Proceedings of Science and Mathematics
Faculty of Science, Universiti Teknologi Malaysia
Vol. 9, 2022, page 240-249

Investigation of Firefly Algorithm using Multiple Test Functions for Optimization Problem

Muhammad Naqiuddin Irfan Heiqal Azizan, See Pheng Hang
Department of Mathematical Sciences, Faculty of Science
Universiti Tekonologi Malaysia, 81310 Johor Bahru, Malaysia
*Corresponding author: sphang@utm.my

Abstract
Because of their durability, simplicity, and effectiveness in solving difficult optimization problems, bio-inspired optimization approaches have received a lot of attention in recent years. The Firefly Algorithm (FA) is an optimization method with these features. This algorithm is based on collective intelligence algorithms that is the flashing of fireflies' lights. Randomly generated solutions are treated as fireflies in the algorithm, and their brightness is allocated based on their performance on the objective function. The primary purpose of this work is to investigate the convergence in discovering global minima by using the same FA parameters but varying number of populations while when solving numerous test functions as well observe the movement of the fireflies.
Keywords: Firefly algorithm (FA); Test functions, Global minima, Number of populations

1. Introduction

In our daily lives, the basic meaning of optimization is to do better in a field, and in the field of computational intelligence, optimization can be characterized as discovering a parameter in a function that can create the solution better among all possible solutions, and the best value is identified as the optimum solution [7]. Metaheuristic algorithms are used in order to solve optimization problems nowadays. These methods are based on existing processes of a natural biological phenomena. Natural systems are among the most intriguing sources of inspiration for developing new strategies aimed at solving a variety of optimization challenges. Nature-inspired approaches include ant systems, particle swarm optimization, and bee algorithms. These algorithms employ swarm intelligence characteristics. As a result, they are based on living insects or basic interactions between isolated elements [3,5]. Metaheuristic algorithms make specific trade-offs between randomization and local search, since randomization provides a decent approach to move away from local search and toward global search, implying that metaheuristic algorithms are intended to be suited for global optimization.

Dr. Xin-Shi Yang created the Firefly algorithm, which is a metaheuristic algorithm. This algorithm is inspired by the natural behaviour of fireflies, which is centred on the bioluminescence phenomenon. Natural fireflies may produce light due to specific photogenic organs located extremely near to the body surface behind a window of translucent cuticle. This light allows them to communicate with one another as well as attract prey and other fireflies. Their number is believed to be over 2,000 firefly species. The majority of them emit brief, repetitive flashes. Their bioluminescent flashing light might be used as part of courting rituals or as a warning signal [3,5,6].

The rest of the paper is organized as follows. In Sec. 2 we briefly describe the application of FA, test functions and efficiency of FA. Section 3 presents structure of FA with pseudocode and briefly describe the movement of fireflies. Section 4 gives experimental results based on the firefly algorithm using test functions, which is used to solve selected optimization problems. Finally, Sec. 5 summarizes the conclusions.

This research aims to (1) observe the initial and final locations of the fireflies after n-th iteration and (2) find out the convergence in finding global minima for test functions.

2. Literature Review

2.1. Application of Fireflies Algorithm

FA and its variants have been utilised to tackle a variety of optimization and classification problems, as well as a number of engineering challenges. FA has been used to the following kinds of optimization problems: continuous, multimodal, constrained, multi-objective, dynamic, and stochastic optimization. It's also been used in machine learning, data analysis, and neural networks to address categorization problems. Finally, the firefly algorithms are used in almost every technical field. In this review, we emphasised image recognition, industrial optimization, wireless sensor networks, transceivers, corporate optimization, automation, semantic web, chemistry, and civil engineering [1].

The FA is used to solve continuous optimization issues in the bulk of extant articles. In the majority of cases, well-known optimization function benchmarks were applied. In order to provide a complete picture of this area, FA has been used to solve mixed continuous/discrete systemic optimization problems taken from the literature regarding welded beam design, pressure vessel design, helical compression spring design, reinforced concrete beam designs, stepped cantilever beam design, and car side impact design. FA outperforms other metaheuristic algorithms including particle swarm optimization, genetic algorithms, simulated annealing, and differential evolution, according to the optimization findings. Despite FA's high efficiency, oscillating behaviour was observed as the search process approached the ideal design. The general behaviour of FA may be enhanced by significantly lowering the randomization parameter as the optimization progressed [2].

