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Solving Mixed Convection Boundary Layer Flow of Viscoelastic Nanofluid Past Over a Sphere in Presence of Heat Generation Effect

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Abstract
The scope of the research focuses on the mixed convection boundary layer flow of viscoelastic nanofluid past over a sphere in presence of heat generation. For this study, the Tiwari-Das model has been considered where the copper (Cu) has been chosen as the nanoparticle and Carboxymethyl cellulose solution (CMC) has been chosen as the base fluid. Besides, the BVP4C in MATLAB has been used to develop a simulation to investigate the flow behaviour of the nanofluid model as influenced by viscoelastic parameters and heat characteristics. The relationship between several parameter such as velocity and temperature profile as well as skin friction and heat transfer have been analyzed graphically.

Keywords: Mixed convection; boundary layer flow; viscoelastic; nanofluid; sphere; heat generation effect; BVP4C

1. Introduction

Nanotechnology has applications in almost every industry and allows for the creation of new enabling science. It has been established in many industrial and engineering applications for a long time. In recent years, investigation of nanofluid flow has gained considerable interest by the researchers owing to an increase in the implementations in different fields of technology and various types of applications. Thermal conductivity and dynamic viscosity are two important properties of nanofluids that indicate heat transfer and flow [1]. As a result, nanofluids are widely used to improve mechanical cooling systems, such as engines, nuclear reactors and electronics, as well as various types of heat sources.

Nanofluids have received a lot of attention in recent years because of their potential use as an enhanced thermophysical heat transfer fluid and their importance in applications including drug delivery and oil recovery. Nanofluids are defined as the addition of a small quantity of nanometer-sized particles with a nominal size of less than 100 nm to base fluids such as oil, water, biofluids, ethylene, and lubricants [2]. As nanofluids have enhanced thermo-physical properties, their application as a heat transfer fluid is limitless. These improved properties are primarily due to Brownian motion, which gives it an advantage over conventional fluids [3].

Since then, the growth of nanofluid mathematical modelling has become a popular field of study and many researchers have been studying mixed convection flow in nanofluids. For instance, Mahat *et al*. [4] studied about the mixed convection boundary layer flow past a horizontal circular cylinder in viscoelastic nanofluid with constant wall temperature. Then, RamReddy *et al.* [5] researched about the Soret effect on mixed convection heat and mass transfer in the boundary layer region of a semi-infinite vertical flat plate in a nanofluid under the convective boundary conditions. In research from Buschmann *et al.* [6], stated that regardless of nanoparticle concentration or composition, the heat transfer enhancement given by nanofluids equals the increase in the thermal conductivity of the nanofluid as compared to the base fluid.

2. Literature Review

Mixed convection is the combination of two convection flows which are forced convection and natural or also known as free convection. Forced convection is a fluid flow that is caused by an external or outside factor where there is motion within a fluid caused due to an external source such as a blower for a gas or a pump for a liquid. On the other hand, natural or free convection is a type of mechanism where the fluid motion does not involve any external or outside factor but only based on density differences in the fluid. Also, it is a motion within a fluid caused due to density variations that are known as buoyancy forces. Mixed convection boundary layer flow of nanofluids has stimulated several researchers due to its widespread prevalence in industrial sectors especially in engineering, where including cooling and heating are two processes that are important for increasing the efficiency of a thermodynamic system. Recently, Tibaut *et al.* [7] have conducted a numerical solution of mixed convection of a nanofluid in a circular pipe with different numerical models and considering the viscosity of a nanofluid must be assigned in the pipe flow.

A nanofluid is a liquid that contains nanometer-sized particles known as nanoparticles. Water, ethylene glycol, and oil are examples of common base fluids. For the past decades, the scientists and engineers have made phenomenal discoveries regarding nanoparticles and nanofluid flow past over a cylinder/sphere. For instance, some nanofluids exhibit superior thermal properties such as anomalously high thermal conductivity at low nanoparticle concentrations, strong temperature and size dependent thermal conductivity, a nonlinear relationship between thermal conductivity and concentration, and a threefold increase in the critical heat flux at a small particle concentration of the order of 10 ppm [8]. Mohamed *et al.* [9] extended this study and stated that nanofluids have been discovered to have improved thermophysical properties such as thermal conductivity, thermal diffusivity, viscosity, and convective heat transfer coefficients when compared to base fluids such as oil or water.

