Developing Excel User Interface for Keller Box Method with Unsteady Two-Dimensional Boundary Layer Flow

Nurul Najiha Fadzil, Anati Ali*
Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia
*Corresponding author: anati@utm.my

Abstract

The purpose of this study is to investigate the computational of two-dimensional boundary layer flow equation in Microsoft Excel and develop the user interface for Keller Box Method (KBM). Keller Box Method (KBM) is a method used to solve nonlinear equations. A case study involving a boundary layer flow problem consisting of several two-dimensional governing equations is selected. The governing equations were solved numerically using the Keller Box method first to develop the user interface and to compare the result obtained with Microsoft Excel. The dimensionless governing equations from the case study are transformed into first-order differential equations. Then, the Centered Finite Difference Method (CFDM) is applied to the first-order differential equation. Newton's Linearization method is used to transform the nonlinear equations into linear equations. The IF formula is mainly used to determine the variable inputted and produce results based on the manual calculation. In the output section of Microsoft Excel, each output is simplified and the operations are classified into different cases considering the number of variables, the power of the variable, the existence of prime, and variable type. For the Newton Linearization part in Microsoft Excel, variables for delta are determined by using the IF formula on the number of variables and the type of variables exist in the operation. The interface template is then verified with multiple two-dimensional equations from past research to prove that the user interface and results are in agreement and valid.

Keywords: Keller Box Method; two-dimensional boundary layer flow; IF formula

1. Introduction

Ludwig Prandtl proposed the concept of boundary layer. Boundary layer has been applied in aerodynamic for prediction of skin friction drag, heat transfer between wall and fluid flows. Knowing that it can applied on other range of problem, many researchers been study on boundary layer and their findings had impact on the industry and engineering such as the automobiles industry and airplanes industry.

The boundary layer is a thin layer of viscous fluid in contact with a moving stream that has a different flow velocity from the wall to the borders. The liquid or gas has the tendency to cling to the surface when it is boundary layer. In aerodynamic, boundary layer flow happened on aircraft wing when the airflow from surrounding flow hitting the surface of the wing that can affect the take-off or the landing of the aircraft. The wings of the aircraft are designed in a form that no separation of boundary layer occur when there is air resistance. In automobiles industry, boundary layer involves in the design of the body. A car with streamlined body can reduce the form of drag and no separation between boundary layer of air flow and the body surface.

Keller Box Method (KBM) is a meticulous and mostly execute finite difference method namely Keller Box Method (KBM). This method can give the most accurate outcome but the calculation is tedious especially for higher order partial differential equation. Hence, developing a user interface for Keller Box Method (KBM) can be useful and practical for research on boundary layer flow.

Boundary layer flow problems can be solved in many methods either in the numerical way or analytical way. In this research paper, we are focusing on G-Jitter Induced Natural Convection Nanofluid Flow with Mass Transfer in The Stagnation Point Region of a Three-Dimensional Body by Ahmad [1]. Moreover, the Keller Box Method (KBM) used to solve the governing equation numerically are subjected until Newton's Linearization method. Excel template based on the Keller Box Method (KBM) were created with the purpose of efficiency of the future research.

2.   Literature Review

2.1.3. Boundary Layer Flow
A boundary layer flow is described as a region near the surface that is subject to strong wall frictional forces. There have been many distinct forms of boundary layer flow proposed. There are various unstable boundary-layer problems in which the oscillations in the external flow must be taken into consideration. Both the magnitude and the direction of these oscillations may alter. A simpler situation is when the external flow just shifts in amplitude rather than direction. Yang [2] researched on unsteady boundary layer flow of a fractional viscoelastic over a stretching sheet of a symmetrical rectangular area. It was tested in their research the effects of the fractional parameter γ and the time-fractional parameter α in response to y and found that when the space fractional parameter is increased, the thickness of the velocity boundary layer increases.