A unique multi-objective FA was created for multi-objective optimization that enlarged FA for directly producing the Pareto optimal front [8]. Before being used to solve design optimization benchmarks in industrial engineering, this technique was tested on a subset of multi-objective functions from the research with convex, non-convex, and discontinuous Pareto fronts. The proposed algorithm was tested against other multi-objective optimization algorithms such as the vector evaluated genetic algorithm (VEGA), nondominated sorting genetic algorithm-II (NSGA-II), multi-objective differential evolution (MODE), differential evolution for multi-objective optimization (DEMO), multi-objective bees' algorithms (Bees), and strength Pareto evolutionary algorithm (SPEA), and the results showed that it is a satisfactory work optimizer.

To recapitulate, since its beginning in 2008, Firefly Algorithm has substantially expanded its application fields. There is hardly no domain in which the Firefly Algorithm has not been employed. Moreover, the algorithm's development zones are highly dynamic, with new applications appearing on a daily basis. This approach has been shown to handle multi-modal problems effectively, has a fast convergence rate, may be used as a general, worldwide problem solver as well as a local search heuristic, and is relevant to any problem domain [1].

2.2. Many Local Minima Test Functions

Test functions known as artificial landscapes, are useful for evaluating optimization algorithm properties such as convergence rate, precision, robustness and general performance. Some test functions for objective functions for single-objective optimization instances are offered here to give a sense of the many conditions that optimization algorithms must deal with when dealing with these types of problems. Some of the many local minima test functions: Ackley function and Drop-Wave Function [4] were used to solve optimization problem by using FA.

2.1.1. Ackley Function

The Ackley function is commonly used in optimization algorithm testing. It has a virtually flat outside region and a big hole in the centre in its two-dimensional form, as illustrated in the Figure 1. The function can trap optimization algorithms, particularly hill climbing algorithms, in one of its numerous local minima [9]. On a 2-dimensional domain it is defined by:

$$f(x) = -a \cdot exp\left(-b\sqrt{\frac{1}{d}\sum_{i=1}^{d} x_i^2}\right) - exp\left(\frac{1}{d}\sum_{i=1}^{d}\cos(cx_i)\right) + a + exp(1), \quad (1)$$

where variable values suggested are: $a = 20$, $b = 0.2$ and $c = 2\pi$. The hypercube is commonly used to assess the function $x_i \in [-32.768, 32.768]$, for all $i = 1, \dots, d$, although it might be limited to a smaller domain and it has a global minimum at $f(x^*) = 0$, where $x^* = (0, \dots\dots, 0)$.

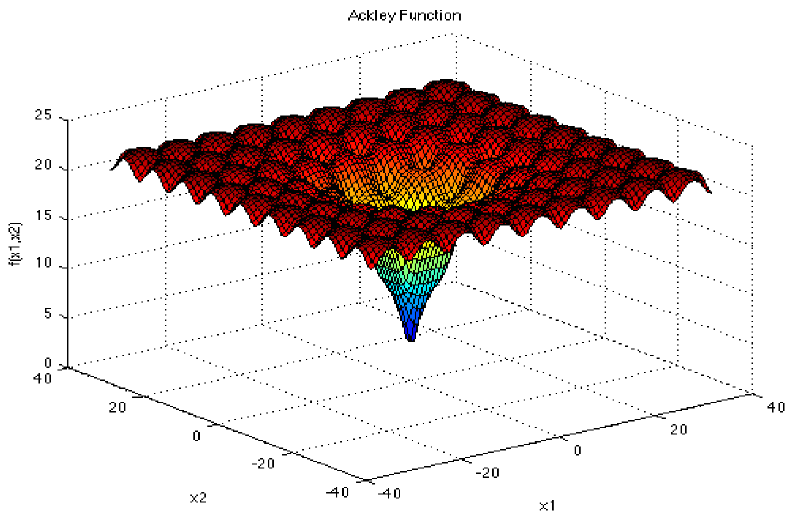

Figure 1: Ackley Function

2.1.2. Drop-Wave Function

Drop-Wave function is quite complex, with growing ripples like an item thrown upon a liquid surface. The drop wave function has a global optimum and many local optimum areas as illustrated in the Figure 2. As a result, there is a considerable risk of misleading search agents [9]. The function on a smaller region displays its 'wave' properties. On a 2-dimensional domain it is defined by:

$$f(x) = -\frac{1+\cos\left(12\sqrt{x_1^2+x_2^2}\right)}{0.5(x_1^2+x_2^2)+2}, \quad (2)$$

where the function is usually computed on the square $x_i \in (-5.12, 5.12)$, for all $i = 1, 2$ and it has four global minima are located at $f(x^*) = -1$, at $x^* = (0, 0)$.

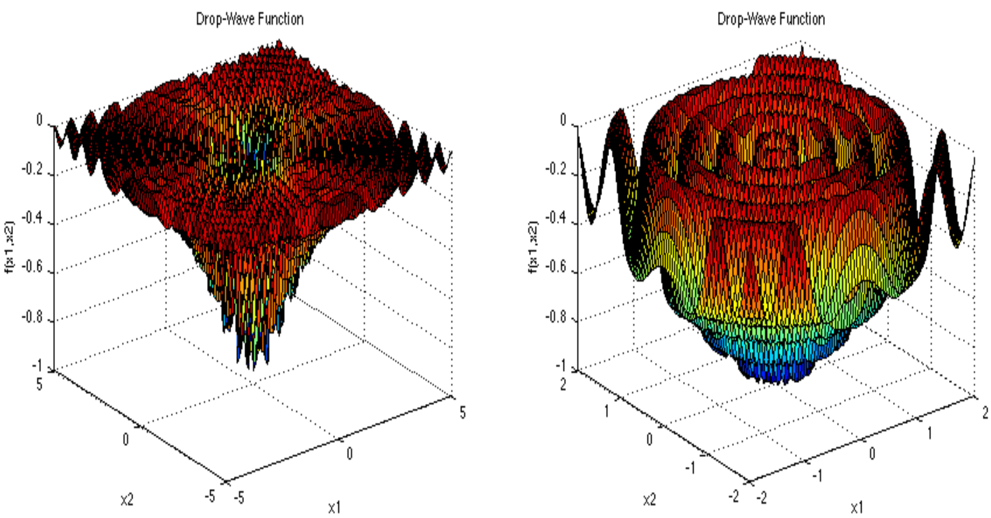

Figure 2: Drop-Wave Function

From these test functions, Ackley function is a multimodal function while Drop-Wave Function is an unimodal function. Since the multimodal function were non-convex, there may be one global optimum and one or more local or deceptive optima. Alternately, there may be multiple global optima, i.e.

These test functions as mentioned above have possibility of becoming caught in one of their numerous local minima. Despite these test functions have numerous local minima, some of these functions has been used as a performance test issue for optimization techniques in mathematics and it's an excellent example of a non-linear multimodal function commonly employed for testing optimization methods.

2.3. The Efficiency of Firefly Algorithm

The FA may automatically split its population into subsets because local attraction is higher than long distance attraction. As a result, the firefly approach can deal naturally and efficiently with nonlinear system, multi-modal optimization issues. Aside from that, the firefly approach doesn't need to use historical individual best si* or a global best g*. This eliminates any early convergence drawbacks, such as those described in PSO. Furthermore, because the FA does not use velocities, there is no concern about velocity as there is with PSO [1].

Second, if the population size is far bigger than the number of phases, the fireflies can find all of the optima at once. The average distance between groups of fireflies that may be viewed by neighbourhood groups is governed by the formula 1/y. As a consequence, depending on a specified, average distance, a complete population may be classified into subgroups. If x = 0 in the worst-case scenario, the whole population will not disperse. It is particularly suited to nonlinear system, multimodal optimization issues due to its independence subdivision capacity [8].

3. Methodology

3.1. Structure of Firefly Algorithm

The FA is based on a scientific formula that states that light intensity I diminishes as the square of the distance r^2 grows. Because the other firefly's flash is brighter than its own, the firefly is lured to it. The attractiveness of the light is proportional to its intensity. Light absorption causes the light to diminish as the distance from the source of light increases. The pseudocode for the general FA [5] was described in the following section.

