Proceedings of Science and Mathematics
Faculty of Science, Universiti Teknologi Malaysia

Vol. 9, 2022, page 175-184

# A User Interface Development for a Block Tridiagonal Algorithm in a Two-Dimensional Keller Box Method

**Mohamad Abdul Rasman Ahmad*, Anati Ali**
Department of Mathematical Sciences, Faculty of science
Universiti Teknologi Malaysia, 81310 Johor Bahru, Malaysia
*Corresponding author: mohamadabdulrasman@graduate.utm.my

## Abstract

The purpose of this study is to develop a user interface as part of the solving process of a boundary layer flow problem using the Keller Box method (KBM). To begin with, a case study involving free convection boundary layer flow near a stagnation point, which consists of several governing equations, is selected. The manual Keller Box method is applied first to the problem in order to form the user interface. The governing equations are reduced to the first order equations by introducing new variables and a finite central difference method will discretize the system of equations later. The resulting product on the coefficient matrix from the discretization procedure is then linearized using Newton's method. The product of a linear system of equations is then used to form the block tridiagonal matrix. The interface is then formed for users to easily obtain the calculation and resulting equations after inputting the governing equations and boundary conditions into the interface template. The template is then verified with various past research with similar case of boundary layer flow problems to show that it can be used and the results are in agreement.

**Keywords:** Keller Box method; User Interface; Block Tridiagonal matrix

## 1. Introduction

Many Scholars have developed an interest in boundary layer flow in recent years since it may be used for a wide range of scientific subjects as well as various engineering and industrial operations. Boundary layer flow has several uses, particularly in the design of diverse things that must overcome fluid flow for smooth operation. Certain examples of applications that use boundary layer flow are airplanes and automobiles. Mathematically, these boundary layer flow issues may be solved using a variety of ways. The findings are obtained by solving its governing equations numerically or analytically. In this research, we focus on numerically solving the created governing equations using Keller Box method (KBM) and developing an interface for future research users.

Many previous researchers have investigated fluid boundary layer flow. It has been discovered that there are certain problems in manually using the Keller Box Method, particularly for issues involving multiple equations. Thus, the primary goal here is to create a type of system interface for future study that will employ the Keller Box method. To acquire the output method results, users will simply need to enter the transformed non-dimensionless partial differential equation, the equation form introducing the new variable and the boundary conditions.

This research is focused on the g-jitter Free Convection Flow of Nanofluid in the Three-Dimensional Stagnation Point Region. Furthermore, the Keller Box method used in this research is limited to Newton's method of linearization and writes them in the Block tridiagonal matrix form. Following that, an Excel template will be used to provide a system interface for future usage of the Keller Box Method.

## 2. Literature Review

### *2.1. Boundary Layer Flow*

A boundary layer is defined as the fluid layer in the direct area of a bounding surface that contain significant viscous effects. Many different types of boundary layer flow have been considered. Rosali et. al [5] researched on unsteady boundary layer stagnation point flow. They analyzed the unsteady boundary layer stagnation point flow and heat transfer towards a stretching sheet by considering the porosity. As a result, it is found that the skin friction coefficients decrease whereas the local Nusselt number increases with the increase in permeability parameter.

Kebede et al. [6] studied on heat and mass transfer in unsteady boundary layer flow of Williamson nanofluid. They applied the homotopy analysis method (HAM) to obtain analytic series solutions for the unsteady boundary layer flow of Williamson nanofluid in a porous medium. The validity of the results is also verified by comparing them with previous results under some restricted assumptions and found to be in excellent agreement. They found that heat transfer rate can be speeded up by increasing the unsteadiness parameter or by decreasing the porosity of the medium, thermal radiation, or destructive chemical reaction. Mass transfer rate can also be enhanced by increasing the thermal radiation, constructive chemical reaction, or unsteadiness parameter.

### *2.2. Finite Difference Method*

The finite difference method (FDM) is an approximate method for solving partial differential equations. It has been used to solve a wide range of problems. These include linear and nonlinear, time independent and dependent problems. This method can be applied to problems with different boundary shapes, different kinds of boundary conditions, and for a region containing a number of different materials.

Pandy et al. [8] studied a finite difference approach for solving elliptic boundary value problems numerically. They look at a finite difference approach for numerically solving an elliptic equation system in a square domain. Whereas the typical second-order accurate technique is used. They provide unique finite difference approximations for systems of elliptic equations, resulting in a difference equation that perfectly fulfils the boundary requirements.

### *2.3 The Keller Box Method*

Most of the problems arising in science and engineering are nonlinear. They are inherently difficult to solve. The Keller Box method is one of the methods that can solve numerically the nonlinear equation. Among the numerical techniques for solving problems, the Keller-Box method is an implicit method for which a set of differential equations are reduced to the system of first-order differential equations.

Swalmeh et al. [9] studied on heat transfer flow of Cu-water and Al2O3-water micropolar nanofluid about a solid sphere in the presence of natural convection using Keller Box method. The nonlinear partial differential equations of the boundary layer are first transformed into a non-dimensional form and then solved numerically using an implicit finite difference scheme known as Keller Box method. They found that this method seems to be the most flexible of the common methods and, despite recent developments in other numerical methods, remains a powerful and very accurate approach for parabolic boundary layer flows. Cebeci concluded that the Keller Box method is also easily adaptable to solve equations of any order and unconditionally stable on the solutions.

### *2.4 The User Interface Development*

User interface development include the creation of websites, online apps, mobile applications, and software. It is essential to the software development life cycle [SDLC]. A user interface is a two-way interaction between a user and a computerized system that is designed expressly for the use of input in software. The user interface's purpose is to make user interaction as easy and efficient as feasible in terms of achieving user goals. According to past study, researchers often employ a variety of Math Works programmes such as Fortran, MATLAB, Maple, and other appropriate software to get their results. However, the techniques necessary to utilize these tools may be too difficult for researchers to understand. As a result, in this study, we take an Excel-based approach to building a user interface.

