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Adaptive Runge Kutta for Solving Log Aesthetic Curve

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Abstract
In this study, Log Aesthetic Curve equation is solved with Adaptive Runge Kutta Fehlberg method. This Fehlberg method is compared with Classical Fourth Order Runge Kutta in term of computational speed and error. Firstly, the Log Aesthetic Curve equation is formulated by synthesizing a curvature function that create a constant gradient which its coordinates of the intersection of y-axis with shape parameter will determine the type of Log Aesthetic Curve. After undergoing manipulation and integration process, Log Aesthetic Curve is obtained and easily converted into first ordinary differential equation with initial condition as its limits. As such, this equation is computed by using numerical method. The advantage of Fehlberg method is based on error control and the adjustment of the time step to maintain the error at a certain threshold. By comparing with Classical Runge Kutta, it shows that Adaptive Runge Kutta has better performance in terms of computational speed. This study ends with a detailed investigation method to compute Log Aesthetic Curve

Keywords: Fourth Order Runge Kutta; Fehlberg Method; Computational Speed; Error Control;

1. Introduction

In Roman times, curves and surfaces were developed for shipbuilding. Advances in mapping this in the form of mathematical equations and algorithms have increased dramatically with the introduction of numerical control on milling machines in the 1950s. In the 1960s, automobile and aircraft design was two important areas where mathematicians worked on the curves used for effective and efficient design. LAC was introduced to beautify products which consisting of spiral families such as clothoids, logarithmic spirals, circular involutes, and Neilsense spirals. In the 1970s, Nutbourne et al. investigated the curve composition in which the curve is derived from a well-defined curvature function. Then, there are two enthusiasts Meek & Walton who have made significant changes to the clothoid due to design intent.

Log Aesthetic Curve is such a visually appealing curves which has been evolved the use of monotonic curvature profile. One of the good-sized capabilities of LAC is that its curvature will increase or decreases monotonically. This belonging would possibly help in resource withinside the layout of ship hulls and breakwaters if the connection among LAC and hydrodynamics became made clear. In order to get monotonic layout to provide aesthetic shape, the designer want to undergo a fairing technique after preliminary layout. The concept of the use of spirals households with monotonic curvature profiles has been progressing considering the fact that clients are attracted via way of means of the classy look of the goods before distinguishing its distinct practical capabilities. Thus, the classy shapes of merchandise dictate to the achievement of the economic product which include designing of highway or railway layout. Also, many civil engineers used one of the types of LAC which is clothoid to construct a very good motorway or railway layout.

In addition, LAC is also used for shape completion issues. In the design environment, objects' outlines may not exist or may be excluded. These scenarios are commonly referred to as shape completion problems or curve completion problems. Complete objects can continue to be used in modelling, network analysis, computer- assisted construction, and manufacturing process applications, so it's important to complete the shape of the missing object while maintaining its aesthetic appeal.

After Yoshida and Saito proposed G1 algorithm that interactively provides LAC for the creation of aesthetic shapes, LAC research began to move rapidly. The Log Aesthetic Curve has undergone a variety of practical and experimental studies. The computational time to render the LAC can be reduced by expressing the LAC in an analytical format, for example as an incomplete gamma function (IGF). The analytical format is values to calculate the accuracy of Log Aesthetic Curve.

In addition, this study describes the classical fourth order as one of the best ways to solve ordinary differential equations when the exact solutions are difficult to compute. Therefore, the numerical method will give approximation to the problem. Runge-Kutta method is an effective and widely used method for solving the initial value problem of differential equations. This method can also be used to build an accurate higher-order numerical method by the function itself, without the needs for derivatives of higher-order functions. Researchers are constantly improving the way they solve ordinary differential equations. Thus, there is a way to improve theRunge- Kutta method, called adaptive Runge-Kutta. A good ODE solver may have adaptive controls that make iterative changes to the step size that can make the calculation more efficient (Wai et al., 2021). The purpose of adaptive step size control is to achieve a certain given accuracy of the solution with minimal computational effort. There are various types of Adaptive Runge Kutta methods, which are the Fehlberg method, the Sarafyan method, the Kutta Merson method, the Cash-Karp method, and the Dormand-Prince method.

Therefore, this study aims to perform a numerical analysis of log-aesthetic curves using the classical Fourth Order Runge-Kutta and Adaptive method. The numerical approach is performed using the programming software, MATLAB to simplify the calculations. Then, the approximation solution from both methods will be compared in computation time in which method will have lesser time while computing the solution.

