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Nonlinear Grey Bernoulli Model vs Grey Model For Electrical Demand Forecasting

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Abstract
Electrical demand forecasting has received much attention in recent years. Many models have been utilized to obtain better forecasting. Grey theory is a widely used approach to construct models in order to provide better forecast results. The grey model (GM (1,1)) and the nonlinear grey Bernoulli model (NGBM (1,1)) are two types of grey forecasting models in grey theory. The purpose of this study is to find out which grey forecasting model between GM(1,1) NGBM(1,1) is more accurate to obtain the forecasting result of electrical demand. The ordinary least square (OLS) method is used to obtain the developing coefficients *a* and *b*. The solving methods as Generalized Reduced Gradient (GRG) Nonlinear method have been proposed in this study to optimize the value of the power index *n* of NGBM(1,1). Mean absolute percentage error (MAPE) is used to evaluate the performance of these models. Simulative results of the proposed models are performed using different sizes of sample data from 1965 to 2019 in five countries in Southeast Asia. The acquired results classified that GM(1,1) and NGBM(1,1) have excellent MAPE using a 5-year data compared to 50-year data. Furthermore, the case study to investigate the accuracy of these models using different sizes of data revealed that the simulative values of each model approach the actual values with 5-year data flexibly. In addition, results of forecasting values are compared between GM(1,1) and NGBM(1,1) and it was discovered that NGBM(1,1) showed a superior forecasting performance compared to GM(1,1). NGBM(1,1) can aid decision-makers in developing policies and better measures in the future.
Keywords: Grey Model; Nonlinear Grey Bernoulli Model; Forecasting; Electric Demand; MAPE

1. Introduction

Forecasting is a vital part of any country's planning. Accurate forecasts contribute to the growth of any economy. Therefore, each government has established a statistics department, which is responsible for collecting and evaluating national economic data. The information is then used by policymakers to forecast future trends in a country. Forecasting demand is also significant in several other fields, such as power generation.

In this era, electricity demand forecasting is an important factor for governments to formulate energy policies and adjust industrial structures [1]. Globally, energy demand is expanding faster than renewables. Forecasting demand for power generation is essential because it allows for production planning and scheduling [2].

Traditional statistical forecasting method is used to find the forecasted value before various studies are conducted by a lot of researchers to improve the accuracy of the prediction. Nevertheless, this method is unable to generate accurate forecast with small, uncertain and inadequate data [3]. Thus, numerous of research and models have been conducted and designed to solve the problem.

Despite the fact that various grey forecasting models have been proposed and developed, the outcome of the forecasting is limited to a single forecasting value which has limited information. Several research has attempted to improve this situation by establishing a forecasting model that produces enhanced interval forecast results

2. Literature Review

2.1. Electrical Demand Forecasting

In developing countries, proper electricity demand forecasting is crucial to energy planning. Economics theories and views identify energy as one of the most important production factors in industries, and forecasting its future demand is an important part of macroplanning in industry and energy [4].

Demand forecasting should be accomplished over a broad time span for economically efficient operation and control of power systems [5]. Generally, the required demand forecasting can be categorized into short-term, medium-term and long-term forecasts.

It is important to plan for electricity supply with the minimal waste possible for any electric utility since electricity is difficult to store. An overestimation of consumption would result in overspending on idle capacity, while underestimating can lead to increased operating costs for energy providers and potential utility outages [6]. Thus, having an accurate model of electricity usage is essential to avoid costly mistakes.

2.2. Models used in demand forecasting

Over the past decade, electricity demand planning has been developed to accurately predict future demand. Electricity demand is difficult to store as its demand changes continually. Consequently, the supply must meet the demand in order to full use of electricity energy and provide equal supply and demand. Also to maximize the use of electricity.

A wide variety of models based on data mining have been proposed to forecasts future electricity demand based on time series technique. Artificial neural network (ANN) models [7], fuzzy logic approach [8], support vector regression models [9] and Grey forecasting models.

As electricity demand forecasting has developed over recent decades, artificial intelligence has become increasingly important [10]. Artificial intelligence forecasting models study from a significant amount of historical data rather than relying on the explicit relationship between electricity demand and its affecting elements [11].

