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Geometrical Transformation of a Yoshimoto Cube

Wan Heng Fong*, Dickson Lui
Department of Mathematical Sciences, Faculty of Science
Universiti Teknologi Malaysia, 81310 Johor Bahru, Malaysia
*Corresponding author: fwh@utm.my

Abstract
A Yoshimoto cube is a geometrical artwork that transforms into another form and returns to its original form without reversing the transformation. According to past research, this property of a Yoshimoto cube relates to the structure of the Yoshimoto cube. The objectives of this research are to identify the structures and properties of a Yoshimoto cube, to obtain the transformation of a Yoshimoto cube through orthographic projection, and to analyse the transformation of a Yoshimoto cube in different forms by using the properties of hinged dissection. The scope of this research is to investigate the transformation of a Yoshimoto cube from a 2×2×2 cube into a 1×2×4 cuboid. In this research, the six different forms of a Yoshimoto cube are presented. By using orthographic projection of a Yoshimoto cube in each different form, the two-dimensional shadow is obtained. It is found that the relationship between the shadow of a Yoshimoto cube relates to the hinge dissection. The significance of this research is to provide insights on the properties of transformable objects with hinges and to understand their transformation.
Keywords: Yoshimoto cube; Orthographic projection; Hinge dissection; Transformation

1. Introduction

A Yoshimoto cube is a geometrical artwork invented by Naoki Yoshimoto in 1971 which consists of many interesting properties. First, a Yoshimoto cube is a cube that can be separated into two stellated rhombic dodecahedra [1]. Second, a Yoshimoto cube is an object that is capable to transform into another shape and turn it back to its original form. A Yoshimoto cube is an object that contains many components, each component is essential for the transformation. Thus, there must be a mechanism of a Yoshimoto cube that resulted in this kind of transformation. One of the mechanisms of the transformation involved in the transformation of the Yoshimoto cube from a cube to a cuboid is the hinge.

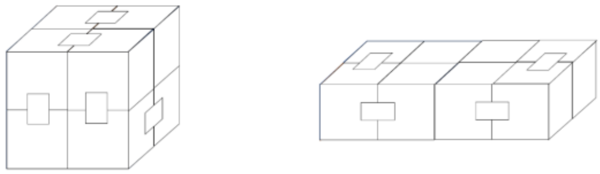
Many researchers have studied this interesting object, Escher [2] found that there is another similar artwork that was demonstrated by John Conway in 1996 called Conway's cubes. This cube also shares some similar properties which include the transformation from a cube into a cuboid. After that, Komatsu and Shizukawa [3] found out that the hinge plays an important role in the transformation of a Yoshimoto cube. The transformation sequence and procedure of a Yoshimoto cube are found and each different hinge plays the role in the transformation.

In this research, the objectives are to identify the structures and properties of a Yoshimoto cube, to obtain the transformation of a Yoshimoto cube through orthographic projection, and to analyse the transformation of a Yoshimoto cube in different forms by using the properties of hinged dissection. The significance of this research is to provide insights into the properties of transformable objects with hinges and to understand their transformation. Then, the research is able to enhance the understanding of the reader on the relationship between the transformation of a Yoshimoto cube and its two-dimensional shadow obtained from the orthographic projection.

2. Literature Review

2.1. Yoshimoto Cube

A Yoshimoto cube is a 2×2×2 cube built of eight small cubes stacked together. A Yoshimoto cube also is an example of a cube ring and it has the connectors between cubes which are called hinge edges [3]. There is a special property of the Yoshimoto cube in which it can transform from a cube to a cuboid. Figure 1 shows the cube form and the cuboid form of a Yoshimoto cube.



A Yoshimoto cube in a cube form. A Yoshimoto cube in a cuboid form.

Figure 1 The cube form and the cuboid form of a Yoshimoto cube.

Another interesting property of the Yoshimoto cube is that it is an infinitely open body [2]. It is called infinitely open because the cube is able to perform a transformation back to its original form without reversing the transformation that had been done. Apart from this, a Yoshimoto cube is also an artwork that can be constructed into an origami which can implemented in mathematical learning process [4,5]. Furthermore, Yoshimoto cube is also used as an idea in architecture [6].

