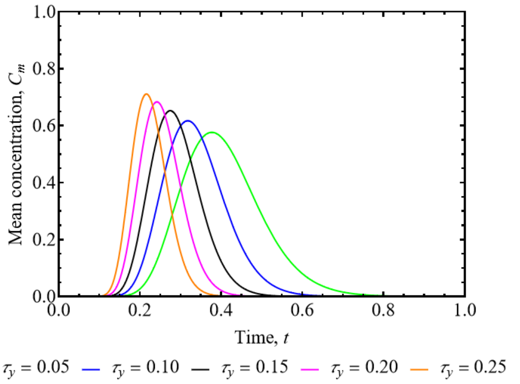

Figure 10: Variation of mean concentration, C_m for different values of yield stress, τ_y without the presence of varying viscosity, μ_B $(E_t = 3, F = 1, Pe = 1, Z = 4, z_s = 1.4, R(z) = 0.98)$

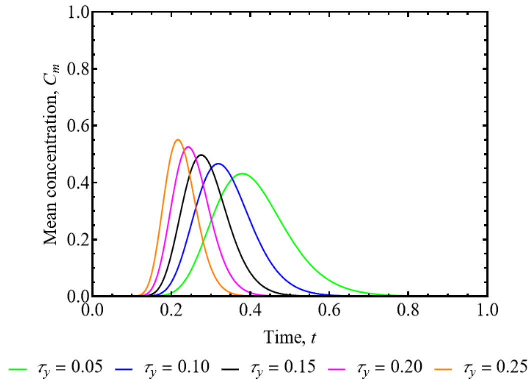

Figure 11: Variation of mean concentration, C_m for different values of yield stress, τ_y with the presence of varying viscosity, μ_B $(E_t = 3, F = 1, Pe = 1, Z = 4, K = 0.2, n = 2, z_s = 1.4, R(z) = 0.98)$

4 Conclusion

The Bingham fluid model is a mathematical model that has been developed to analyze the concept of dispersion solute in the blood flow. The effect of varying viscosity through a tapered stenosed artery on the dispersion of solute in the Bingham fluid is investigated. The effective axial diffusivity and relative axial diffusivity of solute dispersion were calculated using the generalized dispersion model. The higher the yield stress, decrease the velocity of blood and the presence of varying viscosity can disturb the velocity of the blood flow. The presence of varying viscosity in mean concentration disturbs the mean concentration of solute and the highest mean concentration exhibits the most effective concentration. *Mathematica* is used to find the solution for solute concentration, dispersion function and mean concentration in a tapered stenosed artery. Graphical results are represented using various parameter given in the *Mathematica*.

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