2.3. Grey Forecasting Model and Electricity Demand Forecasting

Traditional forecasting models are not able to achieve accurate and effective forecasting with limited, uncertain, or incomplete data sets [3]. To solve forecast problems with incomplete and small data sets, the Grey Systems Theory is applying by developing the Gray Forecasting Model [12]. The objective of this system and its application aim to bridge the gap between social science and natural science [12]. Models built with grey theory provide better predictive results. Grey model (GM(1,1)) and nonlinear grey Bernoulli model (NGBM(1,1)) are two types of grey prediction models in grey theory.

2.3.1. Grey Model and Nonlinear Grey Bernoulli Model

Grey theory is primarily described by AGO, in which the technique is intended to reduce the randomness of raw data [13]. Based on the least-square method and the first-order linear differential equation, the grey forecasting model generates excellent forecasting results [13]. As a result, the model has been widely used in a variety of fields, especially in forecasting energy consumption.

The grey model has been studied by several researchers for improving the accuracy of forecasts. The NGBM(1,1) is a differential equation with non-linear parameters n is increasingly used. When compared to linear grey models, this nonlinear model can produce more satisfying results [14]. This is because the exponential value of n does not require a specific integer [15].

3. Methodology

3.1. Research Data

Annual electricity demand data is gathered from the World Bank's website. The historical data refers to the electricity demand in Indonesia, Malaysia, the Philippines, Thailand, and Singapore between 1965 and 2019. Data was used to construct the proposed model and for forecasting. The results obtained will be compared to obtain a good solution between the two methods.

3.2. Grey Model

The following operation describes the method used to construct the model. Suppose that the original series of data with n entries as follow

$$X^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\right), \quad x^{(0)}(t) \geq 0, \quad t = 1,2,\ldots,n \tag{1}$$

where raw matrix $X^{(0)}$ is non-negative original historical time series data.

$$X^{(1)} = \left(x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)\right) \tag{2}$$

$X^{(1)}$ is constructed by Accumulated Generating Operation (AGO) where

$$x^{(1)}(t) = \sum_{t=1}^{n} x^{(0)}(t), \qquad t = 1,2,\ldots,n \tag{3}$$

Form the Grey differential equation with one variable $x^{(0)}$ as follows

$$x^{(0)}(t) + az^{(1)}(t) = b, \tag{4}$$

where

$$z^{(1)}(t) = \frac{1}{2}\left(x^{(1)}(t) + x^{(1)}(t-1)\right), \quad t = 2,3,\ldots,n, \tag{5}$$

is the mean generation of consecutive neighbors sequence of $X^{(1)}$. The unknown a and b are parameters in Grey System theory where a is called a developing coefficient and b stands for grey input. The coefficient of t is called independent variables and $x^{(0)}(t)$ is a grey derivative which maximizes the information density for a given series to be modelled.

From (3.4) it can be rewritten into matrix form

$$\boldsymbol{Y}_N = \mathbf{B}\hat{a} \tag{6}$$

where

$$\mathbf{B} = \begin{bmatrix} -z^{(1)}(2) & \mathbf{1} \\ \vdots & \vdots \\ -z^{(1)}(n) & \mathbf{1} \end{bmatrix}, \tag{7}$$

$$\boldsymbol{Y}_N = \begin{bmatrix} x^{(0)}(2) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}, \tag{8}$$

$$\hat{a} = \begin{bmatrix} a \\ b \end{bmatrix}. \tag{9}$$

The developing coefficients a and b in (3.9) can be solved by using ordinary least square (OLS) method

$$\hat{a} = \begin{bmatrix} a \\ b \end{bmatrix} = (\mathbf{B}^{\mathrm{T}}\mathbf{B})^{-1}\mathbf{B}^{\mathrm{T}}\boldsymbol{Y}_N, \tag{10}$$

and substitute the parameter estimation vector into the first-order differential equation of GM (1,1) model in (3.11) below.