The problem of mixed convection boundary layer flow past over a sphere has been the subject of research for many years due to its importance in understanding the phenomenon and its extensive engineering applications. Rashad [10] studied numerically about the flow of the natural convection boundary layer along a sphere embedded in a porous medium filled with a nanofluid. In addition, the study came out with an increase in the values of Lewis number that results in flow acceleration as represented by increases in the velocity profiles due to a reduction in the nanoparticle buoyancy effect. In similar research, Kasim *et al*. [11] studied the mixed convection flow of viscoelastic fluid over a sphere under convective boundary condition embedded in porous medium.

The study of mixed convection flow past over a sphere was done by Salleh *et al.* [12]. The study investigated the mixed convective flow over a solid sphere with Newtonian heating, and their findings revealed that the velocity and temperature distribution profiles decrease as the Prandtl number increases. Furthermore, Tham *et al.* [13] studied numerically the steady mixed convection boundary layer flow about a solid sphere with a constant surface temperature embedded in a porous medium saturated by a nanofluid containing gyrotactic microorganisms a stream flowing vertically upwards for both cases of a heated and cooled sphere. Salleh *et al.* [14] investigated mixed convection boundary layer flow about a solid sphere with Newtonian heating. Besides, Abdul El-Aziz [15] investigated the effects of variable viscosity on mixed convection flow along a semi-infinite unsteady stretching sheet of viscous dissipation.

Convection is the heat transfer that occurs between a moving fluid and a surface when both are at different temperatures. In general, convection is formed by two mechanisms which are convection caused by random motion of molecules or also known as diffusion and energy transferred by fluid movement. Several researchers have started to study the effects of heat generation on mixed convection flow in the last few decades. The constant heat flux solution for mixed convection boundary layer viscoelastic fluid was discussed and obtained that the boundary layer for a viscoelastic fluid is less delayed in comparison with a Newtonian fluid [16]. Oahimire and Olajuwon [17] studied the heat and mass transfer effects on an unsteady flow of a chemically reacting micropolar fluid over an infinite vertical porous plate. Also, Singh *et al.* [18] studied the effects of thermal radiation on mixed convection flow of a micropolar fluid from an unsteady stretching surface with viscous dissipation and heat generation/absorption. This investigation was extended to the effect of heat generation on mixed convection flow in nanofluids over a horizontal circular cylinder [19]. They carried out the result when the mixed convection parameter was increased, the velocity profile increased, and the temperature profile decreased. Additionally, as the Prandtl number parameter increases, the velocity and temperature profiles decrease.

3. Mathematical Formulation

The mixed convection boundary layer flow past over a sphere of radius a, where placed in a viscoelastic nanofluid with convective boundary condition and in presence of heat generation effect is considered. It is assumed that the velocity outside the boundary is $\bar{u}_e(\bar{x})$ and the temperature of the ambient nanofluid is T_∞, while the constant temperature of the surface of the sphere is T_w. It is assumed that $T_w < T_\infty$ corresponds to a cooled sphere and $\frac{1}{2}U_\infty$ is the velocity of the constant free stream.

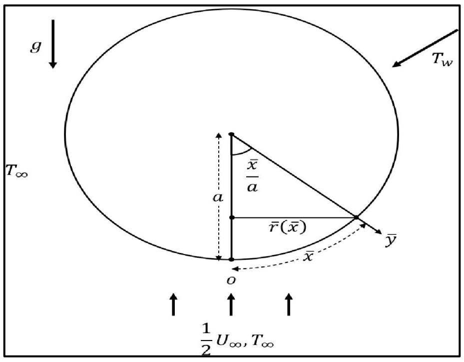

Figure 1 Coordinates system and the Flow model

The governing equations for this problem are as follows, based on the above assumptions and the model of the nanofluid proposed by Tiwari and Das [20] and Merkin [21].