2.2. Orthographic Projection

Orthographic projection is one type of the parallel projections [7]. Orthographic projection is mostly used for finding the shadow of the object [8,9]. Besides that, orthographic projection is also a useful method to create a two-dimensional figure from a three-dimensional object. Figure 2 shows an object undergoing orthographic projection.

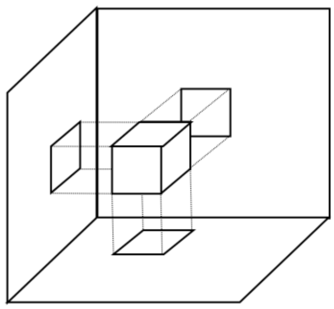


Figure 2 An object undergoing orthographic projection.

The orthographic projection starts with the object contained in a cubical box [10]. Then, the object is projected in six different directions on the six different surfaces of the cubical box. Thus, the shadows of the object are projected on six different sides of the cubical box [10].

2.3. Hinged dissection

Geometric dissection is a rearrangement of a geometric figure from its cutting pieces [11]. A hinged dissection is a geometric dissection where the cutting pieces are linked together with a hinge and the rearrangement is still preserved. Figure 3 shows an example of a two-dimensional hinged piece.

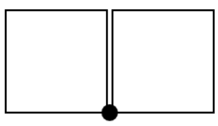


Figure 3 An example of a two-dimensional hinged piece.

A dissection is called swing-hingeable when its pieces are linked together with swing hinges and the pieces are able to form two kinds of figures after swinging in different directions. If the swing hinges are not allowed then it is called dissection unhingeable [12]. The interesting part of the swing hinged dissection is that the dissection can be presented as a two-dimensional figure [13,14]. Thus, the properties of swing-hinged dissection can be applied to any two-dimensional hinge-liked figure.

3. Transformation of a Yoshimoto Cube

3.1. Forms of a Yoshimoto Cube

There are six different forms involved in the transformation of a Yoshimoto cube as presented by Komatsu and Shizukawa [3]. Each form is the product of the rotation of a hinge in the Yoshimoto cube. Figure 4 shows all the six forms of a Yoshimoto cube.