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = b \tag{11}$$

Given the initial condition $\hat{x}^{(1)}(1) = x^{(0)}(t)$, the AGO Grey forecast value, $\hat{x}^{(1)}$ can be attained by

$$\hat{x}^{(1)}(t+1) = \left(x^{(0)}(1) - \frac{b}{a}\right)e^{-at} + \frac{b}{a}, \quad t = 0,1,\ldots,n. \tag{12}$$

Then, the demand forecasted value can be restored by first-order accumulated generating operation (IAGO), define as

$$\hat{X}^{(0)}(t+1) = \hat{x}^{(1)}(t+1) - \hat{x}^{(0)}(t). \tag{13}$$

Therefore, the sequence of reduction is obtained as follow

$$\hat{X}^{(0)} = \left(\hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \dots, \hat{x}^{(0)}(n+1)\right) \tag{14}$$

where $\hat{x}^{(0)}(n+1)$ is the Grey elementary predicting value of $x(n+1)$.

3.2. Nonlinear Grey Bernoulli Model

The non-negative original historical time series data, $X^{(0)}$ and a row matrix $X^{(1)}$ are as same as (1) and (2) in GM (1,1). Bernoulli equation is introduced to replace the traditional grey differential equation in (4). The Bernoulli equation has the following form

$$\frac{d\hat{X}^{(1)}(t)}{dt} + a\hat{X}^{(1)}(t) = b\left[\hat{X}^{(1)}\right]^n, \tag{15}$$

where n is represent a real number and $n \neq 1$. The discrete form of (3.15) as follows

$$x^{(0)}(t) + az^{(1)}(t) = b\left[z^{(1)}(t)\right]^n, \quad t = 2,3,\dots,n \tag{16}$$

which called the basic Grey differential equation of NGBM (1,1) model. When $n = 0$, the equation turns to the traditional GM(1,1), (11). While, for $n = 2$, the equation turns to Grey-Verhulst equation.

In order to achieve a more accurate forecasting model, the value of power index n was optimized using Generalized Reduced Gradient (GRG) Nonlinear method. The GRG nonlinear engine is one of the solving methods beside Simplex Linear Programming and Evolutionary that available in the Microsoft Excel Solver tool.

Parameters a and b are determined by ordinary least square (OLS) method,

$$\hat{a} = \begin{bmatrix} a \\ b \end{bmatrix} = (\mathbf{B}^{\mathrm{T}}\mathbf{B})^{-1}\mathbf{B}^{\mathrm{T}}\boldsymbol{Y}_N, \tag{17}$$

where

$$\mathbf{B} = \begin{bmatrix} -z^{(1)}(2) & \left[z^{(1)}(2)\right]^{\boldsymbol{n}} \\ \vdots & \vdots \\ -z^{(1)}(m) & \left[z^{(1)}(m)\right]^{\boldsymbol{n}} \end{bmatrix}, \tag{18}$$

$$\boldsymbol{Y}_N = \begin{bmatrix} x^{(0)}(2) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}. \tag{19}$$

The corresponding particular solution of (15) together with initial condition $\hat{x}^{(1)}(1) = x^{(0)}(t)$ is

$$\hat{X}^{(1)}(t+1) = \left[\left(x^{(0)}(1)^{1-n} - \frac{b}{a}\right)e^{-a(1-n)t} + \frac{b}{a}\right]^{(1-n)^{-1}}, t = 0,1,\dots,n. \tag{20}$$

3.3. Error Analysis

Further testing is needed to get a better understanding of the difference between forecasting and actual values. To evaluate the accuracy of the model, this study uses error analysis methods, such as absolute percentage error (APE) analysis.

The APE compares the actual and forecast values to evaluate the accuracy at specific time, t. APE is defined as

$$\mathrm{APE} = \varepsilon(t) = \frac{x^{(0)}(t) - \hat{x}^{(0)}(t)}{x^{(0)}(t)} \times 100\%, \quad t = 2,3,\dots,n, \tag{3.21}$$

where $x^{(0)}(t)$ is actual value and $\hat{x}^{(0)}(t)$ is forecasted value. The total model precision can be defined by mean absolute percentage error (MAPE) as follows

$$\mathrm{MAPE} = \varepsilon(avg) = \frac{1}{n-1}\sum_{t=2}^{n}|\varepsilon(t)|, \quad t = 2,3,\dots,n. \tag{3.22}$$

The classification of the model precision based on MAPE [17] is described in Table 1.