$$\frac{\partial}{\partial \bar{x}}(\bar{r}\bar{u}) + \frac{\partial}{\partial \bar{y}}(\bar{r}\bar{v}) = 0. \tag{1}$$

$$\rho_{nf}\left(\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right) = \rho_{nf}\left(\bar{u}_e\frac{\partial \bar{u}_e}{\partial \bar{x}}\right) + \mu_{nf}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + k_0\left(\frac{\partial}{\partial \bar{x}}\left(\bar{u}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right) + \bar{v}\frac{\partial^3 \bar{u}}{\partial \bar{y}^3} + \frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{v}}{\partial \bar{y}^2}\right)$$
$$+g(\rho\beta)_{nf}(T - T_\infty)\sin\left(\frac{\bar{x}}{a}\right). \tag{2}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \alpha_{nf}\frac{\partial^2 T}{\partial \bar{y}^2} + \frac{Q_o}{\left(\rho C_\rho\right)_{nf}}(T - T_\infty), \tag{3}$$

subjected to boundary conditions

$$\bar{u} = 0, \quad \bar{v} = 0, \quad T = T_w, \quad \text{on } \bar{y} = 0, \quad \bar{x} \ge 0,$$
$$\bar{u} = \bar{u}_e(\bar{x}), \quad \frac{\partial \bar{u}}{\partial \bar{y}} = 0, \quad T = T_\infty, \quad \text{as } \bar{y} = \infty, \quad \bar{x} \ge 0, \tag{4}$$

where

$$\alpha_{nf} = \frac{k_{nf}}{(\rho C_\rho)_{nf}}, \quad \alpha_f = \frac{k_f}{(\rho C_\rho)_f}, \quad \mu_{nf} = \frac{\mu_f}{(1-\phi)^{0.25}},$$

$$\rho_{nf} = (1-\phi)\rho_f + (\phi)\rho_s,$$

$$(\rho C_\rho)_{nf} = (1-\phi)(\rho C_\rho)_f + (\phi)(\rho C_\rho)_s,$$

$$(\rho\beta)_{nf} = (1-\phi)(\rho\beta)_f + (\phi)(\rho\beta)_s, \tag{5}$$

$$k_{nf} = k_f \frac{(k_s + 2k_f) - 2\emptyset(k_f + k_s)}{(k_s + 2k_f) - \emptyset(k_f + k_s)}.$$

Here $\bar{x}$ and $\bar{y}$ are the Cartesian coordinates along the surface of the sphere. Besides, $\bar{y}$ is the coordinate measured normal the surface of sphere. Then, $\bar{u}$ and $\bar{v}$ are the velocity components, g is the gravity acceleration, T is the temperature selected fluid, $k_o > 0$ is the constant of the viscoelastic material, ϕ is the nanoparticle volume fraction, v is the kinematic viscosity, β_{nf}, β_f and β_s are the coefficient of the thermal expansions of nanofluid, fluid and sphere, $(C_\rho)_{nf}$, $(C_\rho)_f$ and $(C_\rho)_s$ are the heat capacitance of nanofluid, fluid and sphere, k_{nf}, k_f and k_s are the thermal conductivities of the nanofluid, fluid and sphere , ρ_{nf} , ρ_f and ρ_s are the densities of the nanofluid, fluid and sphere, μ_{nf} and μ_f are the dynamic of the viscosities of nanofluid and fluid and lastly, α_{nf} and α_f are the thermal diffusivities of the nanofluid and fluid. $\bar{u}_e(\bar{x})$ is the local free stream velocity outside the boundary layer and $\bar{r}(\bar{x})$ is the radial distance from the symmetrical axis to surface of the sphere. Both equations are given by

$$\bar{u}_e(\bar{x}) = \frac{3}{2}U_\infty \sin\left(\frac{\bar{x}}{a}\right) \; and \; \bar{r} = (\bar{x}) = a\sin\left(\frac{\bar{x}}{a}\right) \tag{6}$$

Table 1 below shows the thermophysical properties of the base fluid (CMC-water) and the nanoparticle which is Copper (Cu).