The benefit of using Excel is that it has a user-friendly interface that easy for researchers to use. Darren et al. [10] developed an Excel interface for analysis of LC–MS-based metabolomics data. It provides a user-friendly interface for analysis of complex metabolomics datasets without the need for specialist bioinformatics skills, allowing for the rapid production of meaningful, interactive results for biological interpretation of untargeted metabolomics data sets. Grapov et al. [11] developed a graphical user interface to R multivariate analysis tools in Microsoft Excel. They implemented imDEV in Visual Basic for Applications (VBA) and R programming languages and depends on the open-source applications statconnDCOM and RExcel integration as an add-in into MS Excel.

Dimitri [12] has developed an Excel user interface for the Newton's algorithm in the Keller Box method, which considers only the one-dimensional case. This study will extend the work by Dimitri [12] by considering the two-dimensional case and further refine the interface by including the block tridiagonal algorithm.

## 3. Methodology

### *3.1. Keller Box Method*

There are four main steps in the Keller Box Method namely, Reduce the higher order PDE to first order, Finite difference Scheme, Linearization of Newton's method and Tridiagonal block elimination method. The nonlinear partial differential governing equation will be reduced to first-order systems. Then write the difference equation using central differences. Linearize the resulting algebraic equations by Newton's method, and write them in the tridiagonal matrix form.

### *3.1.1 Reduce Higher Order PDE*

We reduce the higher order partial differential equations into first order by
introducing new variables.

| | | | |
|---|---|---|---|
| Let $f' = u$ | Let $h' = p$ | Let $\theta' = s$ | |
| Let $f'' = u' = v$ | Let $h'' = q$ | Let $\theta'' = s'$ | (1) |
| Let $f''' = u'' = v'$ | Let $h''' = p'' = q'$ | | |

### *3.1.2 Finite Difference Scheme*

Finite difference scheme, or also known as centred differential method, converts the produced nonlinear governing equations into first order differential equations, and then replacing the derivatives with its centred difference.

Derivatives of order zero and first orders are transformed into as follows:

$$(\ \ )_{j-\frac{1}{2}}^{i-\frac{1}{2}} = \frac{1}{4}\left[(\ \ )_j^i + (\ \ )_{j-1}^i + (\ \ )_j^{i-1} + (\ \ )_{j-1}^{i-1}\right]$$

$$\left(\frac{\partial(\ \ )}{\partial\eta}\right)_{j-\frac{1}{2}}^{i-\frac{1}{2}} = \frac{1}{2w_j}\left[(\ \ )_j^i - (\ \ )_{j-1}^i + (\ \ )_j^{i-1} - (\ \ )_{j-1}^{i-1}\right] \qquad (2)$$

$$\left(\frac{\partial(\ \ )}{\partial\tau}\right)_{j-\frac{1}{2}}^{i-\frac{1}{2}} = \frac{1}{2d^i}\left[(\ \ )_j^i + (\ \ )_{j-1}^i - (\ \ )_j^{i-1} - (\ \ )_{j-1}^{i-1}\right]$$

$(\ \ )_{j-\frac{1}{2}}^{i-\frac{1}{2}}$ represents the derivative with zero order and $\left(\frac{\partial(\ \ )}{\partial\eta}\right)_{j-\frac{1}{2}}^{i-\frac{1}{2}}$ , $\left(\frac{\partial(\ \ )}{\partial\tau}\right)_{j-\frac{1}{2}}^{i-\frac{1}{2}}$ represents the first order derivative.

*3.1.3 Newton's Method of Linearization*

Once all the derivatives have been changed for every equation. Newton's method of linearization is applied. In order to linearize the equation, the iterations are introduced to obtain the new system by letting:

$$(\ )_j \rightarrow (\ )_j + \delta(\ )_j \qquad\qquad (\ )_{j-1} \rightarrow (\ )_{j-1} + \delta(\ )_{j-1} \tag{3}$$

In these systems there may exist quadratic $(\delta^2)$ terms. Since these values are too small and are close to zero, they are considered insignificant and can be ignored, the resulting equations are then sorted with those in delta $(\delta)$ terms on the left side and other terms on the right side.

*3.1.4 Block Tridiagonal Method*

After linearizing the equations by using Newton's method. The linearized system of equations takes the block tridiagonal structure. It can be written in the form:

$$\begin{pmatrix} [A_1] & [C_1] & & & \\ [B_1] & [A_2] & [C_2] & & \\ \ddots & \ddots & \ddots & & \\ & & [B_{J-1}] & [A_{J-1}] & [C_{J-1}] \\ & & & [B_J] & [A_J] \end{pmatrix} \begin{pmatrix} [\delta_1] \\ [\delta_2] \\ \vdots \\ [\delta_{J-1}] \\ [\delta_J] \end{pmatrix} = \begin{pmatrix} [r_1] \\ [r_2] \\ \vdots \\ [r_{J-1}] \\ [r_J] \end{pmatrix}$$

The block tridiagonal matrix can be solved by using the LU method; the block tridiagonal method is repeatedly used until it satisfies the convergence criteria. However, this case study only covers up until transforming the linearized equations to tridiagonal matrix form.

*3.2. Keller Box – Excel User Interface*

The selected case study's nonlinear governing equations will be used as the interface's input. Calculations are accomplished by determining the needed data depending on the provided input to construct the final output equation utilising certain 'IF Statements' and formulae. The interface results are analysed and compared to the manual Keller Box calculation results to check that the formula is appropriate. After checking all of the equations and ensuring that no errors remain, the template is tested on other similar past studies that are appropriate to guarantee that the formula is still suitable.

## 4. Results and discussion

*4.1 Mathematical Formulation of Case Study*

This study considers the three-dimensional free convection boundary layer flow near stagnation point region which is embedded in viscous nanofluid with effect of g-jitter.