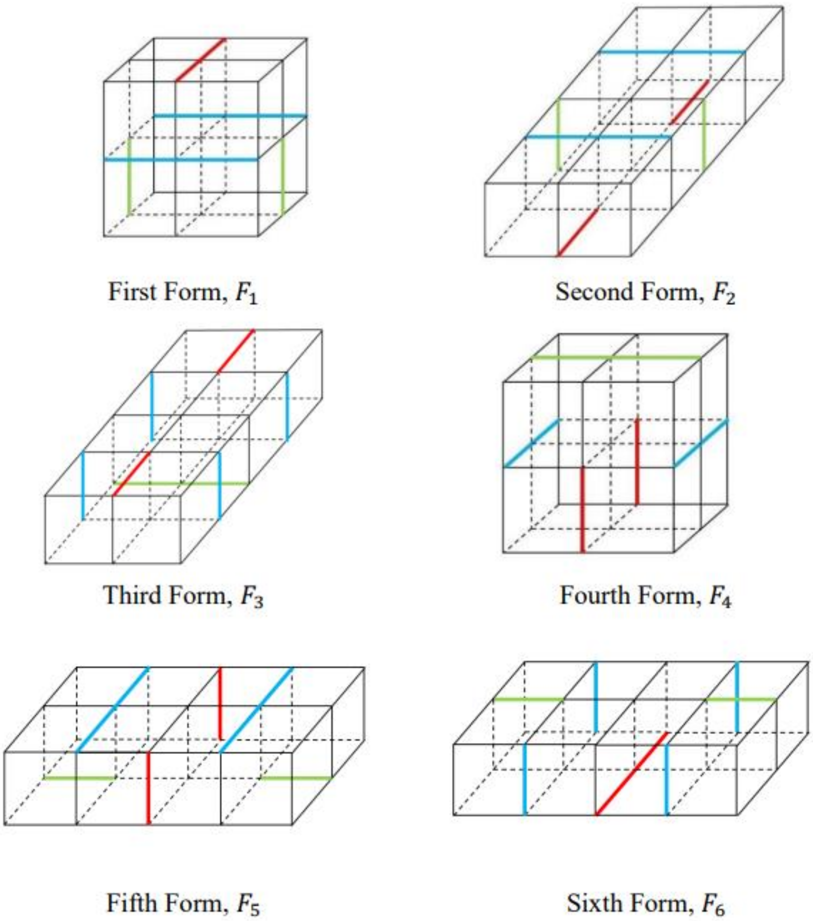


Figure 4 All the six forms of a Yoshimoto cube.

The eight hinges in each form of a Yoshimoto cube are coloured in three different colours, where the colour differentiate the direction of the hinges.

3.2. Transformation Sequence of a Yoshimoto Cube

According to Komatsu and Shizukawa [3], the six different forms of a Yoshimoto cube are shown and every form can be transformed into another form by different transformation of the hinges, Thus, each form can transform back into the next form which ultimately forms a cycle of the transformation of a Yoshimoto cube. Figure 5 shows the cycle of the transformation of a Yoshimoto cube.

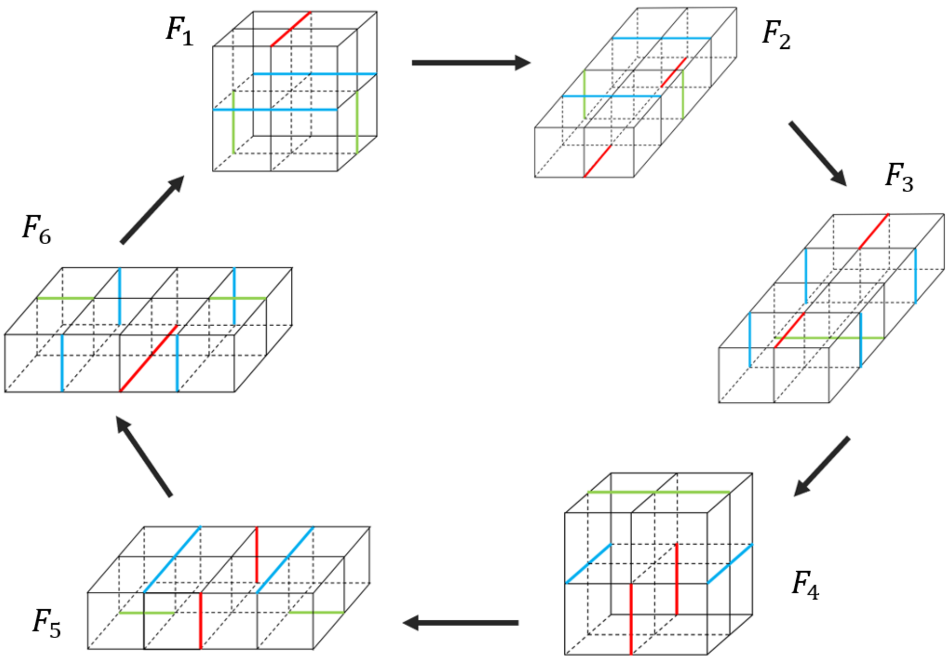
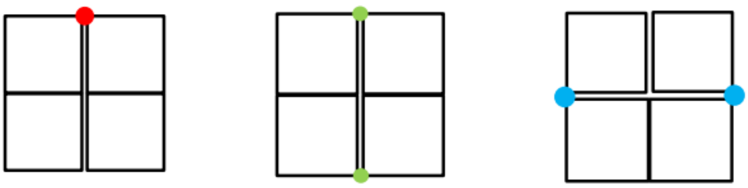


Figure 5 The cycle of the transformation of a Yoshimoto cube.

3.3. Orthographic Projection and shadows of a Yoshimoto Cube

The orthographic projection is one of the methods to determine the hinge movement of the Yoshimoto cube from a two-dimensional perspective. The orthographic projection can obtain all the three orthogonal directions of each Yoshimoto cube form. The obtained two-dimensional faces of a Yoshimoto cube form are called shadows.