Table 1 Thermophysical properties of base fluid and nanoparticles.

Physical Properties	$\rho(kgm^{-3})$	$C_\rho(Jkg^{-1}K^{-1})$	$k(Wm^{-1}K^{-1})$	$\beta \times 10_5(K^{-1})$
Based fluid	997.1	4179	0.613	21
Copper (Cu)	8933	385	401	1.67

Continuity equation, momentum equation and energy equation are transformed into dimensionless equations by substitute the dimensionless variables. The dimensionless variables are defined as:

$$x = \frac{\bar{x}}{a}, \quad y = Re^{1/2}\left(\frac{\bar{y}}{a}\right), \quad u = \frac{\bar{u}}{U_\infty}, \quad v = Re^{1/2}\left(\frac{\bar{v}}{U_\infty}\right),$$

$$r(x) = \frac{\bar{r}(\bar{x})}{a}, \quad u_e(x) = \frac{\bar{u}_e}{U_\infty}, \quad \theta = \frac{(T - T_\infty)}{(T_w - T_\infty)}, \tag{7}$$

where $Re = \frac{U_\infty a}{v}$ is the Reynolds number. Substitution of Equation (7) into Equation (1) to (3) leads to the following dimensionless equations,

$$\frac{\partial}{\partial \bar{x}}(ru) + \frac{\partial}{\partial \bar{y}}(rv) = 0. \tag{8}$$

$$\left((1-\phi) + \phi\frac{\rho_s}{\rho_f}\right)\left(u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y}\right) = \left((1-\phi) + \phi\frac{\rho_s}{\rho_f}\right)\left(u_e\left(\frac{\partial u_e}{\partial x}\right)\right) + \left(\frac{1}{(1-\phi)^{0.25}}\right)\left(\frac{\partial^2 u}{\partial y^2}\right) \tag{9}$$

$$+k_o\left[\frac{\partial}{\partial x}\left(\frac{Re}{a^2}\left(u\frac{\partial^2 u}{\partial y^2}\right)\right)+\frac{Re}{a^2}\left(v\frac{\partial^3 u}{\partial y^3}\right)+\frac{Re}{a^2}\left(\frac{\partial u}{\partial y}.\frac{\partial^2 v}{\partial y^2}\right)\right]+\left((1-\phi)+\phi\frac{(\rho\beta)_s}{(\rho\beta)_f}\right)\lambda\theta\sin x.$$

$$\left((1-\phi)+(\phi)\frac{\left(\rho C_\rho\right)_s}{\left(\rho C_\rho\right)_f}\right)\left(u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial x}\right)=\frac{1}{Pr}\left[\frac{\left(k_s+2k_f\right)-2\emptyset(k_f-k_s)}{\left(k_s+2k_f\right)+\emptyset(k_f-k_s)}\left(\frac{\partial^2\theta}{\partial y^2}\right)\right]+\gamma\theta. \quad (10)$$

the boundary condition become:

$$u=0,\quad v=0,\quad \theta'=-1,\quad \text{on } y=0,\quad x\geq 0,$$
$$\bar{u}=u_e(x)=\frac{3}{2}\sin x,\quad \frac{\partial u}{\partial y}=0,\quad \theta=0,\quad \text{as } y=\infty,\quad x\geq 0. \quad (11)$$

where

$$K=\frac{k_0U_\infty}{\alpha\rho_f v},\quad Pr=\frac{v}{\alpha},\quad \gamma=\frac{Q_o a}{\left(\rho C_\rho\right)_f U_\infty},$$
$$\lambda=\frac{Gr}{Re}=\frac{g\beta_f(T_w-T_\infty)a}{U_\infty^2},\quad Gr=\frac{g\beta_f(T_w-T_\infty)a^3}{v_f^2}. \quad (12)$$