The produced partial differential governing equations from Kamal et al. (2019) research paper is as follows:

$$C_1 f''' + C_2[(f+h)f'' - f'^2] + C_3[1 + \varepsilon cos(\pi\tau)]\theta = C_2 \Omega \frac{\partial f'}{\partial \tau} \tag{4}$$

$$C_1 h''' + C_2[(f+h)h'' - h'^2] + C_3 c[1 + \varepsilon cos(\pi\tau)]\theta = C_2 \Omega \frac{\partial h'}{\partial \tau} \tag{5}$$

$$\frac{C_4}{Pr}\theta'' + (f+h)\theta' = \Omega \frac{\partial \theta}{\partial \tau} \tag{6}$$

$$C_1 = \frac{1}{(1-\phi)^{2.5}},$$

$$C_2 = \left(1 - \phi + \frac{\phi\rho_s}{\rho_f}\right), \tag{7}$$

$$C_3 = \left(1 - \phi + \frac{\phi\rho\beta_s}{\rho\beta_f}\right),$$

$$C_4 = \frac{k_{nf}/k_f}{\left[1 - \phi + \frac{\phi\left(\rho C_p\right)_s}{\left(\rho C_p\right)_f}\right]},$$

with the boundary condition

$$f(\tau, 0) = f'(\tau, 0) = 0,\ h(\tau, 0) = h'(\tau, 0) = 0,\ \theta(\tau, 0) = 0, \qquad f' \to 0 \qquad h' \to 0 \qquad \theta \to 0\ \ as\ \ \eta \to \infty \tag{8}$$

*4.2. Reduce the Higher order PDE*

Use and substituting the new variables (1) into the (4-6) equations respectively gives:

$$f' = u \tag{9}$$

$$u' = v \tag{10}$$

$$h' = p \tag{11}$$

$$p' = q \tag{12}$$

$$\theta' = s \tag{13}$$

$$C_1 v' + C_2[(f + h)v - u^2] + C_3[1 + \varepsilon cos(\pi\tau)]\theta - C_2\Omega\frac{\partial u}{\partial \tau} = 0 \tag{14}$$

$$C_1 q' + C_2[(f + h)q - p^2] + C_3 c[1 + \varepsilon cos(\pi\tau)]\theta - C_2\Omega\frac{\partial p}{\partial \tau} = 0 \tag{15}$$

$$\frac{C_4}{Pr}s' + (f + h)s - \Omega\frac{\partial \theta}{\partial \tau} = 0 \tag{16}$$

the boundary conditions become:

$$f(\tau, 0) = u(\tau, 0) = 0, \qquad h(\tau, 0) = p(\tau, 0) = 0,\ \ \theta(\tau, 0) = 0, \qquad u \to 0 \qquad p \to 0 \qquad \theta \to 0\ \ as\ \ \eta \to \infty \tag{17}$$

*4.3. Finite Difference Scheme*

Now that the equations are first order, we apply the finite difference scheme for every variable in the equations as previously stated in equation (2) and apply them into equation (9-16), thus form the following:

The finite difference form for equations (9-13) at midpoint $\left(\tau^i, \eta_{j-\frac{1}{2}}\right)$

$$f_j^i - f_{j-1}^i - \frac{w_j}{2}\left(u_j^i + u_{j-1}^i\right) = 0 \tag{18}$$

$$u_j^i - u_{j-1}^i - \frac{w_j}{2}\left(v_j^i + v_{j-1}^i\right) = 0 \tag{19}$$

$$h_j^i - h_{j-1}^i - \frac{w_j}{2}\left(p_j^i + p_{j-1}^i\right) = 0 \tag{20}$$

$$p_j^i - p_{j-1}^i - \frac{w_j}{2}\left(q_j^i + q_{j-1}^i\right) = 0 \tag{21}$$

$$\theta_j^i - \theta_{j-1}^i - \frac{w_j}{2}\left(s_j^i + s_{j-1}^i\right) = 0 \tag{22}$$

The finite difference form for equations (14-16) at midpoint $\left(\tau^{i-\frac{1}{2}}, \eta_{j-\frac{1}{2}}\right)$,

$$\begin{aligned} C_1\left(v_j^i - v_{j-1}^i\right) + \frac{C_2 w_j}{8}\left[\left(f_j^i + f_{j-1}^i + h_j^i + h_{j-1}^i\right)\left(v_j^i + v_{j-1}^i\right) - \left(u_j^i + u_{j-1}^i\right)^2\right] \\ + \frac{C_3 w_j}{2}[1 + \varepsilon cos(\pi\tau)]\left(\theta_j^i + \theta_{j-1}^i\right) - \frac{C_2 \Omega w_j}{d_i}\left(u_j^i + u_{j-1}^i\right) = R_j^i \end{aligned} \tag{23}$$

$$\begin{aligned} C_1\left(q_j^i - q_{j-1}^i\right) + \frac{C_2 w_j}{8}\left[\left(f_j^i + f_{j-1}^i + h_j^i + h_{j-1}^i\right)\left(q_j^i + q_{j-1}^i\right) - \left(p_j^i + p_{j-1}^i\right)^2\right] \\ + \frac{C_3 c w_j}{2}[1 + \varepsilon cos(\pi\tau)]\left(\theta_j^i + \theta_{j-1}^i\right) - \frac{C_2 \Omega w_j}{d_i}\left(p_j^i + p_{j-1}^i\right) = T_j^i \end{aligned} \tag{24}$$

$$\frac{C_4}{Pr}\left(s_j^i - s_{j-1}^i\right) + \frac{w_j}{8}\left(f_j^i + f_{j-1}^i + h_j^i + h_{j-1}^i\right)\left(s_j^i + s_{j-1}^i\right) - \frac{\Omega w_j}{d_i}\left(\theta_j^i + \theta_{j-1}^i\right) = K_j^i \tag{25}$$

The boundary condition takes the form

$$f_0^i = 0, u_0^i = 0, h_0^i = 0, p_0^i = 0, \theta_0^i = 0, u_J^i = 0, p_J^i = 0, \theta_J^i = 0 \tag{26}$$

