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Numerical Solution for Rubella Transmission With Vaccination Effect Using Adaptive Runge-Kutta Method

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Abstract

The purpose of this study is to investigate the application of Runge-Kutta method Order 4 (RK4) and Runge-Kutta Fehlberg method (RK45) in analyzing the spread of Rubella's disease with the effect of vaccination by using MATLAB programming. Rubella is an infectious disease caused by a virus that is spread through the respiratory tract. In some cases, diseases can become endemic, which is a situation in which a disease spreads over a long period of time in a specific location. Ordinary differential equation (ODE), the Runge-Kutta method Order 4 (RK4) was used as the benchmark for the proposed algorithm. A model structure selection based on differential equation in the Suspectible, Exposed, Infected, Recovered and Suspected (SEIR) model has been proposed in this study to reduce case population of Rubella disease and to compared the accurancy between the RK4 and RK45 . The Runge–Kutta method is a common and effective method for solving initial-value problems in differential equations. The Runge–Kutta method can be used to build high order accurate numerical methods from functions without the necessity for high order derivatives of functions. The RK4 is the most often used RK, and it is so accurate that computer systems designed to solve differential equations will use it by default. The RK45 is a numerical analysis algorithm for solving ordinary differential equations that includes a procedure for determining whether the right step size being used. Results were compared between RK4 and RK45. It was discovered that the percentage between two method is slightly different in most cases, RK45 performs better than RK4 in terms of the accurancy of the approximate solution of the spread of Rubella's disease with the effect of vaccination. So it can be concluded that RK45 is better than RK4 in predicting the state rate of Rubella disease case population in SEIR model. For the future research can used the RK45 to solve any ODE because of the smallest step size.

Keywords: Runge-Kutta method Order 4 (RK4) ; Runge-Kutta Fehlberg method (RK45) ; Rubella disease ; SEIR model ; MATLAB programming

1. Introduction

Rubella is a contagious viral infection caused by the Rubella virus, also known as Togavirus, and belonging to the Rubivirus genus. Rubella is a virus-borne infectious disease that spreads through the respiratory tract. In some situations, diseases can become endemic, which is a condition in which a disease spreads over along period of time in a specific area.

To solve the Rubella disease in this research will use the mathematical model which is SEIR model by using Runge-Kutta method. This SEIR model has determined four populations. Susceptible (S) refers to the number of human populations that are susceptible to a disease, Exposed(E) refers to the number of human populations that are showing or have symptoms of being infected with a disease, Infected (I) with the disease, and Recovered (R) refers to the number of human populations that have recovered from the disease [1].

This research aims to (1) obtain a solution of the spread of Rubella's disease with the effect of vaccination by using Runge-Kutta method order 4 (RK4), (2) obtain a solution of the spread of Rubella's disease with the effect of vaccination by using Runge-Kutta Fehlberg method (RK45) and (3) calculate and analyse the accurancy of the approximate solution of the spread of Rubella's disease with the effect of vaccination for both method.

2. Literature Review

2.1. The Rubella Transmission with Vaccination Effect

The Measles Rubella (MR) vaccination is the most effective way to avoid this disease. Rubella infection is a major cause of miscarriage and birth abnormalities in nations that do not have a vaccine control programme (2). More than 11,000 suspected measles cases are reported each year through surveillance activities, with laboratory confirmation results indicating that 12–39 % of them are lab confirmed measles and16–43 % are definite rubella. According to the research, the study analyses the SEIR model in measles with the effect of vaccines and migration, which shows the effect of permanent vaccines on measles spread using the Routh Hurwith method, which is used to evaluate the stability of the endemic equilibrium point (1).

The model in this paper employs four populations: suspected, exposed, infected, and recovered. The model analysis employs the Runge-Kutta method, with data operating as the initial value of the number of cases of Rubella disease and parameter values operating as the number of cases of Rubella disease. As a result, this model can help enhance preventive methods in managing the number of infected cases or people exposed to Rubella's disease by providing interpretation or information and predictions of the number of cases of Rubella spread over the next several years.