Here the K is the dimensionless viscoelastic parameter, γ is the heat generation parameter, Pr is the Prandtl number, λ is the constant mixed convection parameter and Gr is known as Grashof number. The following variables are being assumed to solve Equations (8), (9) and (10) together with the boundary conditions (11):

$$\psi=xr(x)f(x,y),\quad \theta=\theta(x,y), \quad (13)$$

where ψ is the stream function defined as follows,

$$u=\frac{1}{r}\frac{\partial\psi}{\partial y},\quad v=-\frac{1}{r}\frac{\partial\psi}{\partial x}, \quad (14)$$

and also considering,

$$u_e(x)=\frac{\bar{u}_e(\bar{x})}{U_\infty}=\frac{3}{2}\sin x,\quad r(x)=\sin x. \quad (15)$$

Here, $u_e(x)$ is the local free stream velocity outside the boundary layer and $r(x)$ is the radial distance from the symmetrical axis to the surface of the sphere. The stream function in Equation (13) is automatically satisfy for continuity equation. Next, by substituting the Equation (13) into Equation (8) to (10), and considering Equation (14) and (15), the equation obtained as follows:

$$\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial^2\psi}{\partial x\partial y}=0. \quad (16)$$

$$\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)\left(x\frac{\partial^3 f}{\partial x\partial y^2}+\left(-\frac{\cos x}{\sin x}\right)\frac{\partial^2 f}{\partial y^2}+x\cos x\,\frac{\partial^2 f}{\partial y^2}\right)+\left(x\frac{\sin x}{\cos x}+1\right)\left(\frac{\partial^2 f}{\partial y^2}-\frac{\partial^2 f}{\partial y^2}f\right)$$
$$=\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)\frac{9}{4}\frac{\sin x\cos x}{x}+\frac{1}{(1-\phi)^{2.5}}\left(\frac{\partial^3 f}{\partial y^3}\right)$$
$$+K\left(x\frac{\partial^2 f}{\partial x\partial y}\frac{\partial^3 f}{\partial y^3}+2\left(1+x\frac{\cos x}{\sin x}\right)\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+x\frac{\partial^4 f}{\partial x\partial y^3}\frac{\partial f}{\partial y}-x\frac{\partial^4 f}{\partial y^4}\frac{\partial f}{\partial y}\right. \quad (17)$$

$$-x\frac{\partial^3 f}{\partial x\partial y^2}\frac{\partial f}{\partial y^2}-2\left(x\frac{\cos x}{\sin x}\right)\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}-\left(1+x\frac{\cos x}{\sin x}\right)\frac{\partial^4 f}{\partial y^4}\frac{\partial f}{\partial y}-\left(x\frac{\cos x}{\sin x}+1\right)\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial y^2}\right)$$
$$+\left((1-\phi)+\phi\frac{(\rho\beta)_s}{(\rho\beta)_f}\right)\lambda\theta\frac{\sin x}{x}.$$

$$\left((1-\phi)+(\phi)\frac{(\rho C_\rho)_s}{(\rho C_\rho)_f}\right)\left(x\frac{\partial f}{\partial y}\frac{\partial\theta}{\partial x}\pm\left(x\frac{\partial f}{\partial y}+f\left(1+\frac{x}{\sin x}\cos x\right)\right)\frac{\partial\theta}{\partial y}\right)$$
$$=\frac{1}{Pr}\left[\frac{(k_s+2k_f)-2\phi(k_f-k_s)}{(k_s+2k_f)+\phi(k_f-k_s)}\left(\frac{\partial^2\theta}{\partial y^2}\right)\right]+\gamma\theta. \tag{18}$$

the boundary conditions become,

$$f=0,\qquad \frac{\partial f}{\partial y}=0,\qquad \theta'=-1,\qquad \text{on } y=0,\qquad x\geq 0,$$
$$\frac{\partial f}{\partial y}\to\frac{3}{2}\frac{\sin x}{x},\qquad \frac{\partial^2 f}{\partial y^2}=0,\qquad \theta\to 0,\qquad \text{as } y\to\infty,\qquad x\geq 0. \tag{19}$$