### *4.3. Linearization using Newton's Method*

$$\delta f_j^i - \delta f_{j-1}^i - \frac{w_j}{2}\left(\delta u_j^i + \delta u_{j-1}^i\right) = (r_1)_j \tag{27}$$

$$\delta u_j^i - \delta u_{j-1}^i - \frac{w_j}{2}\left(\delta v_j^i + \delta v_{j-1}^i\right) = (r_2)_j \tag{28}$$

$$\delta h_j^i - \delta h_{j-1}^i - \frac{w_j}{2}\left(\delta p_j^i + \delta p_{j-1}^i\right) = (r_3)_j \tag{29}$$

$$\delta p_j^i - \delta p_{j-1}^i - \frac{w_j}{2}\left(\delta q_j^i + \delta q_{j-1}^i\right) = (r_4)_j \tag{30}$$

$$\delta \theta_j^i - \delta \theta_{j-1}^i - \frac{w_j}{2}\left(\delta s_j^i + \delta s_{j-1}^i\right) = (r_5)_j \tag{31}$$

$$\begin{aligned} (a_1)_j \delta v_j^i + (a_2)_j \delta v_{j-1}^i + (a_3)_j \delta f_j^i + (a_4)_j \delta f_{j-1}^i + (a_5)_j \delta h_j^i + (a_6)_j \delta h_{j-1}^i + (a_7)_j \delta u_j^i \\ + (a_8)_j \delta u_{j-1}^i + (a_9)_j \delta \theta_j^i + (a_{10})_j \delta \theta_{j-1}^i = (r_6)_j \end{aligned} \tag{32}$$

$$\begin{aligned} (b_1)_j \delta q_j^i + (b_2)_j \delta q_{j-1}^i + (b_3)_j \delta f_j^i + (b_4)_j \delta f_{j-1}^i + (b_5)_j \delta h_j^i + (b_6)_j \delta h_{j-1}^i + (b_7)_j \delta p_j^i \\ + (b_8)_j \delta p_{j-1}^i + (b_9)_j \delta \theta_j^i + (b_{10})_j \delta \theta_{j-1}^i = (r_7)_j \end{aligned} \tag{33}$$

$$(c_1)_j\delta s_j^i + (c_2)_j\delta s_{j-1}^i + (c_3)_j\delta f_j^i + (c_4)_j\delta f_{j-1}^i + (c_5)_j\delta h_j^i + (c_6)_j\delta h_{j-1}^i + (c_7)_j\delta\theta_j^i + (c_8)_j\delta\theta_{j-1}^i = (r_8)_j \tag{34}$$

with the boundary conditions take the form

$$\delta f_0^i = 0, \delta u_0^i = 0, \delta h_0^i = 0, \delta p_0^i = 0, \delta\theta_0^i = 0, \delta u_J^i = 0, \delta p_J^i = 0, \delta\theta_J^i = 0 \tag{35}$$

*4.4. Block Tridiagonal matrix*

The block tridiagonal matrix form for the linearized difference equations (27-34) is entertained as:

$$\begin{pmatrix} [A_1] & [C_1] & & & \\ [B_1] & [A_2] & [C_2] & & \\ \ddots & \ddots & \ddots & & \\ & & [B_{J-1}] & [A_{J-1}] & [C_{J-1}] \\ & & & [B_J] & [A_J] \end{pmatrix} \begin{pmatrix} [\delta_1] \\ [\delta_2] \\ \vdots \\ [\delta_{J-1}] \\ [\delta_J] \end{pmatrix} = \begin{pmatrix} [r_1] \\ [r_2] \\ \vdots \\ [r_{J-1}] \\ [r_J] \end{pmatrix}$$

that is when $j = 1$

$$[A_j] \begin{bmatrix} \delta v_0 \\ \delta q_0 \\ \delta s_0 \\ \delta f_1 \\ \delta v_1 \\ \delta h_1 \\ \delta q_1 \\ \delta s_1 \end{bmatrix} + [C_j] \begin{bmatrix} \delta u_1 \\ \delta p_1 \\ \delta\theta_1 \\ \delta f_2 \\ \delta v_2 \\ \delta h_2 \\ \delta q_2 \\ \delta s_2 \end{bmatrix}$$

when $j = J$

$$[B_j] \begin{bmatrix} \delta u_{J-2} \\ \delta p_{J-2} \\ \delta\theta_{J-2} \\ \delta f_{J-1} \\ \delta v_{J-1} \\ \delta h_{J-1} \\ \delta q_{J-1} \\ \delta s_{J-1} \end{bmatrix}$$

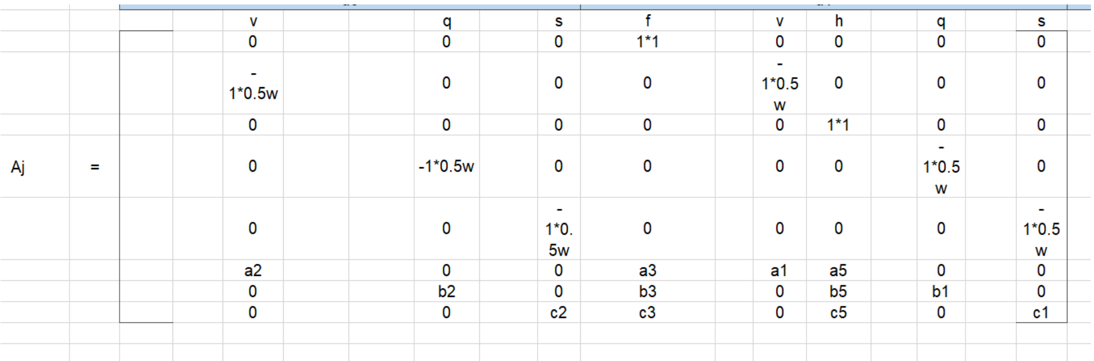
where the element is defined as

$$[A_j] = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ d & 0 & 0 & 0 & d & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & d & 0 & 0 & 0 & 0 & d & 0 \\ 0 & 0 & d & 0 & 0 & 0 & 0 & d \\ a_2 & 0 & 0 & a_3 & a_1 & a_5 & 0 & 0 \\ 0 & b_2 & 0 & b_3 & 0 & b_5 & b_1 & 0 \\ 0 & 0 & c_2 & c_3 & 0 & c_5 & 0 & c_1 \end{bmatrix} \quad [B_j] = \begin{bmatrix} 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & d & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & d & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & d \\ 0 & 0 & 0 & a_4 & a_2 & a_6 & 0 & 0 \\ 0 & 0 & 0 & b_4 & 0 & b_6 & b_2 & 0 \\ 0 & 0 & 0 & c_4 & 0 & c_6 & 0 & c_2 \end{bmatrix}$$