Table 1: MAPE classification of model precision

MAPE(%)	**≤10**	**10~20**	**20~50**	**≥50**
Classification	Excellent	Good	Reasonable	Unacceptable

4. Results and discussion

4.1. The Accuracy of Models with Varying Sample Sizes

The accuracy of models depends on the amount of samples data used. Therefore, the size of samples is a crucial issue in order to obtain sufficient result. The data consists of the 5 countries in Southeast Asia which are Malaysia, Indonesia, Philippine, Thailand and Singapore. Different sample size is applied to construct different GM(1,1) and NGBM(1,1) models.

The MAPE results for electrical demand in Malaysia using GM(1,1) and NGBM(1,1) is 69.599% as the power index n optimized by GRG Nonlinear method is $n = 0$. The MAPE ≥50%, implies that the models are classified as unacceptable. For electrical demand in Thailand, the proposed models achieved a MAPE of 61.577%, which also ≥50%. The proposed models gave the MAPE of Indonesia's electrical demand of 47.741%. From the results, the models are classified as reasonable. Meanwhile, MAPE for Philippines' electrical demand lies between 10% to 20% and Singapore ≤10%, indicate that the models' precision is excellent and good. This means that the models are not relevant to the large sample as the results obtained are chaos.

However, models with 25 and 15 year of data indicate an excellent MAPE, that is ≤10%, which signifies highly forecast ability of the models and with 5 year of data yielded lowest MAPE compared with the other models. Thus, both models are highly accurate with small data. As a result, analysis on GM(1,1) and NGBM(1,1) accuracy can be conducted to the next step.

4.2. TheAccuracy of Grey Forecasting Models

The simulative results obtained by GM(1,1) and NGBM(1,1) are compared with the actual values in order to determine which models perform with high accuracy results. Figure 1 to Figure 5 show the comparison of actual values, GM(1,1) and NGBM(1,1) with 5 years of data from 2010 to 2014.

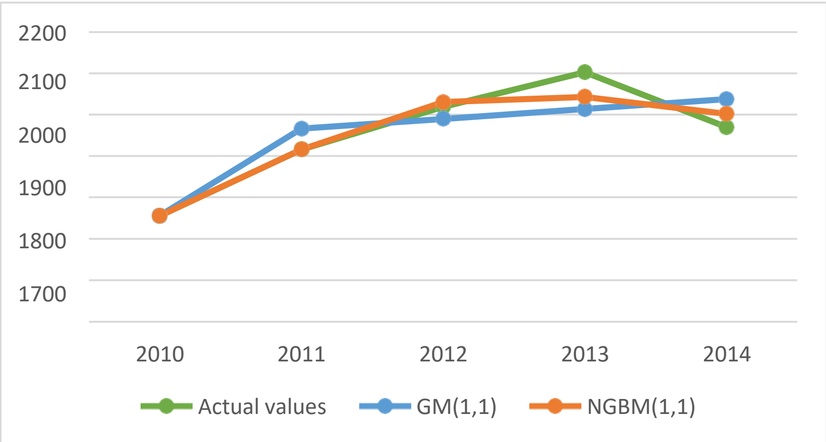

Figure 1 Curves of Actual and Simulative Values of Indonesia's Electrical Demand

From Figure 1, although the simulative value for electrical demand in 2012 using GM(1,1) is quite close to actual value, there are big differences between results of other years for the period 2013 and 2014.