2.2. Runge-Kutta Method Order 4 (RK4)

The Runge-Kutta method (RK) was developed in early 1900 by Carl Runge(1856-1927) and Martin Kutta (1867- 1944), two German mathematicians, in order to find the solution to a differential equation in the field of atomic spectra (3). The RK method expands the concept used in the Modified Euler's method. Aside from that, the RK method is a series of implicit and explicit iterative methods for approximating the solutions of ordinary differential equations in numerical analysis. Even though their computational costs are much higher, implicit methods have grown far more important than explicit methods. The reason is because implicit procedures are less constrained in their performance by stability constraints (4).

The Runge-Kutta method order 4 (RK4) is the most often used Runge-Kutta method (RK), and it is so accurate that computer systems designed to solve differential equations will use it by default (5). One of the benefits of fourth order Runge-Kutta methods is the ease with which the step size can be changed. It was also claimed that high order Runge-Kutta formula approaches can achieve extremely accurate approximations of differential equation solutions at a lower computational cost than low order methods.

2.3. Runge-Kutta Fehlberg Method (RK45)

The Runge–Kutta–Fehlberg method (RK45) is a numerical analysis algorithm for solving ordinary differential equations. In 1969, Erwin Fehlberg, a well-known mathematician, modified the well-known RK method to create it (3). This method includes a procedure for determining whether the right step size h is being used. Two different approximations for the solution are made and compared at each step. If the two responses are almost same, the approximation is acceptable. The step size is adjusted if the two responses do not agree to a defined accuracy. The step size is increased if the answers agree to more significant digits than required.

3. Mathematical Model and Runge-Kutta Formula

3.1. Mathematical Model

Based on the above assumptions, the SEIR type model diagram for the spread of Rubella can be seen in Figure 1:

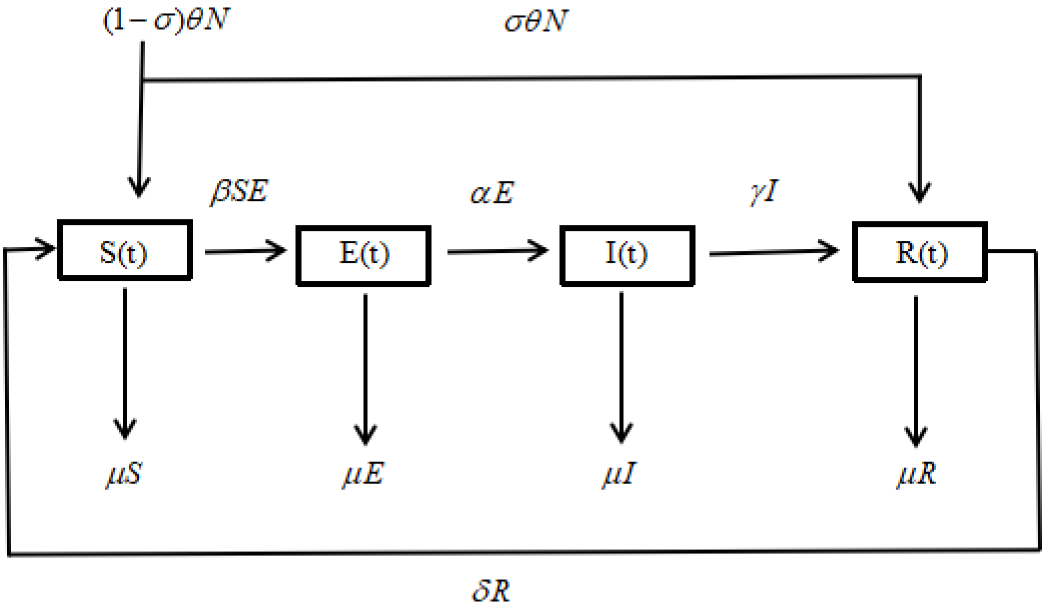

Figure 1 SEIR Model on the Spread of Rubella

From the mathematical model, the system of the spread of Rubella's disease with the effect of vaccination of non-linear differential equations as follow :

$$\frac{dS}{dt} = (1-\sigma)\theta N + \delta R - \beta SE - \mu S \tag{1}$$

$$\frac{dE}{dt} = \beta SE - \alpha E - \mu E \tag{2}$$

$$\frac{dI}{dt} = \alpha E - \gamma I - \mu I \tag{3}$$

$$\frac{dR}{dt} = \gamma I + \sigma\theta N - \delta R - \mu R \tag{4}$$

where

N	-	The number of population
S	-	The number of human populations that are susceptible to a disease

