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Solving Mixed Convection Boundary Layer Flow of Viscoelastic Nanofluid Past over a Sphere in Presence of Viscous Dissipation

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Abstract
This purpose of this study was to examine the flow of viscoelastic nanofluid past over a sphere in mixed convection flow with the presence of viscous dissipation. A mathematical model based on Tiwari-Das nanofluid model is developed. Carboxymethyl cellulose solution (CMC) is chosen as a base fluid and Copper (Cu) as dispersing nanoparticles. The constant mixed convection boundary layer flow around an isothermal sphere with radius a, where a is immersed in a viscoelastic nanofluid with convective boundary condition (CBC). The governing boundary layer partial differential equations are transformed into non-dimensional form then solved numerically by using BVP4C solver in MATLAB. The velocity and temperature profiles with different values of viscous dissipation parameter, viscoelastic parameter, Prandtl number, mixed convection parameter and nanoparticles volume fraction are graphically presented.

Keywords: Mixed convection; boundary layer flow; nanofluid; sphere; viscous dissipation; viscoelastic; MATLAB BVP4C solver.

1. Introduction

Nanotechnology has been widely used in the industry due to the fast progress in technology in all fields of life as it can save energy and reduce cost of production. One of the most likely applications of nanotechnology is to produce nanoparticles of high thermal conductivity and mixing with base fluids and transfer energy forming what are called nanofluids. Nanofluids have high transfer characteristics compared with based fluid as they are assembled by mixing the base fluid of low thermal conductivity with solid nanoparticles of high thermal. Much research has been conducted to analyse the nanofluid dynamic and thermal conductivity process related to mixed convection.

During past few years, many researchers' studies about boundary layer flow. According to Gupta and Gupta [2], the stretching process is not linear in all genuine scenarios because of it has unique physical and chemical features, nanofluid is now widely used in industry. Hayat *et al.* [3] examined the boundary layer flow of viscoelastic nanofluid across a stretching cylinder with mixed convection. This interest of study is inspired by practical uses in large industrial such as biomedical, transportation, electronics, vehicles, fuel cells and hybrid-powered engines. In addition, based on Abbas and Madgy [27] the term of nanofluids can be defined as the addition of small quantity of nanometer-sized particles nominally less than 100 nm into base fluids such as oil, water, biofluids, ethylene and lubricants as Choi [5] invented the term when he added micro nanoparticles to a base fluid to boost thermal conductivity.

In all studies stated, the viscous dissipation is neglected. According to Gebhart [4], the first researcher who studied about viscous dissipation in free convection flow, viscous dissipation is noticeable when the produced kinetic energy exceeds the quantity of heat transported.

The topic of nanofluids has grown in relevance in recent years due to its applications in different fields of science, engineering, and technology, particularly in material processing, chemical and nuclear industries, geophysics, and bioengineering. Nanofluids' unique features make them potentially valuable in this inquiry. As a result, it is worth mentioning that the phenomena observed in viscoelastic nanofluids are not only essential in wide range of technologies application, but also present various interesting challenges to researchers in applied mathematics and engineering.

The primary objective of this study is to better understand the behaviour of viscoelastic nanofluid flow past over a sphere by establishing a mathematical model and developing simulations by using BVP4C solver in MATLAB. Velocity and temperature profiles are developed in this study to determine the flow behaviour as influenced by the viscous dissipation parameter, viscoelastic parameter, Prandtl number, mixed convection parameter, and nanoparticle volume fraction.

2. Literature Review

Nanofluids are obtained by dispersing nanoparticles and dispersant, when present, in a base fluid. In the past few decades, convective heat transfer in nanofluid has become huge interest in fluid flow study due to its wide use in industries. In 1995, the quantity of nanoparticles was first suggested by Choi [5] for augmenting thermal properties of pure fluid. Then, Tham *et al.* [17] study on the mixed convection flow over a solid sphere embedded in a porous medium filled by a nanofluid containing gyrotactic microorganisms as one of the characteristics of nanoparticles is high thermal at very low concentration. In 2014, Sathyamurthy *et al.* [15] study on nanofluid use to enhance the solar energy by replacing the conventional fluid with nanofluid and the research conducted by comparing numerous papers to achieve the aim that using nanofluid can enhance the solar energy. A study by Khrisna [14] shows that the nanofluid provides better surface roughness by reducing the coefficient of friction which leads to a lower cutting temperature.

A study on the mixed convection flow past over a sphere has attracted the attention of many researchers due to its important industrial applications such as liquid film for the condensation procedure, crystal growth, food production and paper, glass manufacturing and others. There were numerous numerical analysis studies on the mixed convection flow over the sphere with different methods. One of the methods is the transformed boundary layer equations were solved numerically using implicit finite difference schemes [17] [24]. Next, the study on the heat transfer of nanofluids has been widely conducted by researchers. One of the studies is for the flow of time dependent mixed convection fluid with heat transmission over an elongated permeable surface using flow condition and this study included numerical and analytical investigation [28].

The studies as mentioned, the viscous dissipation neglected. Based on Gebhart [4], the first researchers that studies on the viscous dissipation in free convection flow, that when the induced kinetic energy gets large in comparison to the amount of heat transferred, viscous dissipation becomes significant. Graphs and tabular representations are used to illustrate velocity, temperature, skin friction coefficient, and local Nusselt number profiles for various parameters considered on a study of the MHD stagnation-point flow and heat transfer characteristics of an electrically conducting nanofluid over a vertical permeable shrinking sheet in presence of viscous dissipation using Runge-Kutta-Felhberg method [26].