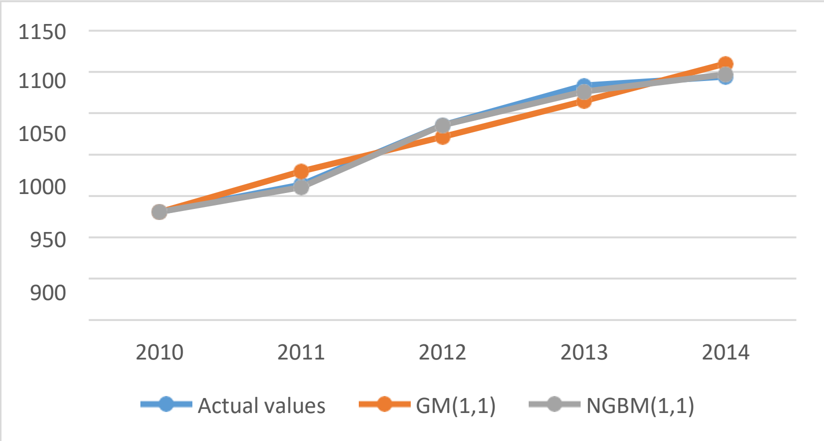

Figure 2 Curves of Actual and Simulative Values of Malaysia's Electrical Demand

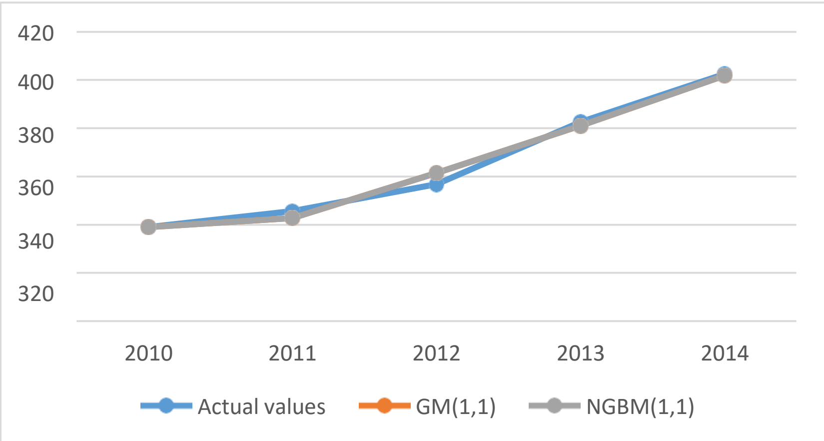

Figure 3 Curves of Actual and Simulative Values of Philippine's Electrical Demand