2. Literature Review

Miura (2005) proposed a new type of curve called Log Aesthetic Curve. This curve was the result of an extension of the study by Harada et al. through the lognormal distribution of curvature (LDDC). Harada et al. (1999) when trying to study the characteristics of curves used in automobile design, he proposed LDDC as a tool for studying the aesthetic value of curves. LDDC is the position of the frequency interval between the radius of curvature and the corresponding length. The LDDCs generated for these curves are in the form of straight lines. Therefore, a curve with aesthetic properties produces a linear LDDC. The Log Curvature Graph (LCG) is an analytical plot of LDDC which first published by Kanaya et al. (2003). Miura formulated LAC by synthesizing a curvature function that creates a constant gradient in the LCG. Recently, Ziatdinov etc. LAC is shown in the form of an incomplete gamma function using the classic Runge-Kutta order 4.

Yoshida and Saito (2006) evaluated LAC using numerical integration based on the Gaussian quadrature method (Kronrod, 1964), but when presented in analytical form, numerical integration may be avoided and it is interactive since numerical integration more suitable for interactive applications, which important when generating surfaces that containing many Log Aesthetic Curve Segments. Ziatdinov et al. obtain the resulting parametric equation consists of an incomplete gamma function for which a good approximation exists, thus from that, analytic parametric equation can be get for the logarithmic curve of the tangential angle. This allows to get an accurate representation of the actual value or shape parameter.

Several other researchers have also discussed LAC in terms of its relationship to streamline fluid flow patterns. As already mentioned above, one of the main features of LAC is the monotonous increase or decrease in curvature. This property may be useful in the design of hulls and breakwaters if the relationship between LAC and fluid dynamics is clarified. Recently, Wo, Gobithaasan and Miura (2014) showed how LACs are formed under the influence of magnetic fields and provide physical insights into LACs in electromagnetism. Therefore, this work is the result of a study of LAC patterns in the field of fluid mechanics. Mei Seen Wo et al. describe LAC in the form of pathlines, streamlines, and streak lines, which are the three main flow fields studied in a flow visualization environment, and then how LACs were formed under the influence of pressure through the Euler equations.

There are numerous works on developing surfaces using Log Aesthetic Curve. In 2012, Ziatdinov implemented the generalized version of LAC as a profile curve to generate a surface of revolution called a superspiraloid. Inoue et. Al developed an algorithm that used the LAC as a profile curve to generate a log aesthetic curved surface using the virtual reality (VR) technique. Furthermore, the LAC has also been used to design bi-cubic B-spline surfaces. The fundamental equation of the LAC can also be used as a digital filter to smooth any arbitrary surfaces.

3. Methodology

3.1. Equation of Log Aesthetic Curves

Log Aesthetic Curve can easily be converted into first ordinary differential equation with initial conditions as its limits. This enables to compute Runge-Kutta method with initial values.

$$Y' = f(\theta, Y) = y + \rho(\theta)\sin(\theta), 0 \le \theta \le 3, Y_0 = 0$$

This equation is solved by using 2 methods which are Fourth Order Runge Kutta and Fehlberg Method and then compare the solution from both methods with exact solution.

4. Results and discussion

4.1. Performances of both methods

The performances of both methods in solving Log Aesthetic Curve equation are presented graphically in Figure 4.3.

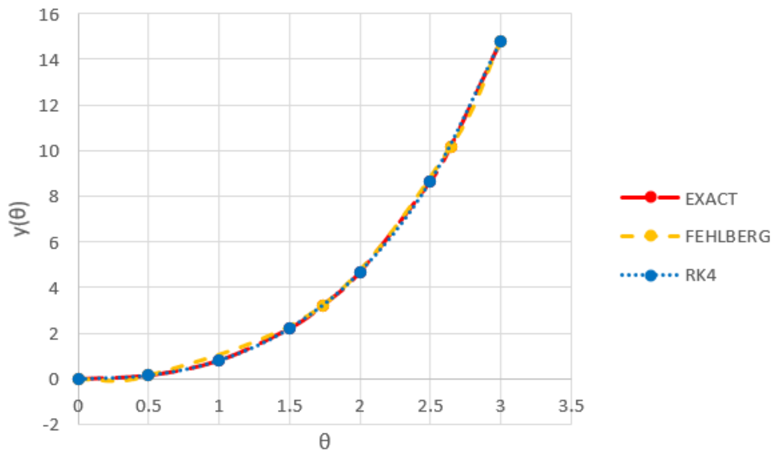

Figure 4.3: Comparison between Fehlberg Method and Fourth Order Runge Kutta with Exact Solution

Conclusion

From Figure 4.3, there is huge difference between Exact Solution and Fourth Order Runge Kutta rather than Fehlberg method and Exact Solution. Thus, Fehlberg method acquire less error rather than Fourth Order Runge Kutta since Fehlberg method is constructed based on method of order four with an error estimator of order five. By performing one extra calculation, the error in the solution can be estimated and controlled by using higher-order embedded method that allows for an adaptive step size to be determined.

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