Mansur [3] investigate the unsteady boundary layer flow of a nanofluid via a stretching or shrinking sheet with convective boundary conditions and parameters such as acceleration, stretching or shrinking, convective, Brownian motion, and thermophoresis were analyzed of their effect over the local Nusselt number. The findings suggest that dual solutions exist for both stretching and shrinking scenarios.

Khan [4] study wire coating analysis solution of magnetohydrodynamic (MHD) flow and heat transfer of radiative viscoelastic fluid with temperature-dependent viscosity by analyzing the effect of magnetic parameter, thermal radiation parameter temperature parameter. The governing equation in this paper was solved by the homotopy analysis method (HAM) then Adomian Decomposition Method (ADM) and the ND-Solve method are used to verify the generated results.

2.1.4. Finite Difference Method
Once the governing equations become first-order differential equations, we can proceed with the discretization of the centered Finite Difference Method (FDM). Centered Finite Difference Method (FDM) is ideal as it would give second-order accuracy. Here, we introduce the step length which corresponds to the ith interval. It is namely known centered Finite Difference Method (FDM) because the step length of an interval is added and divided by two.

The central difference approximation is introduced for each of the related variables. The variables in the first-order differential equation that we have will be replaced with the centered finite approximation. Liu et   al. [5] study an unstable asymptotic coupled model solved using a block-centered finite difference approach where the second-order error estimates in discrete norms are derived on nonuniform rectangular grids for both pressure and velocity.

2.1.5. Keller-Box Method
Keller Box method is one of the methods to solve boundary layer flow equations that had been proven by past research that it is a method that gives the most accurate results. However, the Keller Box method is a tedious way to solve the equation and can be challenging. The Keller Box approach works by substituting the first derivative of an additional variable for the higher spatial derivatives in the equation.
nonlinear differential equations are the optimal homotopy analysis method (OHAM) and the Keller Box method.

Kudenatti [6] studied on boundary layer flow of a viscous fluid and heat transfer over a wedge. Two approaches are used in this study such that the full nonlinear equations are solved numerically

using the Keller-box method and solved in the asymptotic limit. In terms of estimating wall shear stress, temperature gradient, velocity, and temperature profiles, both methodologies accord well.

2.2. Microsoft Excel User Interface Development

The creation of the Microsoft Excel user interface revolves around arranging the IF function according to the input and the desired output. The user interface involves the users to key in the input and the programme automatically calculates and produces the outcome.

Wang [7] study on absorption kinetic were based on Excel software, a user interface (UI) for solving kinetic models was created and offered as supplemental material, which will assist readers in simulating the adsorption kinetic process. The Excel User Interface (UI) developed by Wang is considered more advance where Solver Add-in are used. The User Interface (UI) allows users to choose one kinetic model and they can input their data.

Dimitri [8] studies on developing Excel User Interface (UI) for one-dimensional boundary layer flow. The study shows the result obtained from Excel User Interface (UI) and the manual calculation are in agreement.

3. Methodology

3.1. Keller Box Method

The Keller-Box Method involve four numerical calculation steps. By introducing new variables, the dimensionless and nonlinear partial differential governing equation will transform into first-order system of equation. At variable spaced mesh points, the derivatives in nonlinear differential equations are replaced by a finite difference method. As a consequence, the nonlinear differential equations system is transformed into a system of nonlinear algebraic equations. Using the Newton Linearization method, the nonlinear system of algebraic equations is linearized.

3.1.1 Reduce Higher-Order PDE

The system of higher-order partial differential equations is reduced to first order by introducing new variables for the derivative.

$$
\begin{aligned}
&f' = u, f'' = v, (f'')' = v', \\
&h' = w, h'' = b, (h'')' = b' \\
&\theta' = p, \theta'' = p' \\
&\Phi' = q, \Phi'' = q'
\end{aligned}
\tag{1}
$$

3.1.2. Finite Difference Scheme

The discretization uses a centered finite difference scheme on the boundary layer of nanofluid with boundary conditions. Finite difference scheme used will be centered exactly halfway between the grid points in a such way that it gives second order accuracy.