At the lower stagnation point of the sphere, $x \approx 0$, Equations (17) and (18) reduce to the following ordinary differential equations:

$$\frac{1}{(1-\phi)^{2.5}}f'''+\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)\left(2ff''-f'^2+\frac{9}{4}\right)+K\left(2f'f'''-ff''''-f'^2\right)$$
$$+\left((1-\phi)+\phi\frac{(\rho C_\rho)_s}{(\rho C_\rho)_f}\right)\lambda\theta=0, \tag{20}$$

$$\frac{1}{Pr}\left[\frac{(k_s+2k_f)-2\emptyset(k_f-k_s)}{(k_s+2k_f)+\emptyset(k_f-k_s)}\right]\theta''+2\left((1-\phi)+\phi\frac{(\rho C_\rho)_s}{(\rho C_\rho)_f}\right)f\theta'+\gamma\theta=0, \tag{21}$$

and the boundary conditions become,

$$f=0,\qquad f'(0)=0,\qquad \theta'(0)=-1,\qquad \text{on } y=0,\qquad x\geq 0,$$
$$f'(y)\to\frac{3}{2},\qquad f''(y)=0,\qquad \theta(y)\to 0,\qquad \text{as } y\to\infty,\qquad x\geq 0. \tag{22}$$

Next, in the following chapter, the Equations (20) and (21), along with the boundary conditions (22), will be implemented into the MATLAB algorithm to determine the behaviour of the nanofluid flow by investigating the velocity and temperature profiles.

4. Solving Viscoelastic Nanofluid Past Over a Sphere in Presence of Heat Generation

In order to solve using a BVP4C solver in MATLAB, the function must be written that represents the equation as a system of first-order equations, as well as a function for the boundary conditions and an initial guess function. The BVP4C solver then solves the problem using these three inputs. The basic syntax in the BVP4C solver is $sol = bvp4c(odefun, bcfun, solinit, options)$ which integrates a system of differential equations, $odefun$ in the form of $y' = f(x,y)$ with $bcfun$, the boundary conditions and

$solinit$, the initial solution guess. The $options$ in the basic syntax of BVP4C solver is an additional integration setting which is an argument which created using bvp set function. Thus, Equations (20) to (22) must be rewritten as first order differential structures,

$$y_1 = f, \qquad y_2 = f', \qquad y_3 = f'', \qquad y_4 = f''', \qquad y'_4 = f'''',$$
$$y_5 = \theta, \qquad y_6 = \theta', \qquad y'_6 = \theta''. \tag{23}$$

The momentum equation is being rearranged to equation below:

$$y'_4 = \frac{1}{y_1}\left((y_2 y_4 - y_3^2) + \frac{1}{2K}\left((1-\phi) + \emptyset\frac{\rho_s}{\rho_f}\right)\left(2y_1y_3 - y_2^2 + \frac{9}{4}\right) + \frac{1}{(1-\phi)^{2.5}}y_4 + \left((1-\phi) + \phi\frac{(\rho\beta)_s}{(\rho\beta)_f}\right)\lambda y_5\right). \tag{24}$$

The energy equation is also being rearranged as follows:

$$y'_6 = P_r\left(\frac{(k_s + 2k_f) - 2\emptyset(k_f - k_s)}{(k_s + 2k_f) + \emptyset(k_f - k_s)}\right)\left(-2\left((1-\phi) + \phi\frac{(\rho C_\rho)_s}{(\rho C_\rho)_f}\right)y_1y_6 - \gamma y_5\right). \tag{25}$$

with the boundary conditions:

$$y_1(0) = 0, \qquad y_2(0) = 0, \qquad y_6(0) = -1,$$
$$y_2(\infty) \to \frac{3}{2}, \qquad y_3(\infty) = 0, \qquad y_5(\infty) = 0. \tag{26}$$

5. Results and Discussion

The MATLAB algorithm is being used in solving the mixed convection nanofluid flow problem of this study at lower stagnation point. In this section, the algorithm is used to solve and plot the graphs of velocity and temperature profiles as shown in Figure 2 until Figure 5. The system of ordinary differential equations subjected to boundary conditions is being solved by BVP4C solver.