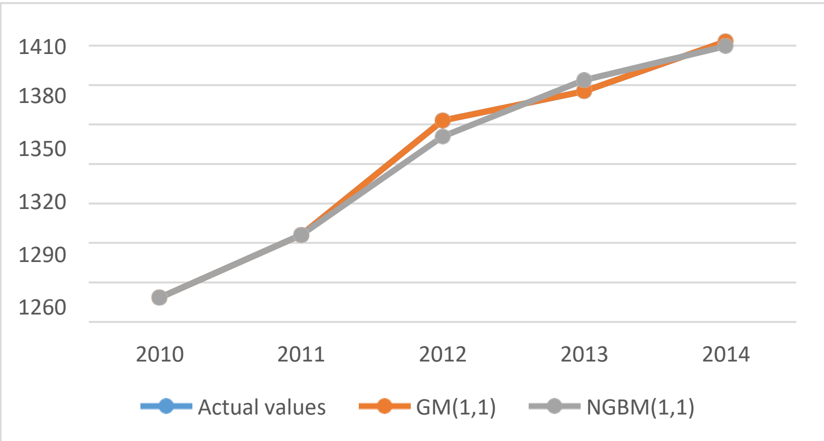

Figure 4 Curves of Actual and Simulative Values of Thailand's Electrical Demand

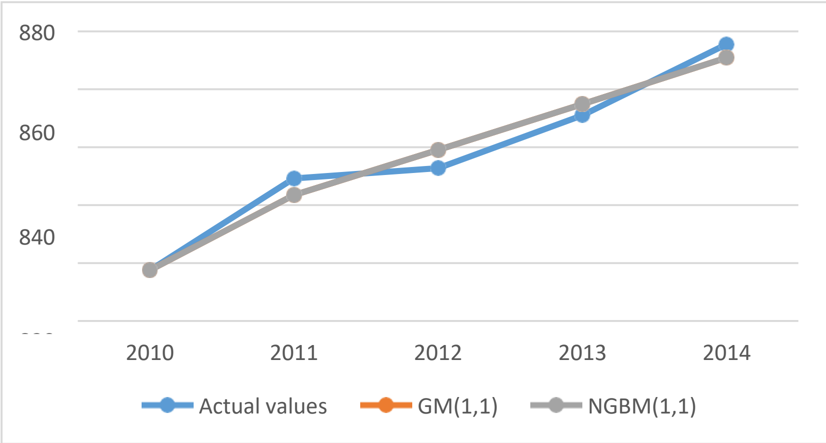

Figure 5 Curves of Actual and Simulative Values of Singapore's Electrical Demand

The simulative values attained by NGBM(1,1) with 5-year data is closely tied with actual value over to GM(1,1). The reason is NGBM(1,1) uses many parameters. The simulative results show that NGBM(1,1) can enhance the model precision though appropriate selection of the power n Therefore, NGBM(1,1) provides more accurate results than GM(1,1). NGBM(1,1) is clearly the superior model for forecasting electrical demand.

4.3.. Forecasting Results

The forecasting results of electricity demand using GM(1,1) and NGBM(1,1) approach with varying sample size displayed in Table 2 to Table 5.

Table 2: Forecasting values of GM(1,1) and NGBM(1,1) model from 55 year

Year	Indonesia				Malaysia			
	GM(1,1)	$\varepsilon(t)\%$	NGBM(1,1), $n=0$	$\varepsilon(t)\%$	GM(1,1)	$\varepsilon(t)\%$	NGBM(1,1), $n=0$	$\varepsilon(t)\%$
2015	2921.71	48.08	2921.71	48.08	1780.11	60.37	1780.11	60.37
2016	3088.65	52.33	3088.65	52.33	1893.24	61.96	1893.24	61.96
2017	3265.13	55.23	3265.13	55.23	2013.57	69.65	2013.57	69.65
2018	3451.7	51.07	3451.7	51.07	2141.54	83.23	2141.54	83.23
2019	3648.92	47.41	3648.92	47.41	2277.64	92.39	2277.64	92.39

	Philippine				Thailand			
	GM(1,1)	$\varepsilon(t)\%$	NGBM(1,1), $n=0.2107$	$\varepsilon(t)\%$	GM(1,1)	$\varepsilon(t)\%$	NGBM(1,1), $n=0$	$\varepsilon(t)\%$
2015	427.02	3.22	404.47	8.33	2209.97	51.67	2209.97	51.67
2016	440.71	8.55	415.32	13.82	2346.68	57.62	2346.68	57.62
2017	454.85	13.94	426.43	19.32	2491.84	64.58	2491.84	64.58
2018	469.44	13.6	437.79	19.43	2645.98	70.1	2645.98	70.1
2019	484.49	13.82	449.41	20.07	2809.66	80.16	2809.69	80.16

	Singapore			
	GM(1,1)	$\varepsilon(t)\%$	NGBM(1,1), $n=0.2303$	$\varepsilon(t)\%$
2015	1075.83	15.55	986.57	5.97
2016	1137.73	17.75	1039.73	7.61

2017	1203.19	20.71	1095.66	9.92
2018	1272.42	27.06	1154.52	15.28
2019	1345.63	36.47	1216.45	23.37