$$
()_j^{i-\frac{1}{2}} = \frac{1}{2}[()_j^i + ()_j^{i-1}]
$$

$$
()_{j-\frac{1}{2}}^i = \frac{1}{2}[()_j^i + ()_{j-1}^i]
$$

$$
\left(\frac{\partial()}{\partial\eta}\right)_{j-\frac{1}{2}}^{i-\frac{1}{2}} = \frac{1}{2w_j}[()_j^i - ()_{j-1}^i + ()_j^{i-1} - ()_{j-1}^{i-1}]
$$

$$
\left(\frac{\partial()}{\partial\tau}\right)_{j-\frac{1}{2}}^{i-\frac{1}{2}} = \frac{1}{2k_i}[()_j^i + ()_{j-1}^i - ()_j^{i-1} - ()_{j-1}^{i-1}]
$$

$$\tag{2}$$

3.1.3. Newton's Linearization Method
  Newton's method of linearization serves the purpose of transforming nonlinear equations into a linear form. Every variable that is in terms of j and j-1 in the equations is transformed as follows:

$$Let \begin{cases} (\ )_j \rightarrow (\ )_j + \delta(\ )_j \\ (\ )_{j-1} \rightarrow (\ )_{j-1} + \delta(\ )_{j-1} \end{cases}$$

  For every equation, the end equation result will be all δ terms on the left side while other terms on the right. Also, $\delta 2$ terms, including $\delta(\ )\delta[\ ]$ ones, are ignored since the values are very small and can be considered insignificant.

3.2. Keller Box- Excel User Interface (KB-EUI)
  Initial input is required to begin creating the user interface in order to produce the final output results. Our initial input in this case is the dimensionless governing equations. To break down the equations and arrange them into an excel spreadsheet, the characteristics of the equations are segregated into distinct rows and columns.

4. Results and discussion

4.1. Mathematical Formulation of Case Study
This study considers natural convection nanofluid flow with mass transfer in the stagnation point region of a three-dimensional body.
  The governing equation from this case study are in dimensionless equation such that

$$C_1 f''' + C_2 (f+h) f'' - C_2 f'^2 + C_3 [1 + \varepsilon \cos(\pi\tau)]\theta + \frac{G_m}{G_r} C_2 [1 + \varepsilon \cos(\pi\tau)]\Phi = C_2 \Omega \frac{\partial f'}{\partial \tau} \tag{3}$$

$$C_1 h''' + C_2 (f+h) h'' - C_2 f'^2 + C_3 [1 + \varepsilon \cos(\pi\tau)]\theta + \frac{G_m}{G_r} C_2 [1 + \varepsilon \cos(\pi\tau)]\Phi = C_2 \Omega \frac{\partial h'}{\partial \tau} \tag{4}$$

$$\frac{1}{P_r} \frac{C_4}{C_5} \theta'' + (f+h)\theta' = \Omega \frac{\partial \theta}{\partial \tau} \tag{5}$$

$$\frac{1}{S_c} C_6 \Phi'' + (f+h)\Phi' = \Omega \frac{\partial \Phi}{\partial \tau} \tag{6}$$

Here Pr and Sc are Prandtl and Schmidt numbers where

$$C_1 = \frac{1}{(1-\phi)^{2.5}}, C_2 = (1 - \phi + \frac{\phi \rho_s}{\rho_f}),$$

$$C_3 = (1 - \phi + \frac{\phi(\rho\beta)_s}{(\rho\beta)_f}), C_4 = \frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)},$$

$$C_5 = (1 - \phi + \frac{\phi(\rho C_p)_s}{(\rho C_p)_f}), C_6 = (\frac{1-\phi}{1+\frac{\phi}{2}})$$

Subject to dimensionless boundary conditions

$$f(\tau,0) = f'(\tau,0) = 0$$
$$h(\tau,0) = h'(\tau,0) = 0$$
$$\theta(\tau,0) = \Phi(\tau,0) = 1$$