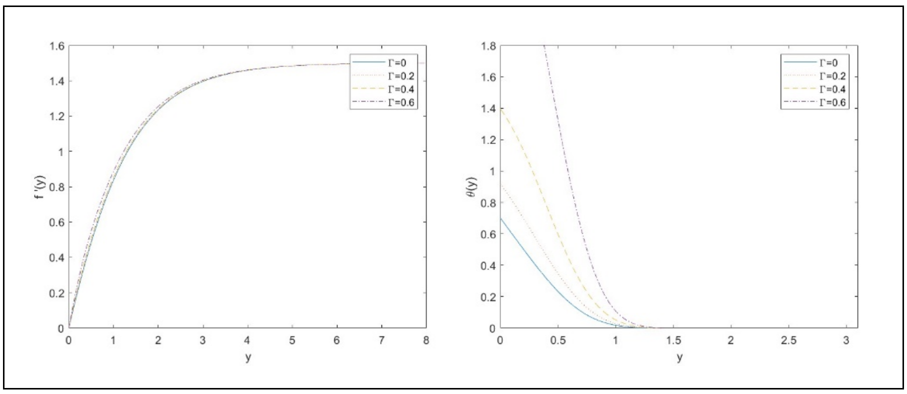

Figure 2 Velocity profile, f' and Temperature profile, θ when Pr $= 6.2$, $K = 1$ and $\lambda = 1$.

The result from Figure 2 shows the existence of the effects for heat generation, γ to the nanofluid velocity and temperatures profiles. It is illustrating the velocity and temperature distributions at stagnation point against different values of the heat generation parameter γ, with constant values of Prandtl number, Pr $= 6.2$, and viscoelastic parameter, $K = 1$. From the observation, the velocity and temperature of the nanofluid distributions rise as the value of the heat generation parameter increases.

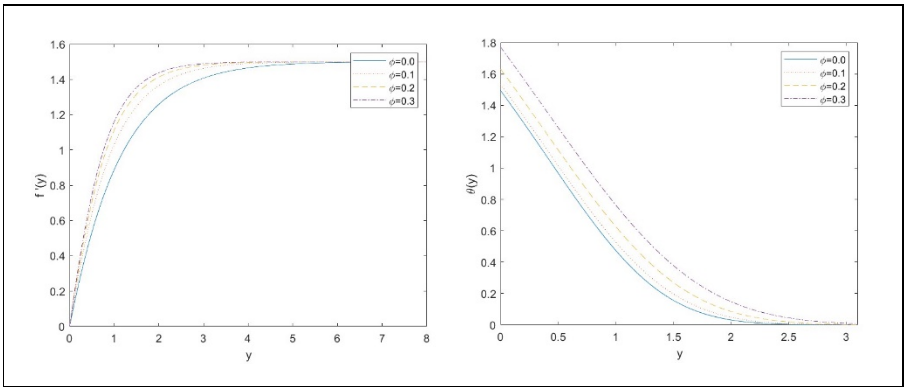

Figure 3 Velocity profile, f' and Temperature profile, θ when $\mathrm{Pr} = 1$, $K = 1$, $\lambda = 1$ and $\gamma = 0.2$.

Figure 3 illustrates the effect of nanoparticles volume fraction, ϕ on the velocity and temperature surface of a sphere in presence of heat generation effect, γ. From the figures, both velocity and temperature profiles increase when the value of nanoparticles volume fraction, ϕ increases. The presence of nanoparticles in the fluid increases the effective thermal conductivity and enhances the heat transfer characteristics. Hence, when the value of ϕ increases, the thermal conductivity increases, and the thermal boundary layer thickness also increases.

