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The Effect of An Aligned Magnetic Field on the Water-Based Nanofluids Over a Stretching Sheet

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Abstract

The effect of an aligned magnetic field on the water-based nanofluids over a stretching sheet is investigated in this study. Three nanoparticles are being considered, namely copper (Cu), aluminium oxide (Al_2O_3), and titanium oxide (TiO_2). An inclined magnetic field is considered with the range starting at 0 to $\frac{\pi}{2}$. The governing equations which consist of partial differential equations are transformed using an appropriate dimensionless variable. Then, the transformed governing equations are solved numerically using the Keller-box method. The numerical solutions of velocity and temperature profiles for different values of nanoparticle volume fraction, Prandtl number, and angle of the aligned magnetic field are presented graphically and discussed in detail. It is found that the increase of the aligned magnetic field has decreased the velocity of the nanofluid flow but increased the temperature profile for all types of nanoparticles. Furthermore, increasing the values of nanoparticles has decreased the velocity of Cu-water nanofluid, but increased the velocity of the Al_2O_3 and TiO_2 nanofluid. Meanwhile, the temperature profile of the nanofluid increase significantly with the increasing value of nanoparticle volume fraction for all types of nanoparticles.

Keywords: Nanofluid; Aligned magnetic field; Boundary layer flow; Numerical solution

1. Introduction

Nanofluids are liquid suspensions of solid nanoparticles with sizes of fewer than 100 nanometers. Materials such as carbide ceramics, metal nitrides, oxide ceramics, metals, carbons, and functionalized nanoparticles are used to make nanoparticles [1]. Since the addition of nanoparticles to liquids considerably improves their thermal conductivity, nanofluids have a wide range of applications in the field of thermal dynamics. Experiments have demonstrated that nanofluids have significantly higher thermal conductivities than pure fluids and are more suited for practical use than existing heat transfer enhancement techniques involving millimeter and/or micrometer-sized particles in fluids [2]. The proposed model by Tiwari and Das [3] investigated the nanoparticle volume fraction, which is an important parameter in understanding the nanoparticle's impact on fluid flow and heat transfer. As a result, while researching the impact and properties of nanoparticles on fluid flow velocity and temperature field, the nanoparticle volume fraction is a crucial factor to consider.

Alsabery *et al.* [4] investigate the natural convection flow of a nanofluid in a partially filled inclined square enclosure with a porous medium. They discovered that a higher concentration of solid volume fraction combined with the thinnest porous layer thickness can influence heat transfer distribution. Mabood *et al.* [5] proposed a mathematical model to investigate the MHD stagnation-point flow and heat transfer characteristics of an electrically conducting nanofluid over a vertical permeable shrinking/stretching sheet in the presence of viscous dissipation. In the same year, Dinarvand and Pop [6] investigate the laminar free-convective flow and heat transfer of an electrically conducting nanofluid in the presence of a transverse magnetic field over a rotating down-pointing vertical cone. In their paper, three water-based nanofluids which are copper, aluminum oxide, and titanium dioxide have been investigated. Recently, Jafar *et al.* [7] observed that the nanoparticle volume fraction is an essential factor to consider while researching the impact and properties on fluid flow velocity and temperature field of MHD radiative nanofluid flow.

According to Narankhishig *et al.* [8], different types of nanoparticles have been discovered by researchers. For example, titanium (II) oxide (TiO), alumina oxide (Al_2O_3), Silicon dioxide (SiO_2), carbon nanotube (CNT), copper oxide (CuO), and iron (II, III) oxide (Fe_3O_4). The majority of convective heat transfer experiments have used oxide nanoparticles at high concentrations, which can improve the viscosity and pumping power of the fluid. Some research studies showed a 60% increase in thermal conductivity for a nanofluid containing water (H_2O) and 5% CuO nanoparticles. Motivated by the study, this research will focus on three types of water-based nanofluids which are copper (Cu-water), alumina oxide (Al_2O_3-water), and titanium oxide (TiO_2-water) under the influence of the aligned magnetic field. Table 1.1 below indicates the thermophysical properties of the base fluid and the nanoparticles [9].