3.1.1. General Firefly Algorithm

Define an initialize benchmark function $f(x)$, $x = (x_i, \dots\dots, x_k)$

Generate initial population of fireflies x_i, $(i = 1,2, \dots\dots, n)$

Determine light intensity for x_i, by calculating $f(x_i)$

Define randomization parameter α, attraction parameter β and light absorption coefficient γ

While $t <$ Maximum Generation

Make a copy of the generated firefly population for move function

For $i = 1$: *n* all *n* fireflies

For $j = 1$: i all *n* fireflies

If $\left(I_j > I_i\right)$,

Move fireflies i and j according to attractiveness

Evaluating new solutions and updating light intensity for next iteration

End if

End for j

End for i

Sorting the fireflies to find the present best

End while

Begin post process on global minima obtained and visualization.

The firefly algorithm starts by spawning a swarm of fireflies, and each firefly in the swarm is distinct. The differentiation is based on the brightness of the firefly. The interior movement of fireflies is governed by their luminosity. The brightness of one firefly is compared to the brightness of the others in the swarm during the iterative process and the difference in brightness leads the firefly to migrate. The beauty of the firefly determines the distance travelled. The best answer so far is constantly updated during the iterative process, which continues until particular stopping criteria are reached. Following the completion of the iterative process, the optimal evaluation solution is identified, and the post-process to acquire the findings is commenced.

3.2. The attractiveness of the firefly

Each firefly's attractiveness β is defined as a monotonically decreasing function of the distance r between two fireflies of any size:

$$\beta(r) = \beta_0 e^{-\gamma r^2}, \tag{3}$$

where β_0 denotes the firefly attractiveness at $r = 0$ and γ denotes the media light absorption coefficient.

3.3. The movement towards attractive firefly

The transition from a firefly i in position x_i to a brighter firefly j in position x_j [4]:

$$x_i(t+1) = x_i(t) + \beta_0 e^{-\gamma r^2}\left(x_i - x_j\right) + \alpha\varepsilon_i, \tag{4}$$

where $\beta_0 e^{-\gamma r^2}\left(x_i - x_j\right)$ is due to the attraction of the firefly x_j and $\alpha\varepsilon_i$ as a randomization parameter; thus, if $\beta_0 = 0$, then it reveals itself to be a simple random movement.

4. Results and discussion

4.1. Convergence of Test Functions

Ackley function and Drop-Wave function were used to evaluate the FA. This chapter depicts the initial and final positions of fireflies for each test function. The minimum cost value was also computed by doing 25 separate runs over a given number of iterations with varied populations set at 5, 15 and 50. The minimum cost value was then observed to differentiate on the number of populations.

4.1.1. Ackley Function

The Ackley function has a nearly flat outer region and a large hole at the centre as shown in Figure 1 and it has one global minima $f(x^*) = 0$ at $x^* = (0, 0)$. The simulation has been made on Ackley function with the different number of populations, $n = 5, 15, 50$.