$$[C_j] = \begin{bmatrix} d & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & d & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ a_7 & 0 & a_9 & 0 & 0 & 0 & 0 & 0 \\ 0 & b_7 & b_9 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_7 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

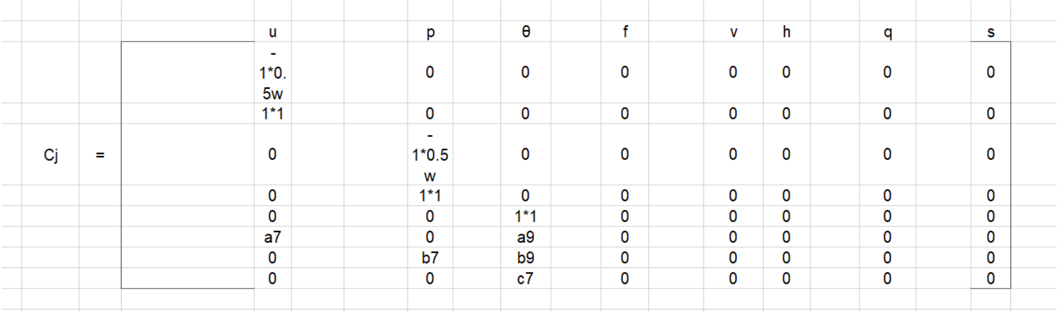
where $d = -\frac{w_j}{2}$

### *4.5. Output matrix in Keller Box – Excel User Interface*



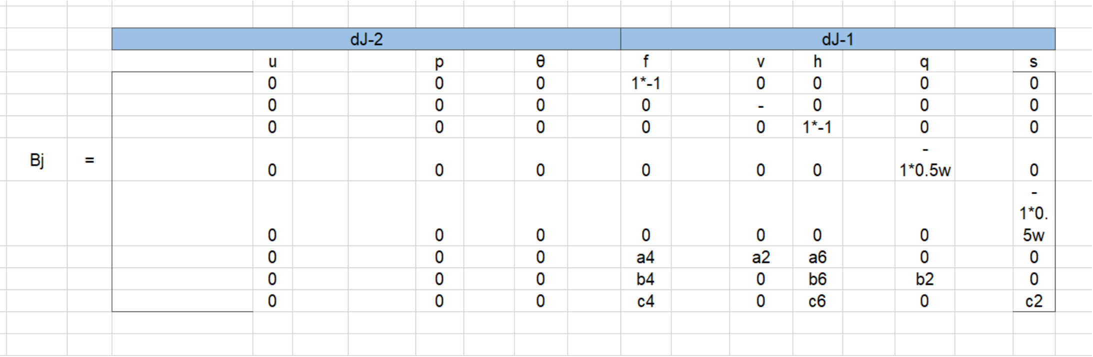
| | | v | q | s | f | v | h | q | s |
|---|---|---|---|---|---|---|---|---|---|
| | | 0 | 0 | 0 | 1*1 | 0 | 0 | 0 | 0 |
| | | -1*0.5w | 0 | 0 | 0 | -1*0.5w | 0 | 0 | 0 |
| | | 0 | 0 | 0 | 0 | 0 | 1*1 | 0 | 0 |
| Aj | = | 0 | -1*0.5w | 0 | 0 | 0 | 0 | -1*0.5w | 0 |
| | | 0 | 0 | -1*0.5w | 0 | 0 | 0 | 0 | -1*0.5w |
| | | a2 | 0 | 0 | a3 | a1 | a5 | 0 | 0 |
| | | 0 | b2 | 0 | b3 | 0 | b5 | b1 | 0 |
| | | 0 | 0 | c2 | c3 | 0 | c5 | 0 | c1 |

Figure 4.1 Output matrix $A_j$



| | | u | p | ѳ | f | v | h | q | s |
|---|---|---|---|---|---|---|---|---|---|
| | | -1*0.5w | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | | 1*1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Cj | = | 0 | -1*0.5w | 0 | 0 | 0 | 0 | 0 | 0 |
| | | 0 | 1*1 | 0 | 0 | 0 | 0 | 0 | 0 |
| | | 0 | 0 | 1*1 | 0 | 0 | 0 | 0 | 0 |
| | | a7 | 0 | a9 | 0 | 0 | 0 | 0 | 0 |
| | | 0 | b7 | b9 | 0 | 0 | 0 | 0 | 0 |
| | | 0 | 0 | c7 | 0 | 0 | 0 | 0 | 0 |