E - The number of human populations showing or showing symptoms of being infected with a disease

I - The number of human populations who have been infected with the disease

R - The number of human population who have recovered from the disease

σ - Vaccinated individual level

β - Level of Rubella-exposed individuals

α - The rate of Rubella-infected individuals

γ - The rate of Rubella-recovered individuals

δ - The rate of Rubella-re-suspected individuals

μ - The rate of death individual

θ - The rate of birth population

3.2. Runge-Kutta Order 4 Method (RK4)

The Runge-Kutta method order 4 (RK4) is the most often used Runge-Kutta method (RK), and it is so accurate that computer systems designed to solve differential equations

The generalization of linear first ODE is :

$$\frac{dy}{dx} = f(x, y) \tag{5}$$

which will give the new initial conditions

$$y(x_0) = y_0 \tag{6}$$

The RK4 formulas for a system of ODE is :

$$y(x_0) = y_0 \tag{7}$$

Where

$$k_1 = hf(x_i, y_i) \tag{8}$$

$$k_2 = hf(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1) \tag{9}$$

$$k_3 = hf(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2) \tag{10}$$

$$k_4 = hf(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_3) \tag{11}$$

3.3. Runge-Kutta Fehlberg Method (RK45)

The RK45 order 4 as in the equation:

$$y_{i+1} = y_i + \frac{25}{216}k_1 + \frac{1408}{2565}k_3 + \frac{2197}{4101}k_4 + \frac{1}{5}k_5 \tag{12}$$

The RK45 order 5 as in the equation:

$$y_{i+1} = y_i + \frac{16}{135}k_1 + \frac{6656}{12825}k_3 + \frac{28561}{56430}k_4 - \frac{9}{50}k_5 + \frac{2}{55}k_6 \tag{13}$$

Where

$$k_1 = hf\,(x_i, y_i)$$
$$k_2 = hf\,(x_i + \frac{1}{4}h, y_i + \frac{1}{4}k_1)$$
$$k_3 = hf\,(x_i + \frac{3}{8}h, y_i + \frac{3}{32}k_1 + \frac{9}{32}k_2)$$
$$k_4 = hf\,(x_i + \frac{12}{13}h, y_i + \frac{1932}{2197}k_1 - \frac{7200}{2197}k_2 + \frac{7296}{2197}k_3)$$
$$k_5 = hf\,(x_i + h, y_i + \frac{439}{216}k_1 - 8k_2 + \frac{3680}{513}k_3 - \frac{845}{4104}k_4)$$
$$k_6 = hf\,(x_i + \frac{1}{2}h, y_i - \frac{8}{27}k_1 + 2k_2 + \frac{3544}{2565}k_3 + \frac{1859}{4104}k_4 - \frac{11}{40}k_5)$$

Next substracting (12) and (13) to measure the error in the optimal step size. The optimal step size can be determined by multiplying the scalar s times the current step size h. The scalar s is

$$s = \left(\frac{tol \times h}{2\,|\,y_{i+1} - x_{i+1}\,|}\right)^{\frac{1}{4}} \approx 0.84\left(\frac{tol \times h}{|\,y_{i+1} - x_{i+1}\,|}\right)^{\frac{1}{4}} \tag{14}$$

where tol is the specified error control tolerance. Even if a constant step size produces a nicer-looking table of values, it is important to understand that it is not the ideal method. If values that are not in the table are required, polynomial interpolation should be applied.