The orthographic projection of each Yoshimoto cube form has six different shadows. There are three pairs of shadows that are the same: the front and back shadow, the left and right shadow, and the top and bottom shadow. Figure 6 shows the shadows of the first form of a Yoshimoto cube.



(a) The front-back shadow

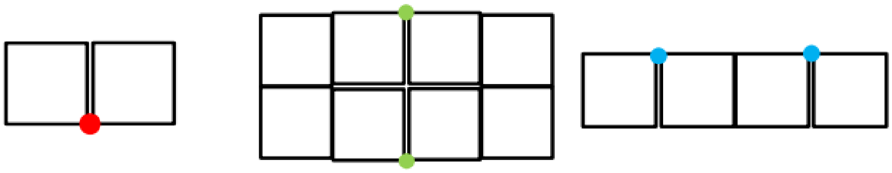
(b) The top-bottom shadow

(c) The left-right shadow

Figure 6 The shadows of the first form of a Yoshimoto cube.

The shadows obtained from the orthographic projection of all forms of a Yoshimoto cube also includes the hinges. The first form of a Yoshimoto cube has three 2×2 square shadows. The first shadow of the first form is the front-back shadow which consists of two 1×2 rectangles that are connected by a red hinge. Then, the second shadow is the top-bottom shadow which has two green hinges and two 1×2 rectangles. The two green hinges of this shadow are connected to the two 1×2 rectangles together. Meanwhile, the third shadow is the left-right shadow which has three pieces and two hinges. The two 1×1 squares of this shadow are connected to a 2×1 rectangle with a blue hinge respectively.

The figures show all the shadows of the second form until the sixth form. First, Figure 7 shows the shadows of the second form. Next, Figure 8 shows the third form. Then, Figure 9 shows the fourth form. Lastly, Figure 10 shows the fifth form and Figure 11 shows the sixth form of a Yoshimoto cube.

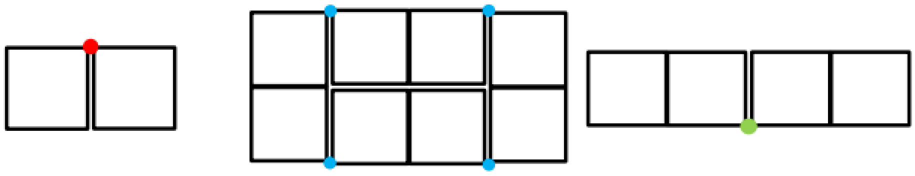


(a) The front-back shadow

(b) The top-bottom shadow

(c) The left-right shadow

Figure 7 The shadows of the second form of a Yoshimoto cube.

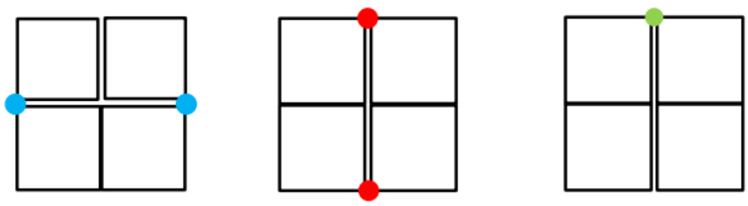


(a) The front-back shadow

(b) The top-bottom shadow

(c) The left-right shadow

Figure 8 The shadows of the third form of a Yoshimoto cube

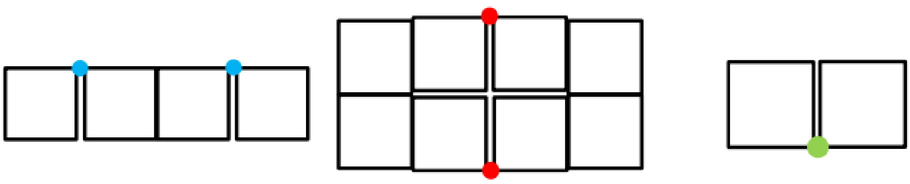


(a) The front-back shadow

(b) The top-bottom shadow

(c) The left-right shadow

Figure 9 The shadows of the fourth form of a Yoshimoto cube.