Table 1.1 The thermophysical properties of the base fluid and the nanoparticles

Physical Properties	**Water**	**Cu**	**Al_2O_3**	**TiO_2**
C_p (J / kg K)	4179	385	765	686.2
ρ (kg / m^3)	997.1	8933	3970	4250
K (W / mK)	0.613	400	40	8.9538
α × 107 (m^2 / s)	1.47	1163.1	131.7	30.7
β × 10−5 (1 / K)	21	1.67	0.85	0.9

Even though using metal-based nanofluids gives good heat transfer enhancement, the method needs a high cost to produce the nanofluids. There are a few reasons why the previous researchers used water-based nanofluids. One of the reasons is that nanoparticles become less expensive and easier to obtain as nanotechnology advances. Loong *et al.* [10] studied the thermal performance evaluation for different types of metal oxide water-based nanofluids. They considered metal oxide-water-based nanofluids because they are easier to prepare and less expensive than metallic-based nanofluids. Narankhishig *et al.* [8] also stated that the integration of nanoparticles within the base fluid is not only beneficial for a high thermal framework but is also less expensive monetarily.

Past researchers who were concerned with the heat transfer problem in the industry have studied the effect of magnetohydrodynamics (MHD) on nanofluids. MHD is based on the idea that magnetic fields can induce currents in a flowing conductive fluid, which causes forces to be exerted on the fluid and modifies the magnetic field itself. Researchers have recently become interested in the influence of aligned angle coupled with magnetic field on the boundary layer flow problem. A magnetic field is applied to the flow stream at an acute angle (0˚-90˚). External magnetic fields are particularly effective at controlling the flow and heat transfer characteristics of magnetic nanofluids by setting their thermal and physical properties. The study of the aligned magnetic field in a nanofluid has piqued the curiosity of many researchers. Overall, it is discovered that increasing the aligned angle increases the magnetic field, which causes the fluid's velocity to decrease and its temperature to rise. As a result, it demonstrates why aligned magnetic fields improve heat transfer in a nanofluid, so benefiting the industry. Rashid *et al.* [1] observed that the magnetic field does not affect the velocity profile for an acute angle, when $\gamma = 0$ and the magnetic field behaves transversely in the case of an acute angle, when $\gamma = \frac{\pi}{2}$ along the flow section. Sreenivasulu *et al.* [11] investigated further into effects of an aligned magnetic field and Navier slips on the free convective radiative flow of nanofluids with carbon nanotubes. They discovered that the fluid flow velocity decreases as the magnetic field angle of inclination increases. When magnetic flux grows as the aligned angle increases, the Lorentz force increases, and the fluid flow is suppressed. Hence, an increase in the angle of inclination raises the temperature of the nanofluid. Furthermore, Srinivasulu & Goud [12] observed that the skin friction coefficient increases with the magnification of the angle of inclination and magnetic parameter due to the Lorentz force, however the mass transfer rate increases as the angle of inclination and magnetic parameter decreases. Motivated by that, this research aims to investigate the effect of aligned magnetic fields on nanofluid flow past a stretching sheet under the influence of types of nanoparticles.

2. Mathematical Formulation

Consider a two-dimensional, incompressible nanofluid flow across a stretching sheet. In this case, water is used as the base fluid and three different types of water-based nanoparticles, namely copper (Cu), alumina (Al_2O_3), and titanium dioxide (TiO_2) are considered. The inclined magnetic field is applied with an acute angle γ, to the surface along the y-axis of strength B_0, with the x-axis parallel to the sheet.
The basic governing equation for this problem is given as:

Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2}-\frac{\sigma}{\rho_{nf}}B_0{}^2 u\sin^2\gamma \tag{2}$$

Energy equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

with boundary equations:

$$u=u_w(x)=ax,\ v=v_w \text{ at } y=0$$

$$T=T_w=T_\infty+cx, \text{ at } y=0,\ T\to\infty, y\to\infty \tag{4}$$

where u,v is the components of the nanofluid's velocity in the x and y axes respectively. μ_{nf} is the dynamic viscosity, ρ_{nf} is the density of the nanofluid, α_{nf} is the thermal diffusivity, and σ is the electrical conductivity. B_0 is the magnetic field strength, T is the fluid temperature and T_∞ is the ambient temperature.