4. Results and discussion

4.1. Comparison of trends RK4 and RK45

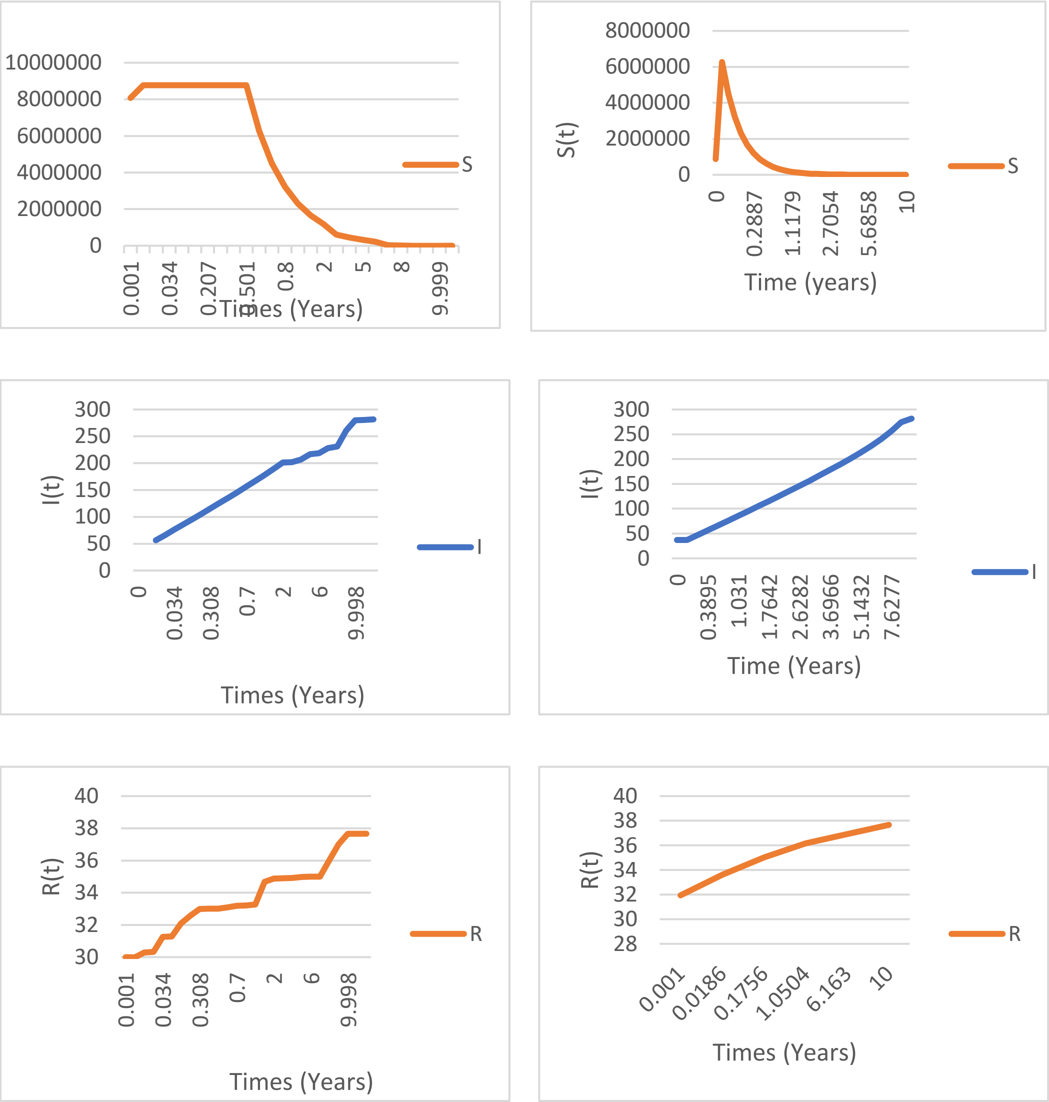

Figure 2 Trends rate of population Rubella diseases by using RK4 and RK45

Based on the trend plot in Figure 1, the Rubella disease in SIR model generates the convergence estimated parameters. The figure shows that RK45 have the good trend to control the the number of cases of Rubella disease spread throughout people over a period of a few years. After get the vaccination, the number of people who have the high risk to get the diseases can be reduce. Therefore, this model can assist in the improvement of methods for reducing the number of infected or exposed Rubella cases.

4.2. Comparison between RK4 and RK45

Table 1 shows the results of Runge-Kutta method order 4 (RK4) and Table 2 shows the result of Runge-Kutta Fehlberg method (RK45) for the Recovered (R) in Rubella diseases.

Table 1: Result values for Recovered (R) in RK4

i	*t*	*R*
0	*0.0000*	*30.0000*
1	*0.0010*	*29.9964*
2	*0.0020*	*29.9970*
…	*…*	*…*
…	*…*	*…*
10000	*9.9990*	*37.6641*
10001	*10.0000*	*37.6646*

Table 2: Result values for Recovered (R) in RK45

i	*t*	*R*
0	*0.0000*	*30.0000*
1	*0.0010*	*31.9375*
2	*0.0186*	*33.5996*
3	*0.1756*	*35.0133*
4	*1.0504*	*36.1702*
5	*6.1630*	*36.9250*
6	*10.0000*	*37.6646*

Based on the both of the Table 1 shows the iteration of the step size for t = 10. In Table, the iteration of I reach until iteration 10001 to get the number of individual group that recovered from Rubella disease in 10 years while in Table 2, the iteration was reduced to 6 iteration. It shows that the efficient of the vaccine to their antibody. As it can be seen in RK45, the step size for each iteration is constant which is h = 0.001. Contrast to the RK4, RK45 provide the optimal step size that different of every iteration. So, RK45 method can reduce the time step to make the calculation shorter than RK4.