(a) The front-back shadow

(b) The top-bottom shadow

(c) The left-right shadow

Figure 10 The shadows of the fifth form of a Yoshimoto cube.

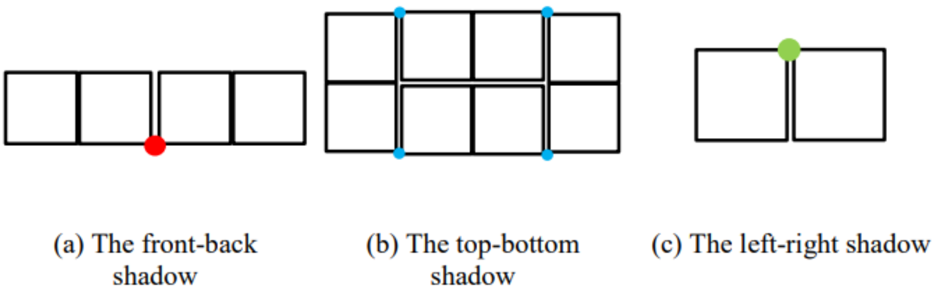


Figure 11 The shadows of the sixth form of a Yoshimoto cube.

These shadows have their own structure and hinges that connect their pieces together. Each form of a Yoshimoto cube has three different shadows that represent the orthogonal direction of the colour hinges.

4. Hinged Dissection of a Yoshimoto Cube

4.1. The Shadow and Hinged Dissection

In hinged dissection, a figure is divided into some smaller pieces and has a hinge that connected the pieces together. This is very similar to the shadow of a Yoshimoto cube where the pieces are connected by hinges as discussed in Figure 6.

4.2. Transformation of Hinged Dissection

The transformation of hinged dissection has two important factors to be considered. The first factor is the shape of the hinged dissection. The second factor is the rotation of the pieces along with the hinges in the hinged dissection. Figure 12 shows the hinged dissection transformation of the shadows of a Yoshimoto cube.

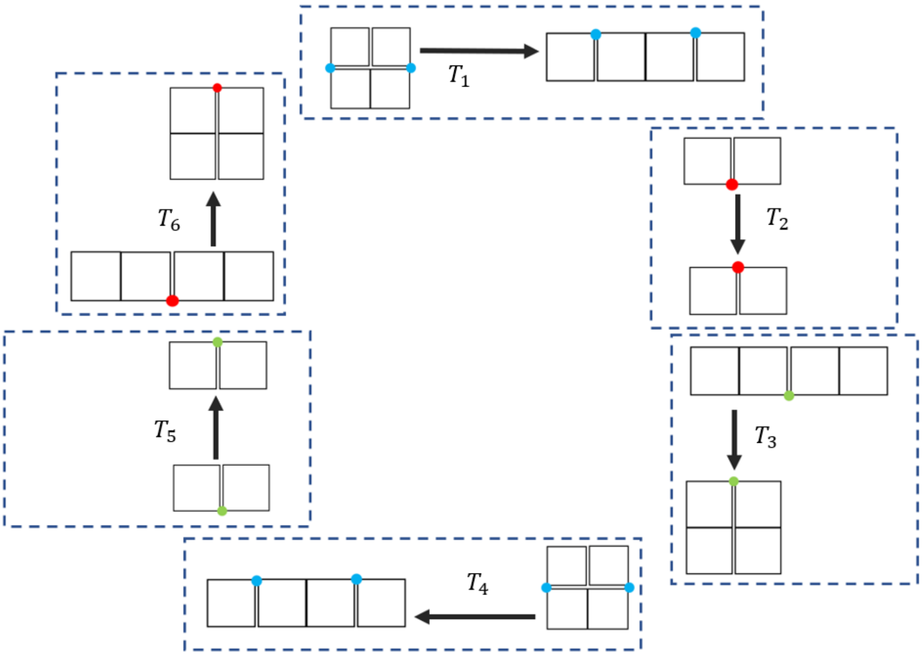


Figure 12 The hinged dissection transformation on the shadows of a Yoshimoto cube.