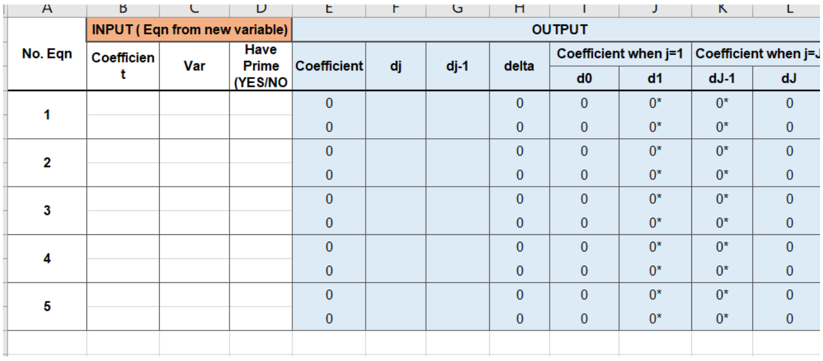
Figure 4.2 Output matrix $C_j$



| | | dJ-2 | | | dJ-1 | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | u | p | ѳ | f | v | h | q | s |
| | | 0 | 0 | 0 | 1*-1 | 0 | 0 | 0 | 0 |
| | | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 |
| | | 0 | 0 | 0 | 0 | 0 | 1*-1 | 0 | 0 |
| Bj | = | 0 | 0 | 0 | 0 | 0 | 0 | -1*0.5w | 0 |
| | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1*0.5w |
| | | 0 | 0 | 0 | a4 | a2 | a6 | 0 | 0 |
| | | 0 | 0 | 0 | b4 | 0 | b6 | b2 | 0 |
| | | 0 | 0 | 0 | c4 | 0 | c6 | 0 | c2 |

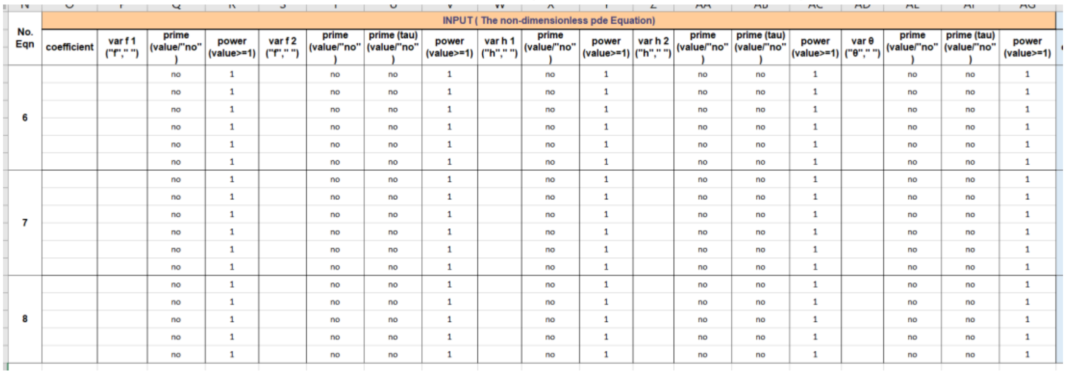
Figure 4.3 Output matrix $B_j$

### *4.6. Keller Box – Excel User Interface Template*



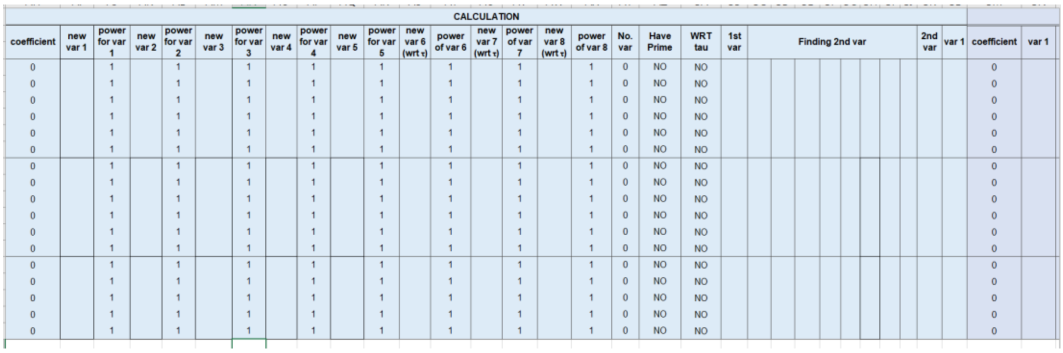
| No. Eqn | INPUT ( Eqn from new variable) | | | OUTPUT | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Coefficien t | Var | Have Prime (YES/NO | Coefficient | dj | dj-1 | delta | Coefficient when j=1 | | Coefficient when j=J | |
| | | | | | | | | d0 | d1 | dJ-1 | dJ |
| 1 | | | | 0 | | | 0 | 0 | 0* | 0* | 0 |
| | | | | 0 | | | 0 | 0 | 0* | 0* | 0 |
| 2 | | | | 0 | | | 0 | 0 | 0* | 0* | 0 |
| | | | | 0 | | | 0 | 0 | 0* | 0* | 0 |
| 3 | | | | 0 | | | 0 | 0 | 0* | 0* | 0 |
| | | | | 0 | | | 0 | 0 | 0* | 0* | 0 |
| 4 | | | | 0 | | | 0 | 0 | 0* | 0* | 0 |
| | | | | 0 | | | 0 | 0 | 0* | 0* | 0 |
| 5 | | | | 0 | | | 0 | 0 | 0* | 0* | 0 |
| | | | | 0 | | | 0 | 0 | 0* | 0* | 0 |