Using the RK45, it is shown that the SEIR model will give interpretation and predictions of the number of cases of Rubella disease spread throughout people over a period of a few years. As a result, this model can assist in the improvement of methods for reducing the number of infected or exposed Rubella cases.

Conclusion

This study has investigated two methods to solve the IVP equations which are Runge-Kutta method Order 4 and Runge-Kutta Fehlberg method. Based on this research, the objectives is to obtain a solution of the spread of Rubella`s disease with the effect of vaccination by using Runge-Kutta method order 4 (RK4) and Runge-Kutta Fehlberg mathod (RK45) and also to calculate and analyse the accurancy of the approximate solution of the spread of Rubella`s disease with the effect of vaccination between these two methods. Additionally, it provides valuable experience in studying and comprehending the two methods for solving the mathematical model of the SEIR model in the transmission of Rubella disease. The first step in constructing this thesis is to conduct a literature review of the two methods that will be employed in this thesis to solve the problem of Rubella disease spread with the effect of vaccination. Then, the problems of the SEIR model are solved manually using the two methods mentioned before. The reason for using the different methods is to find the comparison between two methods and to calculate and analyse the accurancy of the approximate solution. According to the results, both methods can provide accurate solutions. In the SEIR model, the Runge-Kutta Fehlberg method outperformed the Runge-Kutta method Order 4 in predicting the rate of population conditions for Rubella disease.

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References

[1] Abdullah, H., Jeffree, M. S., Atil, A., Syed Abdul Rahim, S. S., Hassan, M. R., Awang Lukman, K., & Ahmed, K. 2021. RUBELLA OUTBREAK AMONG TRAINEES OF NATIONAL SERVICE PROGRAM IN PASIR MAS, KELANTAN, MALAYSIA. Malaysian Journal of Public Health Medicine, 21(1): 268–273.
[2] Arora, G., Joshi, V., & Garki, I. S. 2020. Developments in Runge–Kutta Method to Solve Ordinary Differential Equations. Recent Advances in Mathematics for Engineering, 193–202.
[3] Asri, M., Sidjara, S., Sanusi, W., Side, S., & Pratama, M. I. 2021. Analysis and Solution of The SEIRS Model for The Rubella Transmission with Vaccination Effect using Runge-Kutta Method. Journal of Physics: Conference Series, 1899(1): 012090.
[4] BUTCHER, J. C. 2009. PRACTICAL RUNGE–KUTTA METHODS FOR SCIENTIFIC COMPUTATION. The ANZIAM Journal, 50(3): 333–342.
[5] Christophe Kouakou, N., Serge Kouadio, A., Youssouf, T., Cyrille, G., Daniel Kouadio, E., & Simplice Ncho, D. 2018. Rubella Cases Detection through Measles Surveillance System from 2012 to 2016 in Côte d'Ivoire. International Journal of Tropical Diseases, 1(1).
[6] Romeo, G. 2020. Mathematics for dynamic economic models. Elements of Numerical Mathematical Economics with Excel, 139–215.
[7] Ogunniyi, S., & Faruk, S. 2018. Investigation of Selected Versions of Fourth Order Runge-Kutta Algorithms as Simulation Tools for Harmonically Excited Nonlinear Pendulum. Archives of Current Research International, 14(2): 1–12.
[8] Theodoridis, S., Koutroumbas, K., Holmstrom, L. and Koistinen, P. 2009. Pattern Recognition, Wiley Interdisciplinary Reviews Computational Statistics.
[9] Zaki, M. J., Wong, L., Berry, M. J. A., Linoff, G. S., Hegland, M., Zaki, M. J. and Wong, L. 2003. 'Data Mining Techniques', WSPC/Lecture Notes Series: 9in x 6in, 10(1–2), p. 545.