Table 3: Forecasting values of GM(1,1) and NGBM(1,1) model from 30 year

Year	Indonesia				Malaysia			
	GM(1,1)	$\varepsilon(t)\%$	NGBM(1,1), $n = 0.1183$	$\varepsilon(t)\%$	GM(1,1)	$\varepsilon(t)\%$	NGBM(1,1), $n = 0.2822$	$\varepsilon(t)\%$
2015	2231.16	13.08	2170.18	9.99	1238.69	11.59	1154.32	3.99
2016	2334.41	15.13	2257.53	11.34	1302.67	11.44	1196.39	2.35
2017	2442.45	16.12	2347.97	11.63	1369.96	15.43	1239.35	4.42
2018	2555.48	11.85	2441.63	6.86	1440.72	23.27	1283.22	9.79
2019	2673.74	8.01	2538.65	2.56	1515.14	27.98	1328.04	12.18
	Philippine				Thailand			
	GM(1,1)	$\varepsilon(t)\%$	NGBM(1,1), $n = 0.214$	$\varepsilon(t)\%$	GM(1,1)	$\varepsilon(t)\%$	NGBM(1,1), $n = 0.1973$	$\varepsilon(t)\%$
2015	395.01	10.48	369.37	16.28	1549.05	6.31	1475.05	1.23
2016	404.48	16.07	373.32	22.53	1623.03	9.01	1529.82	2.75
2017	414.18	21.64	377.18	28.64	1700.55	12.31	1586.11	4.76
2018	424.11	21.94	380.93	29.89	1781.76	14.54	1643.95	5.68
2019	434.28	22.76	384.60	31.59	1866.86	19.71	1703.42	9.23

	Singapore			
	GM(1,1)	$\varepsilon(t)\%$	NGBM(1,1), $n = 0.0462$	$\varepsilon(t)\%$
2015	956.38	2.72	946.83	1.7
2016	1004.67	3.98	992.7	2.74
2017	1055.4	5.88	1040.73	4.41
2018	1108.69	10.71	1091.02	8.94
2019	1164.67	18.12	1143.68	15.99

Table 4: Forecasting values of GM(1,1) and NGBM(1,1) model from 20 year

Year	Indonesia				Malaysia			
	GM(1,1)	$\varepsilon(t)\%$	NGBM(1,1), $n = 0$	$\varepsilon(t)\%$	GM(1,1)	$\varepsilon(t)\%$	NGBM(1,1), $n = 0.1364$	$\varepsilon(t)\%$
2015	2163.55	9.66	2170.18	9.99	1163.66	4.83	1154.32	3.99
2016	2251.95	11.06	2257.53	11.34	1210.77	3.58	1196.39	2.35
2017	2343.95	11.43	2347.97	11.63	1259.78	6.14	1239.35	4.42
2018	2439.71	6.78	2441.63	6.86	1310.78	12.15	1283.22	9.79
2019	2539.38	2.59	2538.65	2.56	1363.84	15.2	1328.04	12.18
	Philippine				Thailand			
	GM(1,1)	$\varepsilon(t)\%$	NGBM(1,1), $n = 0$	$\varepsilon(t)\%$	GM(1,1)	$\varepsilon(t)\%$	NGBM(1,1), $n = 0.0942$	$\varepsilon(t)\%$
2015	389.04	10.48	369.37	16.28	1494.73	2.58	1475.05	1.23
2016	397.96	16.07	373.32	22.53	1556.84	4.57	1529.82	2.75

2017	407.08	21.64	377.18	28.64	1621.53	7.09	1586.11	4.76
2018	416.4	21.94	380.93	29.89	1688.9	8.57	1643.95	5.68
2019	425.94	22.76	384.60	31.59	1759.08	12.8	1703.42	9.23

	Singapore			
	GM(1,1)	$\varepsilon(t)\%$	NGBM(1,1), $n = 0.0077$	$\varepsilon(t)\%$
2015	969.69	4.15	946.83	1.7
2016	1021.17	5.69	992.7	2.74
2017	1075.38	7.88	1040.73	4.41
2018	1132.47	13.08	1091.02	8.94
2019	1192.59	20.95	1143.68	15.99

Table 5: Forecasting values of GM(1,1) and NGBM(1,1) model from 10 year

Year	Indonesia				Malaysia			
	GM(1,1)	$\varepsilon(t)\%$	NGBM(1,1), $n = 0.2615$	$\varepsilon(t)\%$	GM(1,1)	$\varepsilon(t)\%$	NGBM(1,1), $n = 0.2031$	$\varepsilon(t)\%$
2015	2062.44	4.53	1930.91	2.135	1157.52	4.28	1106.04	0.36
2016	2087.20	2.94	1842.29	9.142	1206.76	3.23	1106.15	5.37
2017	2112.26	0.42	1744.3	17.074	1258.09	6	1100.04	7.31
2018	2137.62	6.44	1642.05	28.133	1311.6	12.22	1089.32	6.8
2019	2163.29	12.61	1538.93	37.83	1367.39	15.5	1075.13	9.18