Figure 4.4 Input and Output Section for New Equations



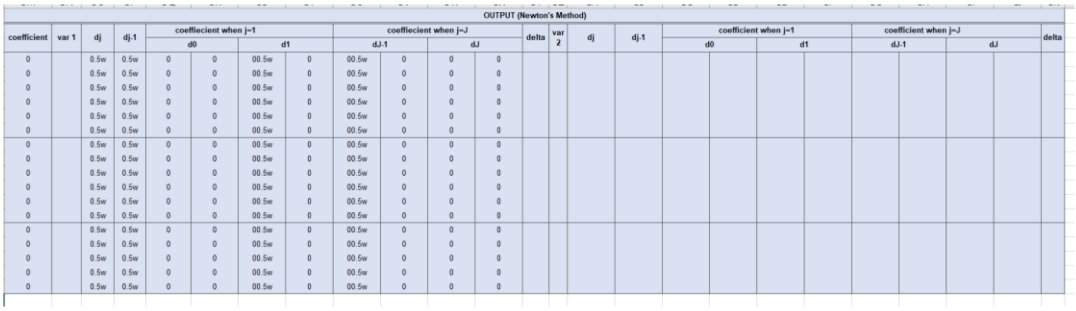
| No. Eqn | INPUT ( The non-dimensionless pde Equation) | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | coefficient | var f 1 ("f"," ") | prime (value/"no" ) | power (value>=1) | var f 2 ("f"," ") | prime (value/"no" ) | prime (tau) (value/"no" ) | power (value>=1) | var h 1 ("h"," ") | prime (value/"no" ) | power (value>=1) | var h 2 ("h"," ") | prime (value/"no" ) | prime (tau) (value/"no" ) | power (value>=1) | var θ ("θ"," ") | prime (value/"no" ) | prime (tau) (value/"no" ) | power (value>=1) |
| 6 | | | no | 1 | | no | no | 1 | | no | 1 | | no | no | 1 | | no | no | 1 |
| | | | no | 1 | | no | no | 1 | | no | 1 | | no | no | 1 | | no | no | 1 |
| | | | no | 1 | | no | no | 1 | | no | 1 | | no | no | 1 | | no | no | 1 |
| | | | no | 1 | | no | no | 1 | | no | 1 | | no | no | 1 | | no | no | 1 |
| | | | no | 1 | | no | no | 1 | | no | 1 | | no | no | 1 | | no | no | 1 |
| | | | no | 1 | | no | no | 1 | | no | 1 | | no | no | 1 | | no | no | 1 |
| 7 | | | no | 1 | | no | no | 1 | | no | 1 | | no | no | 1 | | no | no | 1 |
| | | | no | 1 | | no | no | 1 | | no | 1 | | no | no | 1 | | no | no | 1 |
| | | | no | 1 | | no | no | 1 | | no | 1 | | no | no | 1 | | no | no | 1 |
| | | | no | 1 | | no | no | 1 | | no | 1 | | no | no | 1 | | no | no | 1 |
| | | | no | 1 | | no | no | 1 | | no | 1 | | no | no | 1 | | no | no | 1 |
| | | | no | 1 | | no | no | 1 | | no | 1 | | no | no | 1 | | no | no | 1 |
| 8 | | | no | 1 | | no | no | 1 | | no | 1 | | no | no | 1 | | no | no | 1 |
| | | | no | 1 | | no | no | 1 | | no | 1 | | no | no | 1 | | no | no | 1 |
| | | | no | 1 | | no | no | 1 | | no | 1 | | no | no | 1 | | no | no | 1 |
| | | | no | 1 | | no | no | 1 | | no | 1 | | no | no | 1 | | no | no | 1 |
| | | | no | 1 | | no | no | 1 | | no | 1 | | no | no | 1 | | no | no | 1 |

Figure 4.5 Input Section for the Non-dimensionless Equations



| CALCULATION | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| coefficient | new var 1 | power for var 1 | new var 2 | power for var 2 | new var 3 | power for var 3 | new var 4 | power for var 4 | new var 5 | power for var 5 | new var 6 (wrt τ) | power of var 6 | new var 7 (wrt τ) | power of var 7 | new var 8 (wrt τ) | power of var 8 | No. var | Have Prime | WRT tau | 1st var | Finding 2nd var | 2nd var | var 1 | coefficient | var 1 |
| 0 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | 0 | NO | NO | | | | | 0 | |
| 0 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | 0 | NO | NO | | | | | 0 | |
| 0 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | 0 | NO | NO | | | | | 0 | |
| 0 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | 0 | NO | NO | | | | | 0 | |
| 0 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | 0 | NO | NO | | | | | 0 | |
| 0 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | 0 | NO | NO | | | | | 0 | |
| 0 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | 0 | NO | NO | | | | | 0 | |
| 0 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | 0 | NO | NO | | | | | 0 | |
| 0 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | 0 | NO | NO | | | | | 0 | |
| 0 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | 0 | NO | NO | | | | | 0 | |
| 0 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | 0 | NO | NO | | | | | 0 | |
| 0 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | 0 | NO | NO | | | | | 0 | |
| 0 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | 0 | NO | NO | | | | | 0 | |
| 0 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | 0 | NO | NO | | | | | 0 | |
| 0 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | 0 | NO | NO | | | | | 0 | |
| 0 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | 0 | NO | NO | | | | | 0 | |
| 0 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | 0 | NO | NO | | | | | 0 | |