The expression of thermophysical property for nanofluid as proposed by Tiwari and Das (2007) are given below:

$$v_{nf}=\frac{\mu_{nf}}{\rho_{nf}},\mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}},\rho_{nf}=(1-\phi)\rho_f+\phi(\rho)_s,\alpha_{nf}=\frac{k_{nf}}{(\rho C_p)_{nf}},\frac{k_{nf}}{k_f}=\frac{(k_s+2k_f)-2\phi(k_f-k_s)}{(k_s+2k_f)+\phi(k_f-k_s)},$$

$$(\rho C_p)_{nf}=(1-\phi)(\rho C_p)_f+\phi(\rho C_p)_s,$$

where v_{nf} is the kinematic viscosity, $(\rho C_\rho)_{nf}$ is the heat capacitance of nanofluid, and k_{nf} is the thermal conductivity. The symbol ϕ represents the nanoparticle volume fraction, the subscript f and s are for the base fluid and nanoparticle respectively.

The following similarity transformation are adopted from study by Rawi *et. al* [13] is given as follows:

$$\eta=\left(\frac{a}{v_f}\right)^{\frac{1}{2}}y,\psi(\eta)=(av_f)^{\frac{1}{2}}xf(\eta),\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \tag{5}$$

where η is the similarity variable, f is the dimensionless stream function, θ is the dimensionless temperature and ψ is the stream function defined as:

$$u=\frac{\partial\psi}{\partial y},\ v=-\frac{\partial\psi}{\partial x}. \tag{6}$$

The transformation of the governing equation yields:

$$\frac{f'''(\eta)}{A_1A_2}+f(\eta)f''(\eta)-f'(\eta)\left(f'(\eta)+\frac{M_1}{A_1}\sin^2\gamma\right)=0 \tag{7}$$

$$\frac{A_3}{A_4}\theta''(\eta)+\Pr f(\eta)\theta'(\eta)=0 \tag{8}$$

with boundary condition:

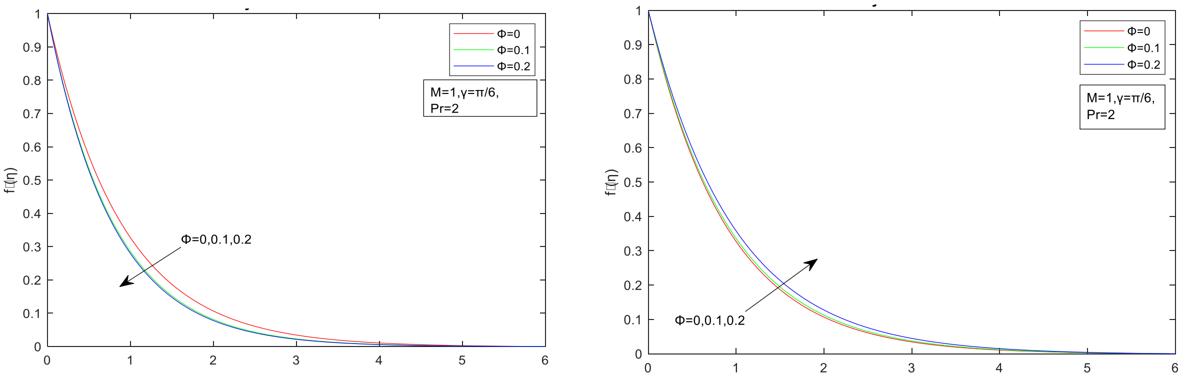
$f(\eta)=0, f'(\eta)=1$ at $\eta=0$ and $f'(\eta)\to 0$ as $n\to\infty$

$\theta(\eta)=1,$ as $\eta=0$ and $\theta(\eta)=0$ at $\eta\to\infty$. (9)