4.2. Linearization using Newton's Method

Finite difference scheme is applied for every variable in the first order equations according to equation (1.3) and we applied them into equation (4) – (7). Hence, we obtain:

The finite difference form for equations (1) and (4) – (7), at midpoint $()_{j-\frac{1}{2}}^{i-\frac{1}{2}}$;

$$f_j^i - f_{j-1}^i - \frac{1}{2w_j}[u_j^i + u_{j-1}^i] = 0 \tag{7}$$

$$u_j^i - u_{j-1}^i - \frac{1}{2w_j}[v_j^i + v_{j-1}^i] = 0 \tag{8}$$

$$h_j^i - h_{j-1}^i - \frac{1}{2w_j}[w_j^i + w_{j-1}^i] = 0 \tag{9}$$

$$\theta_j^i - \theta_{j-1}^i - \frac{1}{2w_j}[p_j^i + p_{j-1}^i] = 0 \tag{10}$$

$$\Phi_j^i - \Phi_{j-1}^i - \frac{1}{2w_j}[q_j^i + q_{j-1}^i] = 0 \tag{11}$$

$$\frac{C_1}{2h_j}(v_j^i - v_{j-1}^i) + \frac{C_2}{16}(f_j^i + f_{j-1}^i + h_j^i + h_{j-1}^i)(v_j^i + v_{j-1}^i) - \frac{C_2}{16}(u_j^i + u_{j-1}^i)^2 +$$
$$\frac{C_3}{4}[1 + \varepsilon\cos(\pi\tau)](\theta_j^i + \theta_{j-1}^i) + \frac{C_2 G_m}{4G_r}[1 + \varepsilon\cos(\pi\tau)](\phi_j^i + \phi_{j-1}^i) - \frac{C_2\Omega}{2k_i}[u_{\tau j}^i + u_{\tau j-1}^i] = R_j^i \tag{12}$$

$$\frac{C_1}{2h_j}(b_j^i - b_{j-1}^i) + \frac{C_2}{16}(f_j^i + f_{j-1}^i + h_j^i + h_{j-1}^i)(b_j^i + b_{j-1}^i) - \frac{C_2}{16}(w_j^i + w_{j-1}^i)^2 +$$
$$\frac{C_3}{4}[1 + \varepsilon\cos(\pi\tau)](\theta_j^i + \theta_{j-1}^i) + \frac{C_2 G_m}{4G_r}[1 + \varepsilon\cos(\pi\tau)](\phi_j^i + \phi_{j-1}^i) - \frac{C_2\Omega}{2k_i}(w_{\tau j}^i + w_{\tau j-1}^i) = T_j^i \tag{13}$$

$$\frac{1}{P_r}\frac{C_4}{2h_j C_5}(p_j^i - p_{j-1}^i) + \frac{1}{16}(f_j^i + f_{j-1}^i + h_j^i + h_{j-1}^i)(p_j^i + p_{j-1}^i) - \frac{\Omega}{2k_i}(\theta_{\tau j}^i + \theta_{\tau j-1}^i) = K_j^i \tag{14}$$

$$\frac{1}{S_c}\frac{C_6}{2h_j}(q_j^i - q_{j-1}^i) + \frac{1}{16}(f_j^i + f_{j-1}^i + h_j^i + h_{j-1}^i)(q_j^i + q_{j-1}^i) - \frac{\Omega}{2k_i}(\phi_{\tau j}^i + \phi_{\tau j-1}^i) = L_j^i \tag{15}$$

$$(a_1)_j\delta v_j + (a_2)_j\delta v_{j-1} + (a_3)_j\delta f_j + (a_4)_j\delta f_{j-1} + (a_5)_j\delta h_j + (a_6)_j\delta h_{j-1} + (a_7)_j\delta u_j$$
$$+ (a_8)_j\delta u_{j-1} + (a_9)_j\delta\theta_j + (a_{10})_j\delta\theta_{j-1} + (a_{11})_j\delta\phi_j + (a_{12})_j\delta\phi_{j-1} = (r_1)_j \tag{16}$$