The shadows of a Yoshimoto cube obtained from the orthographic projection have the same structure as the hinged dissection described. Thus, these shadows can be the initial shape of the hinged dissection. Some of the shadows of each form of a Yoshimoto cube is considered to perform the hinged dissection transformation. Then, these transformations are combined together to show a full picture of the relationship between these transformations and the six different forms of a Yoshimoto cube.

4.3. Transformation Cycle of Hinged Dissection

From these transformations, some of the shadows of the forms are shown. These shadows can group under a category of each form. Figure 13 shows the hinged dissection transformations of the first form, F_1, the second form, F_2, the third form, F_3, the fourth form, F_4, the fifth form, F_5, and the sixth form, F_6.

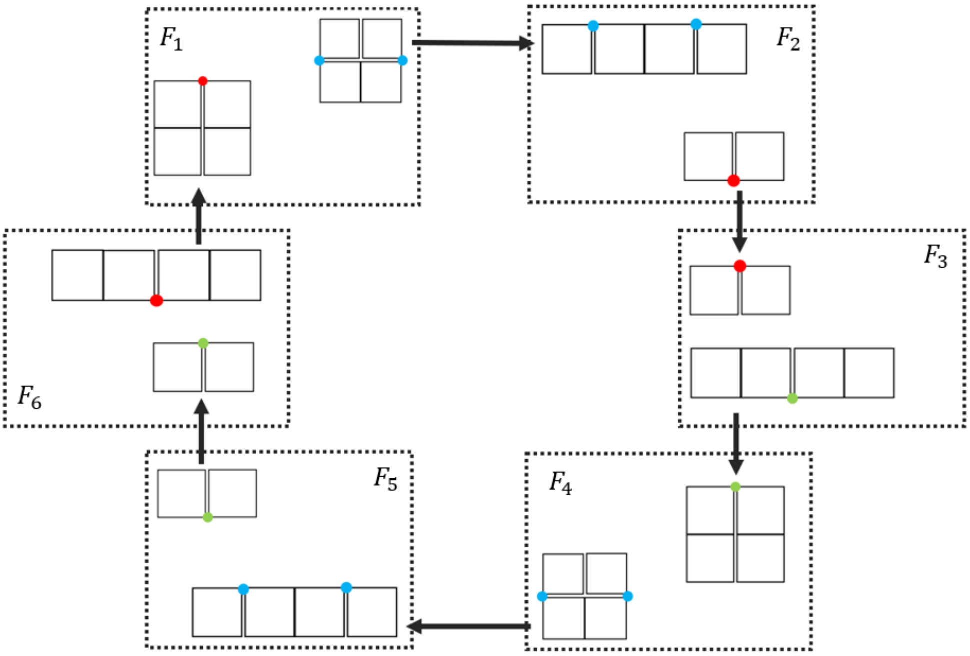


Figure 13 The hinged dissection transformations of the first form, F_1, the second form, F_2, the third form, F_3, the fourth form, F_4, the fifth form, F_5, and the sixth form, F_6.

Since each form is connected by these transformations thus there is a relationship between each form via these transformations. Therefore, the transformation of the hinged dissection becomes the relationship between the two forms and it formed a transformation cycle of hinged dissection.

4.4. Transformation of Hinged Dissection with Backward Transformation

The hinged dissection transformation of T_1, T_2, T_3, T_4, T_5, and T_6 have a backward hinged dissection transformation. Since each form is connected by these transformations, thus there is a relationship between each form via these transformations. Figure 14 shows the hinged dissection transformations of F_1, F_2, F_3, F_4, F_5, and F_6 including backward transformation.