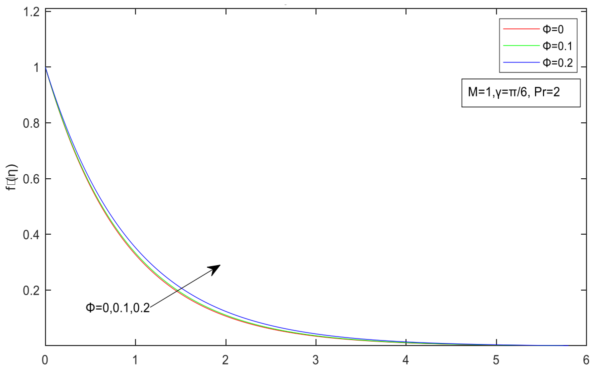
where the constant $A_1=\left(1-\phi+\phi\dfrac{\rho_s}{\rho_f}\right)$, $A_2=\left(1-\phi\right)^{2.5}$, $A_3=\left[\dfrac{(k_s+2k_f)-2\phi(k_f-k_s)}{(k_s+2k_f)+\phi(k_f-k_s)}\right]$, $A_4=\left(1-\phi+\phi\dfrac{(\rho Cp)_s}{(\rho Cp)_f}\right)$, $\Pr=\dfrac{\nu_f}{\alpha_f}$ is the Prandtl number, and magnetic field parameter $M_1=\left(\dfrac{\sigma B_0{}^2}{\rho a}\right)$ are being considered.

3. Results and discussion

The Keller-box method is used to solve the system of nonlinear ordinary differential equations (7) and (8) with respect to boundary conditions (9). The effects of an inclined magnetic field, as well as other physical parameters, on fluid flow characteristics over a stretching sheet are investigated in this section.