Figure 4.6 Calculation Section



| OUTPUT (Newton's Method) | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| coefficient | var 1 | dj | dj-1 | coeffiecient when j=1 | | | | coeffiecient when j=J | | | | delta | var 2 | dj | dj-1 | coefficient when j=1 | | | | coefficient when j=J | | | | delta |
| | | | | d0 | | d1 | | dJ-1 | | dJ | | | | | | d0 | | d1 | | dJ-1 | | dJ | | |
| 0 | | 0.5w | 0.5w | 0 | 0 | 00.5w | 0 | 00.5w | 0 | 0 | 0 | | | | | | | | | | | | | |
| 0 | | 0.5w | 0.5w | 0 | 0 | 00.5w | 0 | 00.5w | 0 | 0 | 0 | | | | | | | | | | | | | |
| 0 | | 0.5w | 0.5w | 0 | 0 | 00.5w | 0 | 00.5w | 0 | 0 | 0 | | | | | | | | | | | | | |
| 0 | | 0.5w | 0.5w | 0 | 0 | 00.5w | 0 | 00.5w | 0 | 0 | 0 | | | | | | | | | | | | | |
| 0 | | 0.5w | 0.5w | 0 | 0 | 00.5w | 0 | 00.5w | 0 | 0 | 0 | | | | | | | | | | | | | |
| 0 | | 0.5w | 0.5w | 0 | 0 | 00.5w | 0 | 00.5w | 0 | 0 | 0 | | | | | | | | | | | | | |
| 0 | | 0.5w | 0.5w | 0 | 0 | 00.5w | 0 | 00.5w | 0 | 0 | 0 | | | | | | | | | | | | | |
| 0 | | 0.5w | 0.5w | 0 | 0 | 00.5w | 0 | 00.5w | 0 | 0 | 0 | | | | | | | | | | | | | |
| 0 | | 0.5w | 0.5w | 0 | 0 | 00.5w | 0 | 00.5w | 0 | 0 | 0 | | | | | | | | | | | | | |
| 0 | | 0.5w | 0.5w | 0 | 0 | 00.5w | 0 | 00.5w | 0 | 0 | 0 | | | | | | | | | | | | | |
| 0 | | 0.5w | 0.5w | 0 | 0 | 00.5w | 0 | 00.5w | 0 | 0 | 0 | | | | | | | | | | | | | |
| 0 | | 0.5w | 0.5w | 0 | 0 | 00.5w | 0 | 00.5w | 0 | 0 | 0 | | | | | | | | | | | | | |
| 0 | | 0.5w | 0.5w | 0 | 0 | 00.5w | 0 | 00.5w | 0 | 0 | 0 | | | | | | | | | | | | | |
| 0 | | 0.5w | 0.5w | 0 | 0 | 00.5w | 0 | 00.5w | 0 | 0 | 0 | | | | | | | | | | | | | |
| 0 | | 0.5w | 0.5w | 0 | 0 | 00.5w | 0 | 00.5w | 0 | 0 | 0 | | | | | | | | | | | | | |
| 0 | | 0.5w | 0.5w | 0 | 0 | 00.5w | 0 | 00.5w | 0 | 0 | 0 | | | | | | | | | | | | | |
| 0 | | 0.5w | 0.5w | 0 | 0 | 00.5w | 0 | 00.5w | 0 | 0 | 0 | | | | | | | | | | | | | |

Figure 4.7 Output Section for Newton's Method

## Conclusion

In this research, we have developed an Excel User Interface that uses the Keller Box method to transform the governing equations. We started by reducing the higher order partial differential equation and applying the Finite Difference Scheme and Newton's method of linearization. Finally, we compared both results and found that they are in good agreement.

**Acknowledgement**

I would like to express my special thanks of appreciation to my supervisor, Dr Anati Ali for supervising and guiding me during my research on this study. Her assistance of giving comments, advice and suggestions during research has contributed to success of this report. I also would like to thank to my beloved family and my fellow friends for the continuous support and encouragement.

## References

[1] Isa, Sharena Mohamad; Mohammad, Nurul Farahain (2017). Boundary Layer Flow of Dusty Fluid on a Stretching Sheet of Another Quiescent Fluid. *Journal of Physics: Conference Series, 819(), 012027–.*

[2] Hamdan, F. R.., Ahmad Kamal, M. H.., Rawi, N. A.., Mohamad, A. Q.., Ali, A.., Ilias,. M. R.., & Shafie, S.. (2020). g-Jitter Free Convection Flow Near a Three-Dimensional Stagnation-Point Region with Internal Heat Generation. *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences*, *67*(1),119–135.

[3] Ahmad Kamal, M. H., Ali, A., & Shafie, S. (2019). g-Jitter Free Convection Flow of Nanofluid in The Three-Dimensional Stagnation Point Region. *MATEMATIKA: Malaysian Journal of Industrial and Applied Mathematics*, *35*(2), 260–270.

[4] Ahmad Kamal, M. H., Ali, A., & Shafie, S. (2019). g-Jitter Free Convection Flow Near a Three-Dimensional Stagnation Point Region in Nanofluid with Heat Generation. *ASM Science Journal,* 26, 125-136.

[5] Rosali, H., Badlilshah, M. N.., Mohamat Johari, M. A.., & Bachok, N. (2020). Unsteady Boundary Layer Stagnation Point Flow and Heat Transfer over a Stretching Sheet in a Porous Medium with Slip Effects. *CFD Letters*, *12*(10), 52–61.

[6] Kebede, T., Haile, E., Awgichew, G., Walelign, T. (2020). Heat and Mass Transfer in Unsteady Boundary Layer Flow of Williamson Nanofluids, *Journal of Applied Mathematics*, vol. 2020, Article ID 1890972, 13 pages,

[7] Mohamed, M. K. A., Noar, N. A. Z. M., Salleh, M. Z., & Ishak, A. (2019). Free convection boundary layer flow on a solid sphere in a nanofluid with viscous dissipation. *Malaysian Journal of Fundamental and Applied Sciences*, *15*(3), 381-388.

[8] Pandey, P.K., Jaboob, S.S.A. (2018). A finite difference method for a numerical solution of elliptic boundary value problems, *Applied Mathematics and Nonlinear Sciences,* 3(1) (2018) 311–320.

[9] Swalmeh, M.Z., Alkasasbeh, H.T., Hussanan, A., Mamat, M. (2018). Heat transfer flow of Cu-water and Al 2 O 3 -water micropolar nanofluids about a solid sphere in the presence of natural convection using keller-box method. *Results in Physics*

[10] Darren J. Creek, Andris Jankevics, Karl E. V. Burgess, Rainer Breitling, Michael P. Barrett, (2012). IDEOM: an Excel interface for analysis of LC–MS-based metabolomics data, *Bioinformatics*, 28(7), 1048–1049.

[11] Dmitry Grapov, John W. Newman, imDEV: A graphical user interface to R multivariate analysis tools in Microsoft Excel, *Bioinformatics*, Volume 28, Issue 17, 1 September 2012, Pages 2288–2290,

[12] Badrul Zaman, D.I (2021). An Excel User Interface for the Newton's Algorithm in Keller Box Method to Solve Dusty Fluid Flow Problem, *Msc Dissertation (UTM).* 1-60.

[13] Tayyaba Shabbir, M. Mushtaq, M. Ijaz Khan, T. Hayat, (2020). Modeling and numerical simulation of micropolar fluid over a curved surface: Keller box method*, Computer Methods and Programs in Biomedicine*, 187, 0169-2607.