$$(c_1)_j\delta b_j + (c_2)_j\delta b_{j-1} + (c_3)_j\delta f_j + (c_4)_j\delta f_{j-1} + (c_5)_j\delta h_j + (c_6)_j\delta h_{j-1} + (c_7)_j\delta w_j$$
$$+ (c_8)_j\delta w_{j-1} + (c_9)_j\delta\theta_j + (c_{10})_j\delta\theta_{j-1} + (c_{11})_j\delta\phi_j + (c_{12})_j\delta\phi_{j-1} = (r_2)_j \tag{17}$$

$$(d_1)_j\delta p_j + (d_2)_j\delta p_{j-1} + (d_3)_j\delta h_j + (d_4)_j\delta h_{j-1} + (d_5)_j\delta f_j$$
$$+ (d_6)_j\delta f_{j-1} + (d_7)_j\delta\theta_j + (d_8)_j\delta\theta_{j-1} = (r_3)_j \tag{18}$$

$$(e_1)_j\delta q_j + (e_2)_j\delta q_{j-1} + (e_3)_j\delta h_j + (e_4)_j\delta h_{j-1} + (e_5)_j\delta f_j$$
$$+ (e_6)_j\delta h_{j-1} + (e_7)_j\delta\phi_j + (e_8)_j\delta\phi_{j-1} = (r_4)_j \tag{19}$$

With the boundary condition:

$$\delta f_0^i = 0, \delta u_0^i = 0,$$
$$\delta h_0^i = 0, \delta w_0^i = 0,$$
$$\delta \theta_0^i = 1, \delta \phi_0^i = 1,$$
$$\delta u_j^i = 0, \delta w_j^i = 0,$$
$$\delta \theta_j^i = 0, \delta \phi_j^i = 0$$

(20)

4.2.4. Excel Keller Box Output

OUTPUT									
coefficient	var 1	vj	vj-1	delta	var 2	dj	dj-1	delta
C1		0.5h	0.5h	v	v			
C2	f	0.125(j-1/2)	0.125(j-1/2)	v	v	v	0.125(j-1/2)	0.125(j-1/2)	f
C2	v	0.125(j-1/2)	0.125(j-1/2)	h	h	h	0.125(j-1/2)	0.125(j-1/2)	v
(-C2)	u	0.125(j-1/2)	0.125(j-1/2)	u	u			
C3(1+ε*cos(ππ))		0.25	0.25	θ				
C2(Gm/Gr)(1+ε*cos(ππ))		0.25	0.25	Φ				
-C2Ω		1/k	1/k	u				
C1		0.5h	0.5h	b	b			
C2	f	0.125(j-1/2)	0.125(j-1/2)	b	b	b	0.125(j-1/2)	0.125(j-1/2)	f
C2	h	0.125(j-1/2)	0.125(j-1/2)	b	b	b	0.125(j-1/2)	0.125(j-1/2)	h
(-C2)	w	0.125(j-1/2)	0.125(j-1/2)	w	w			
C3(1+ε*cos(ππ))		0.25	0.25	θ				
C2(Gm/Gr)(1+ε*cos(ππ))		0.25	0.25	Φ				
-C2Ω		1/k	1/k	w				
(1/Pr)(C4/C5)		0.5h	0.5h	p	p			
1	f	0.125(j-1/2)	0.125(j-1/2)	p	p	p	0.125(j-1/2)	0.125(j-1/2)	f
1	h	0.125(j-1/2)	0.125(j-1/2)	p	p	p	0.125(j-1/2)	0.125(j-1/2)	h
-Ω		1/k	1/k	θ				
(1/Sc)(C6)		0.5h	0.5h	q	q			
1	f	0.125(j-1/2)	0.125(j-1/2)	q	q	q	0.125(j-1/2)	0.125(j-1/2)	f
1	h	0.125(j-1/2)	0.125(j-1/2)	q	q	q	0.125(j-1/2)	0.125(j-1/2)	h
-Ω		1/k	1/k	Φ				