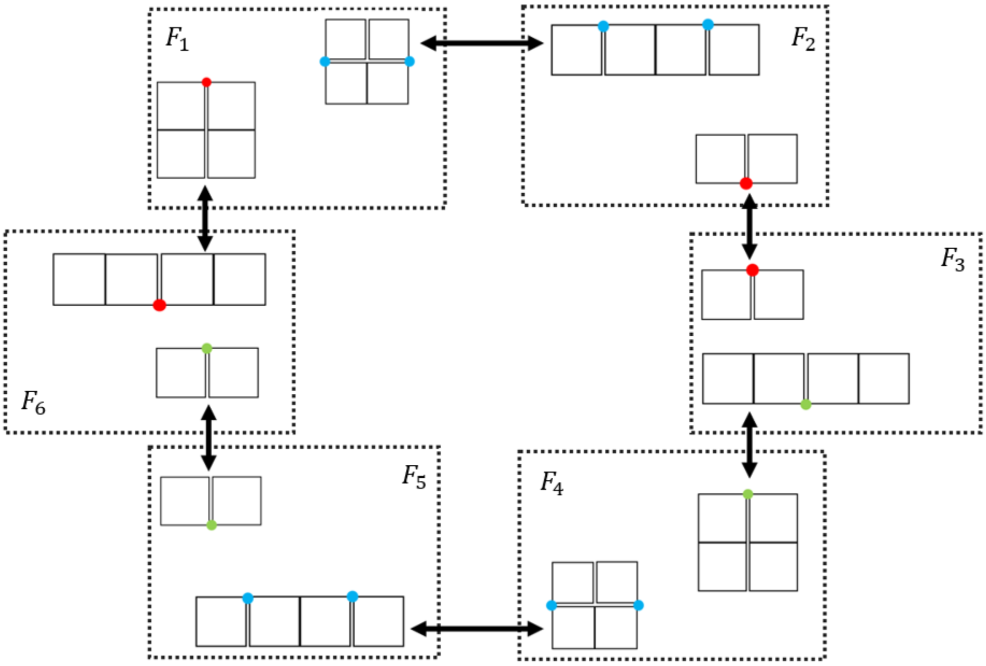


Figure 14 The hinged dissection transformations of F_1, F_2, F_3, F_4, F_5, and F_6 including backward transformation.

The relationship between each form is connected by a transformation of the hinged dissection. From the backward hinged dissection transformation then the new relationship between the hinged dissection transformation and each form is obtained. Each form of a Yoshimoto cube has two hinged dissection transformation that allow it to transform into the next form and the previous form.

4.5 Constraint of Hinged Dissection

The constraints of the hinged dissection are on some of the properties of the hinged dissection and assumption of the transformation of the hinged dissection.

The properties of the hinged dissection are discussed in this section. There are three important properties of a hinged dissection. First, the hinged dissection must have at least a hinge that connects two pieces. Second, the hinged dissection pieces cannot overlap each other. The pieces of the hinged dissection cannot pass through the other piece. Third, the hinged dissection pieces rotate with the center of rotation on the hinge. These are the important properties of a hinged dissection and the transformation of a hinged dissection.

There are some assumptions about the hinged dissection transformation. First, the hinged dissection and the shadow of a Yoshimoto cube are similar. Thus, the first assumption for the hinged dissection transformation is to treat the shadow of a Yoshimoto cube like a hinged dissection. Then, the second assumption is the selection of the initial shape before the hinged dissection transformation. The hinged dissection transformation has many possible shapes of transformation, but the cubical shape is the one that is considered in this research. Next, the third assumption is that, the initial shape of the transformation of the hinged dissection can also be the final shape. The shape can be transformed from the previous shape to another, so as from another shape to the previous shape. Thus, these assumptions are made to relate between the transformation of hinged dissection and the transformation of a Yoshimoto cube.

Conclusion

In this research, some properties of the transformation of a Yoshimoto cube is identified. The transformation of a Yoshimoto cube involves six different forms of a Yoshimoto cube. Then, by relating the shadows of each form to a hinged dissection, the hinged dissection transformation is introduced. Furthermore, by using the properties of the hinged dissection, the hinged dissection transformations of each form are able to obtain a structure similar to the transformation of a Yoshimoto cube. This research also focuses on finding the relationship between a hinged dissection of the shadow of a Yoshimoto cube and the transformation of a Yoshimoto cube itself. The future work for the research is to investigate and identify some other object that is similar to a Yoshimoto cube that can be related to the hinged dissection.

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