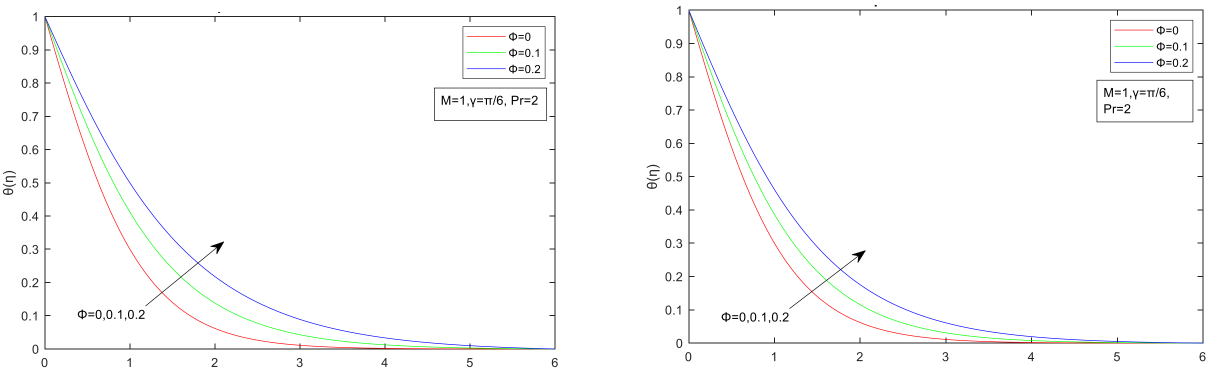


(a) Cu-Water (b) Al_2O_3-Water



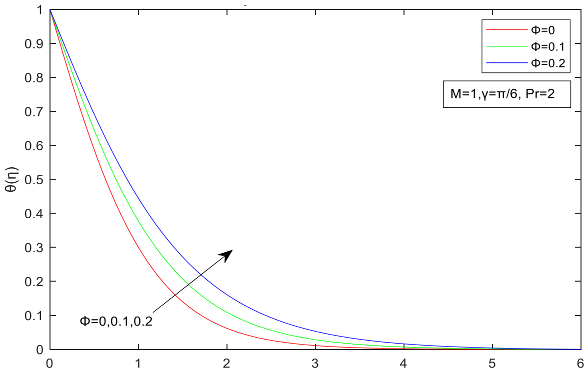
(c) TiO_2-Water

Figure 3.1 Velocity $f'(\eta)$ profile for different value of ϕ



(a) Cu-water

(b) Al_2O_3-Water



(c) TiO_2-Water

Figure 2 Temperature $\theta(\eta)$ profile for different value of ϕ

The effect of nanoparticle volume fraction on the velocity and temperature distributions of Cu-water, Al_2O_3-water, and TiO_2-water is shown in Figure 3.1 and Figure 3.2. The value of magnetic field, M, Prantdl number, Pr, and aligned angle, γ have been fixed by $M = 1, \gamma = \pi / 2, \Pr = 2$. As can be seen, the velocity of the nanoparticles differs with the increase in φ. The increasing value of ϕ decrease the velocity of Cu-water nanoparticle but increase the velocity of Al_2O_3-water and TiO_2-water. The presence of nanoparticles in the base fluid makes it more viscous, slowing the fluid flow velocity. Furthermore, it can be seen that the temperature rises as the volume fraction of Cu-water, Al_2O_3-water, and TiO_2-water increases. The increase of nanoparticle volume fraction causes the fluid's thermal conductivity to improve, resulting in an increase in fluid temperature.

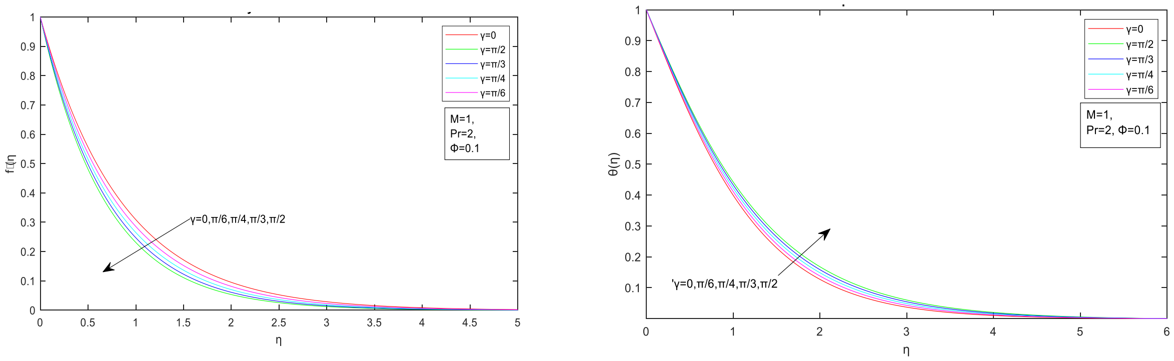


Figure 3 Velocity $f'(\eta)$ and Temperature $\theta(\eta)$ Profile for Different Value of γ

Figure 3 depicts the velocity and temperature profile for each type of nanoparticles for different values of γ. The value of magnetic field, M, nanoparticle volume fraction, ϕ, and Prandtl number, Pr have been fixed by $M=1, \phi=0.1, \mathrm{Pr}=2$. Clearly, the increase of aligned magnetic field angle decreases the velocity profile for Cu-water, Al_2O_3-water, and TiO_2-water nanoparticles. In contrast, the temperature of the nanoparticles increases as the aligned angle increase. Magnetic induction increases as aligned angle values increase along the flow region. Due to various improvements in magnetic induction, a force known as the Lorentz force produces resistance within the boundary layer with an aligned magnetic field. These figures show the physical activity in which the velocity profile decreases and the temperature profile gradually increases for Cu-water, Al_2O_3-water, and TiO_2-water.

Conclusion

In this paper, the effect of aligned magnetic field on the water-based nanofluids over a stretching sheet are considered. It is observed that the velocity of Cu-water decreased when nanoparticle volume fraction increase, while the velocity of Al_2O_3-water and TiO_2-water increase when the volume fraction increase. Meanwhile, the temperature of Cu-Water, Al_2O_3-water, and TiO_2-water increases when the nanoparticle volume fraction increases. Also, the temperature of the nanofluids decrease when the Prandtl number increases. Moreover, the velocity of Cu-Water, Al_2O_3-water, and TiO_2-water decreases when the angle of aligned magnetic field increase. In contrast, the temperature of Cu-Water, Al_2O_3-water, and TiO_2-water increases significantly when the angle of aligned magnetic field increases.

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