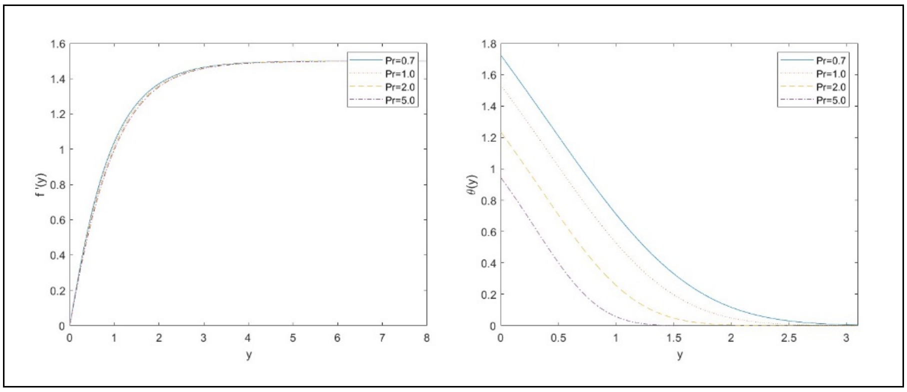

Figure 4 Velocity profile, f' and Temperature profile, θ when $K = 1$, $\lambda = 1$ and $\gamma = 0.2$.

As expected in Figure 4, it is shown that for viscoelastic parameter, *K* = 1 and heat generation effect, γ = 0.2, as Prandtl number increase, the velocity of the fluid and velocity of the gradient is decrease. Also, it shown the same pattern decreasing in the temperature profile as the Prandtl number increase due to the decrement of boundary layer thickness since the decrease of thermal diffusivity leads to the reduction in energy ability.

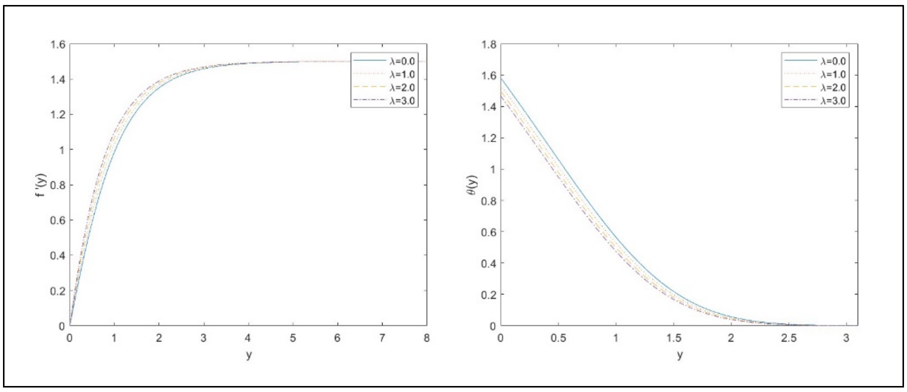

Figure 5 Velocity profile, f' and Temperature profile, θ when $K = 1$, $\mathrm{Pr} = 1$ and $\gamma = 0.2$.

The result for Figure 5 shows the velocity and temperature profiles respectively for different values of mixed convection parameter, λ in presence of heat generation effect, γ. For velocity profile, as the mixed convection parameter, λ increases, the velocity profile also increases. While for temperature profile, it shows the decreasing of the temperature distribution of thermal boundary layer thickness when mixed convection parameter, λ increases due to the energy abilities decreases.

6. Conclusion

The numerical analysis of solving mixed convection boundary layer flow of viscoelastic nanofluid in the presence of heat generation effect is being discussed theoretically and graphically. The governing boundary layer equations were transformed into a non-dimensional form and the resulting nonlinear system of partial differential equations was solved numerically using BVP4C solver in MATLAB. From this study, it has discovered that there are some significant effects of heat generation on the surface of sphere affect the flow and heat transfer characteristics when there are some changes in heat generation, γ, nanoparticles volume fraction, ϕ, Prandtl number, Pr, viscoelastic parameter, K and mixed convection parameter, λ. From the study, the conclusions can be drawn as below:

- As an increase in heat generation, γ, leads to an increase of both velocity of the nanofluid and the temperature distribution.
- It can be concluded that when heat generation effect increase, the velocity and thermal boundary layer thickness also increased. These are expected since the heat generation mechanism creates a layer of hot fluid near the surface.
- The velocity distribution shows an increment with the increase in the nanoparticles volume fraction and the temperature distribution also increased.
- The nanofluid flow and temperature distribution are decreasing as the Prandtl number increase.
- For mixed convection parameter, λ, the velocity of the nanofluid increase with the increase value of λ. While the temperature distribution decreased.

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