Initial locations of n fireflies Locations of n fireflies after 25 iterations

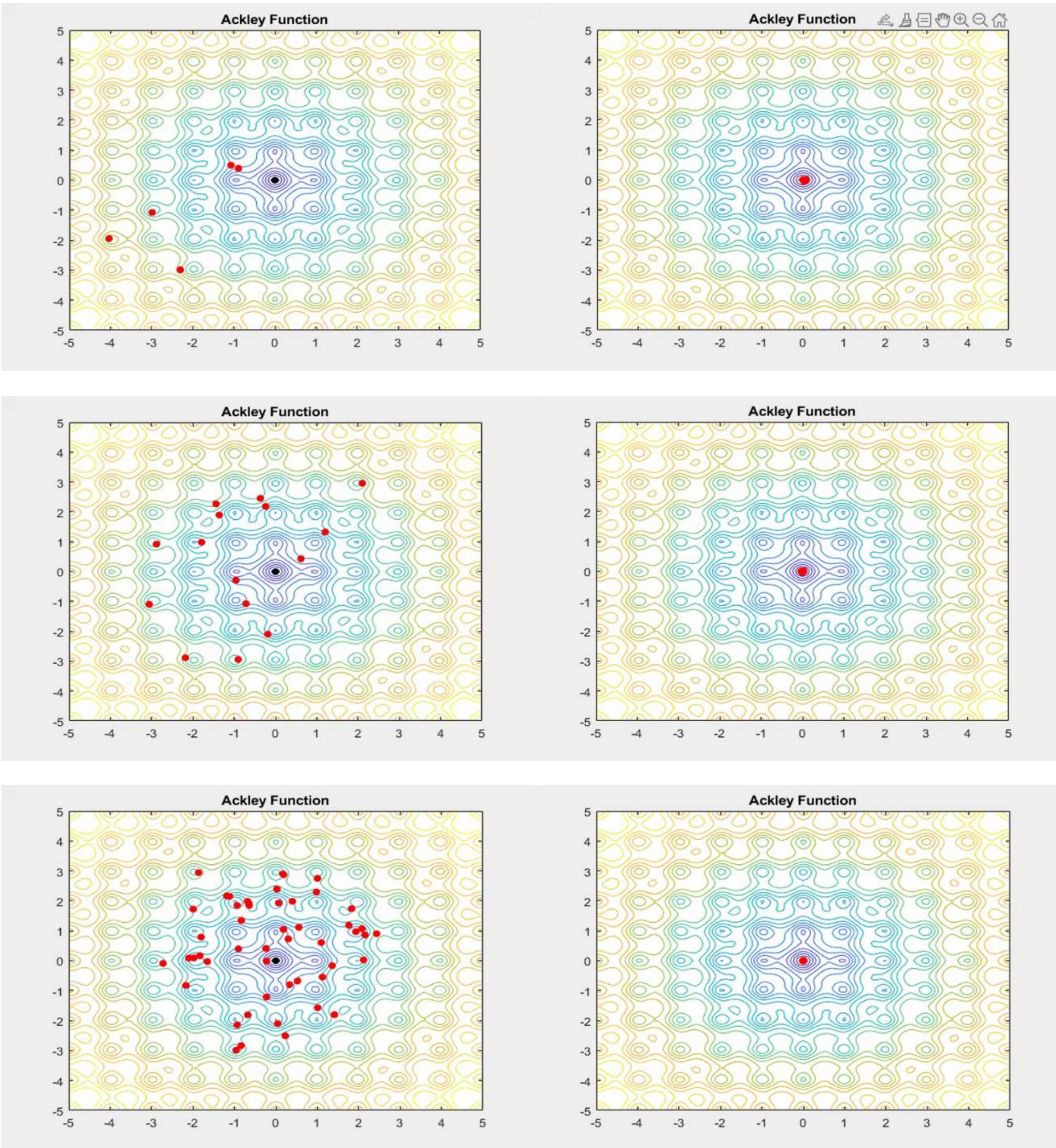

Figure 3: The locations of n fireflies at 25 iterations for Ackley function

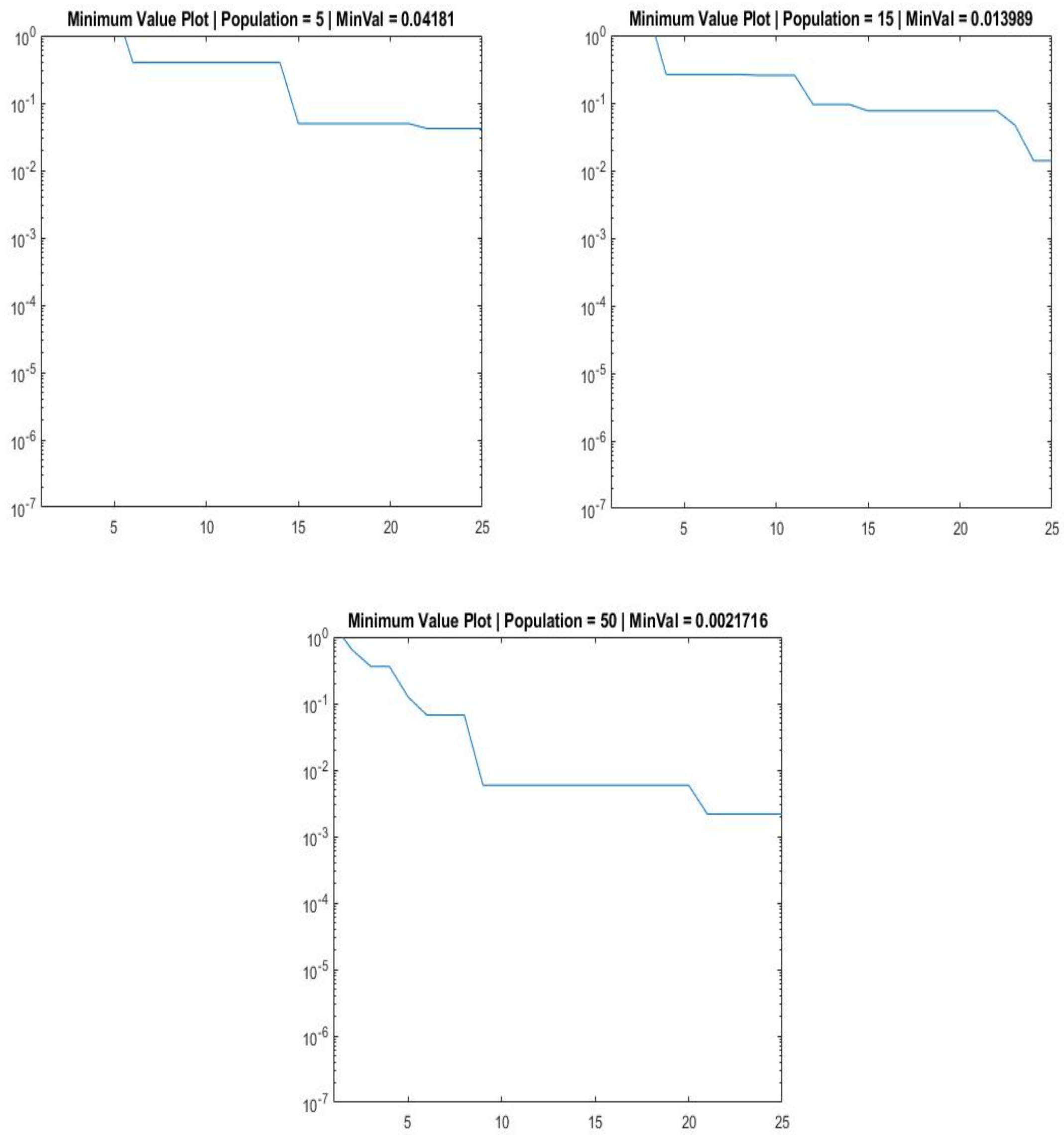

Figure 4: The relationship of objective function with each respective n

As shown in Figure 3, the convergence in finding global minima has been successful since all fireflies converge to global minima $f(x^*) = 0$ after 25 iterations. As the number of populations, n increase, the minimum cost decrease to the known minimum as shown in Figure 4.

4.1.2. Drop-Wave Function

The Drop-Wave function is multimodal, highly complex and it has one global minima $f(x^*) = -1$, at $x^* = (0, 0)$ as shown in Figure 2. The simulation has been made on Drop-Wave function with the different number of populations, $n = 5, 15, 50$.