	Philippine				Thailand			
	GM(1,1)	$\varepsilon(t)\%$	NGBM(1,1), $n = 0$	$\varepsilon(t)\%$	GM(1,1)	$\varepsilon(t)\%$	NGBM(1,1), $n = 0.1419$	$\varepsilon(t)\%$
2015	423.73	3.96	423.73	3.96	1470.74	0.93	1424.55	2.24
2016	446.79	7.29	446.79	7.29	1521.64	2.2	1432.07	3.81
2017	471.1	10.86	471.1	10.86	1574.31	3.98	1434.19	5.28
2018	496.74	8.58	496.74	8.58	1628.79	4.7	1432.22	7.93
2019	523.76	6.84	523.76	6.84	1685.16	8.06	1427.09	8.49

	Singapore			
	GM(1,1)	$\varepsilon(t)\%$	NGBM(1,1), $n = 0$	$\varepsilon(t)\%$
2015	887.37	4.69	887.37	4.69
2016	904.09	6.43	904.09	6.43
2017	921.12	7.59	921.12	7.59
2018	938.48	6.29	938.48	6.29
2019	956.16	3.03	956.16	3.03

Table 6 reveals the comparison between GM(1,1) and NGBM(1,1) in term of the MAPE of simulative and forecasting values.

Table 6: The MAPE of simulative and forecasting results

Data	Country	Fitted		Forecasting	
		GM(1,1)	NGBM(1,1)	GM(1,1)	NGBM(1,1)
10	Indonesia	2.9356	1.2628	5.3875	18.8628
	Malaysia	1.55	0.3228	8.2471	5.8062
	Philippine	0.6689	0.6689	7.5061	7.5061

	Thailand	0.9867	0.443	3.9753	5.5497
	Singapore	0.6039	0.6039	5.6058	5.6058
20	Indonesia	3.2981	3.2981	8.3036	8.3036
	Malaysia	3.2719	1.9124	8.381	2.464
	Philippine	2.8409	2.8409	19.9649	19.9649
	Thailand	2.1566	1.5344	7.1218	2.7419
	Singapore	4.0212	4.0156	10.3509	10.0233
30	Indonesia	4.3991	2.8978	12.8376	8.4748
	Malaysia	6.995	4.1435	17.9412	6.5459
	Philippine	6.0646	3.9232	18.5748	25.7867
	Thailand	5.3704	2.9338	12.3766	4.7293
	Singapore	4.3295	4.2004	8.2816	6.7555
55	Indonesia	47.7415	47.7415	50.8236	50.8236
	Malaysia	69.5985	69.5985	73.5198	73.5198
	Philippine	10.2176	7.4732	10.6266	16.1908
	Thailand	61.5769	61.5769	64.8238	64.8238
	Singapore	14.4109	11.8569	23.5078	12.429

Results from Table 4.14, show that NGM(1,1) yielded the lowest MAPE compared with GM(1,1), which means that NGBM(1,1) reaches the objective of minimising of forecast error and has highly accurate forecasting.

Conclusion

Simulative values of the 5-year data from both models have the lowest MAPE among all other models. However, models with 25 and 15 year data indicate an excellent MAPE of under 10%, which signifies excellent forecasting capability. While models using 55-year revealed that GM(1,1) and NGBM(1,1) are not precise with large samples. The results show that when the sample size is larger, the MAPE changes larger. That is, the smaller the sample model, the more stable the solution of the model. Findings indicates that the accuracy of models valid from 5 to 30 years.

Simulative results show that NGBM(1,1) can enhance the model precision through appropriate selection of the power n. Results show that NGBM(1,1) yield approximately similar simulative values with actual values. Therefore, NGBM(1,1) is clearly the superior model for forecasting electrical demand over GM(1,1). Based on the result of the analysis, forecasting data of the proposed models is same in particular sample sizes. Therefore, another algorithm is needed in order to optimize the value of power index n so that the forecasting result could describe the performances of GM(1,1) and NGBM(1,1)

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