Figure 1 Output of Newton's Linearization Method

4.3. Excel Keller Box User Interface Template

Figure 2 Template of Input Section

Figure 3 Template of Calculation Section Part 1

No. var	Have prime	WRT ()	1st var				Finding 2nd var				2nd var	var 1
0	NO	NO										
0	NO	NO										
0	NO	NO										
0	NO	NO										
0	NO	NO										
0	NO	NO										
0	NO	NO										
0	NO	NO										
0	NO	NO										
0	NO	NO										
0	NO	NO										

Figure 4 Template of Calculation Section Part 2

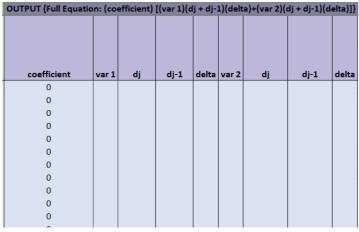| OUTPUT {Full Equation: (coefficient) [(var 1)(dj + dj-1)(delta)+(var 2)(dj + dj-1)(delta)]}							
coefficient	var 1	dj	dj-1	delta	var 2	dj	dj-1	delta
0							
0							
0							
0							
0							
0							
0							
0							
0							
0							
0							

Figure 5 Template of Output Section

Conclusion

The Keller Box method is a numerical scheme that have been used repeatedly by past researcher due to its accuracy and robustness even with various parameters. From this research, an Excel User Interface that uses Keller Box method had been developed where the process of input, calculation and to producing output are taken into deep consideration especially due to the nature of the two-dimensional governing equation. The presence of partial derivative and the number of variables exist in the governing equation can affect the IF formula of the Excel User Interface.

References

[1] Ahmad Kamal, M. H., Rawi, N. A., Ali, A., & Shafie, S. (2019). G-Jitter Induced Natural Convection Nanofluid Flow with Mass Transfer in The Stagnation Point Region of a Three-Dimensional Body. Journal of Physics: Conference Series, 1366(1), 012036. https://doi.org/10.1088/1742-6596/1366/1/012036

[2] Yang, W., Chen, X., Meng, Y., Zhang, X., & Mi, S. (2020a). Numerical Solutions of Unsteady Boundary Layer Flow with a Time-Space Fractional Constitutive Relationship. Symmetry, 12(9), 1446. https://doi.org/10.3390/sym12091446

[3] Mansur, S., & Ishak, A. (2016). Unsteady boundary layer flow of a nanofluid over a stretching/shrinking sheet with a convective boundary condition. Journal of the Egyptian Mathematical Society, 24(4), 650–655. https://doi.org/10.1016/j.joems.2015.11.004.

[4] Khan, Z., Khan, M. A., Siddiqui, N., Ullah, M., & Shah, Q. (2018). Solution of magnetohydrodynamic flow and heat transfer of radiative viscoelastic fluid with temperature dependent viscosity in wire coating analysis. PLOS ONE, 13(3), e0194196. https://doi.org/10.1371/journal.pone.0194196

[5] Liu, W., Cui, J., & Xin, J. (2018). A block-centered finite difference method for an unsteady asymptotic coupled model in fractured media aquifer system. Journal of Computational and Applied Mathematics, 337, 319–340. https://doi.org/10.1016/j.cam.2017.12.035

[6] Kudenatti, R. B., & Jyothi, B. (2019). Two-dimensional boundary-layer flow and heat transfer over a wedge: numerical and asymptotic solutions. Thermal Science and Engineering Progress, 11, 66–73. https://doi.org/10.1016/j.tsep.2019.03.006

[7] Wang, J., & Guo, X. (2020). Adsorption kinetic models: Physical meanings, applications, and solving methods. Journal of Hazardous Materials, 390, 122156. https://doi.org/10.1016/j.jhazmat.2020.122156

[8] Dmitry Grapov, John W. Newman, imDEV: A graphical user interface to R multivariate analysis tools in Microsoft Excel, Bioinformatics, Volume 28, Issue 17, 1 September 2012, Pages 2288–2290.