Initial locations of n fireflies

Locations of n fireflies after 25 iterations

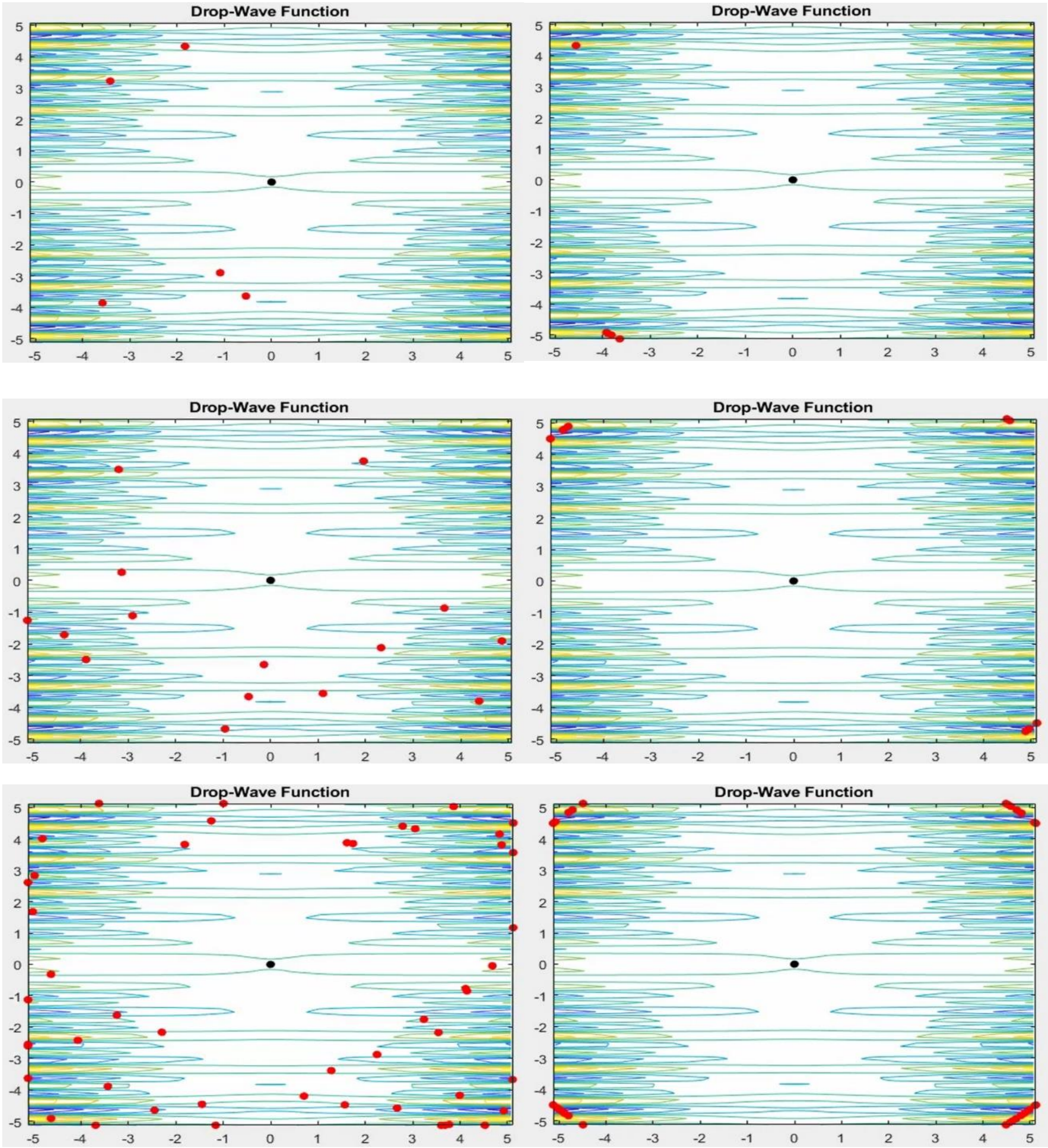

Figure 5: The locations of n fireflies at 25 iterations for Drop-Wave Function

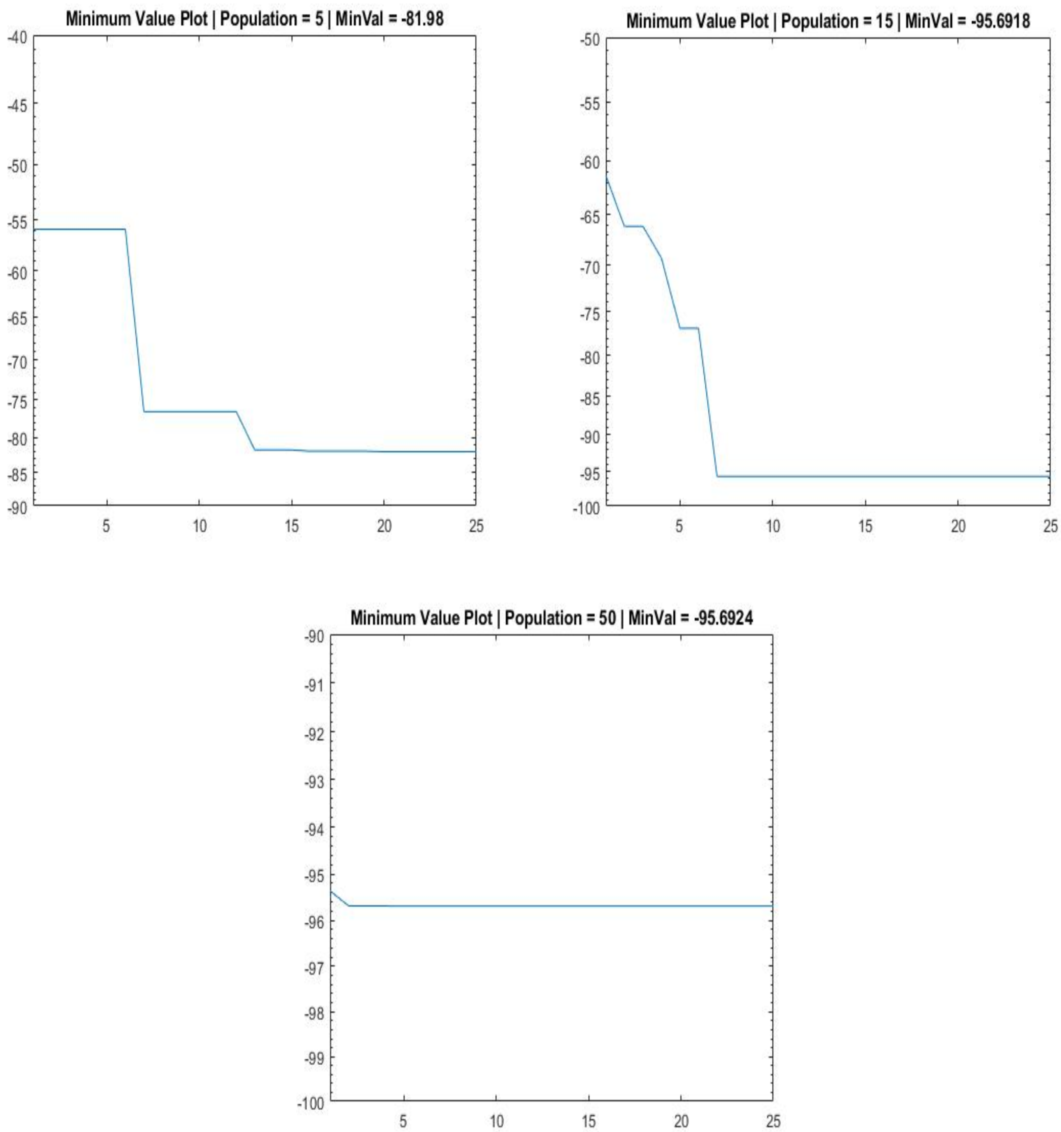

Figure 6: The relationship of objective function with each respective n

As shown in Figure 5, the convergence in finding global minima has been failure since all fireflies doesn't converge to global minima $f(x^*) = -1$ after 25 iterations. As the number of populations, n increase, the minimum cost decrease further away from the known minimum as shown in Figure 6.

Test function	Global minima, $f(x^*)$	Minimum cost		
		$n = 5$	$n = 15$	$n = 50$
Ackley function	0	0.0418	0.0140	0.0022
Drop-Wave Function	-1	-81.98	-95.6918	-95.6924

Table 1: Results for the minimum cost

Conclusion

The computed solutions are of high quality and the obtained minimum cost value obtained for the problems are very close to the known minimum for Ackley function but not for Drop-Wave function. As shown in Table 1 before, the minimum cost value for Ackley function decreases to the known minimum as the number of populations, n increase. However, this is not implied to Drop-Wave function as the number of populations, n increase, the minimum cost value drifted away from the know minimum. The Drop wave function is continuous, multimodal and highly complex function. FA seems to provide a very fast convergence in finding the global minima. However, it can get trapped in local minima when the function has a distant minima. Therefore, the hybrid of deterministic algorithm with metaheuristic algorithm; which is FA can be consider to construct the exact solution and this might can achieve the best solution or using other metaheuristics algorithm such as Bat Algorithm or Particle Swarm Optimization Algorithm.

Acknowledgement

The content is solely the responsibility of the author and does not necessarily reflect the official views of the university. The researcher would like to thank all people who have supported the research.

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