3. Mathematical Model

The constant mixed convection boundary layer flow around an isothermal sphere with radius a, where a is immersed in a viscoelastic nanofluid with convective boundary condition (CBC). Figure 3.1 illustrate the coordinate system and flow model for this study. The velocity beyond the boundary is assumed to be $\bar{u}_e(\bar{x})$, and the temperature of the ambient nanofluid is assumed to be T_∞. T_w is the constant temperature of the sphere's surface. $T_w < T_\infty$, on the other hand, corresponds to a cooled sphere. According to Merkin [27], the free stream velocity is $\frac{1}{2}U_\infty$.

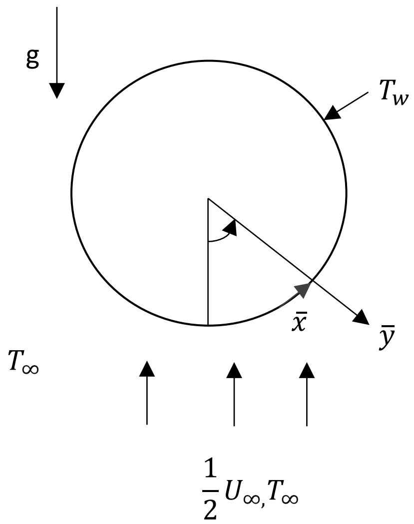

Figure 1: Coordinate system and flow model

By using the nanofluid model from Tiwari and Das [7], the governing equations problem as follows:

$$\frac{\partial(\bar{r}\bar{u})}{\partial\bar{x}} + \frac{\partial(\bar{r}\bar{v})}{\partial\bar{y}} = 0. \tag{1}$$

$$\begin{aligned}\rho_{nf}\left(\bar{u}\frac{\partial\bar{u}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{u}}{\partial\bar{y}}\right) &= \rho_{nf}\left(\bar{u}_e\frac{\partial\bar{u}_e}{\partial\bar{x}}\right) + \mu_{nf}\left(\frac{\partial^2\bar{u}}{\partial\bar{y}^2}\right) \\ &+k_0\left[\frac{\partial}{\partial\bar{x}}\left(\bar{u}\frac{\partial^2\bar{u}}{\partial\bar{y}^2}\right) + \bar{v}\frac{\partial^3\bar{u}}{\partial\bar{y}^3} - \frac{\partial\bar{u}}{\partial\bar{y}}\frac{\partial^2\bar{u}}{\partial\bar{x}\partial\bar{y}}\right] \\ &+g(\rho\beta)_{nf}(T - T_\infty)\sin\left(\frac{\bar{x}}{a}\right).\end{aligned} \tag{2}$$

$$\begin{aligned}(\rho C_p)_{nf}\left[\bar{u}\frac{\partial T}{\partial\bar{x}} + \bar{v}\frac{\partial T}{\partial\bar{y}}\right] &= k_{nf}\frac{\partial^2 T}{\partial\bar{y}^2} + v\left(\frac{\partial\bar{u}}{\partial\bar{y}}\right)^2 - \\ &k_0\left(\bar{u}\frac{\partial\bar{u}}{\partial\bar{y}}\frac{\partial^2\bar{u}}{\partial\bar{x}\partial\bar{y}} + \bar{v}\frac{\partial\bar{u}}{\partial\bar{y}}\frac{\partial^2\bar{u}}{\partial\bar{y}^2}\right),\end{aligned} \tag{3}$$

subjected to boundary conditions

$$\bar{u} = 0,\ \ \bar{v} = 0,\ \ T = T_w\ , \qquad \text{at}\ \ \bar{y} = 0,\ \ \bar{x} \geq 0,$$
$$\bar{u} = \bar{u}_e(\bar{x}),\ \ \frac{\partial \bar{u}}{\partial \bar{y}} = 0,\ \ T = T_\infty\ \ \text{as}\ \ \bar{y} \to \infty, \bar{x} \geq 0, \tag{4}$$

where $\bar{x}$ and $\bar{y}$ are the Cartesian coordinates along the surface of the sphere. While $\bar{y}$ is the coordinate measured normal to the surface of the sphere. Whereas $\bar{u}$ and $\bar{v}$ are the velocity components, $\bar{u}_e(\bar{x})$ is the velocity outside the boundary layer, and $\bar{r}(\bar{x})$ is the radial distance from symmetrical axis to the surface and the equation is given by

$$\bar{u}_e(\bar{x}) = \frac{3}{2} U_\infty \sin\left(\frac{\bar{x}}{a}\right)\ \ \text{and}\ \ \bar{r}(\bar{x}) = a \sin\left(\frac{\bar{x}}{a}\right). \tag{5}$$

Nanofluid terms defined by Tham *et al.* [16] as follows:

$$\alpha_{nf} = \frac{k_{nf}}{(\rho Cp)_{nf}}, \qquad \alpha_f = \frac{k_f}{(\rho Cp)_f}, \qquad \mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}},$$
$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s,$$
$$(\rho Cp)_{nf} = (1-\phi)(\rho Cp)_f + (\phi)_s\ ,$$
$$(\rho\beta)_{nf} = (1-\phi)(\rho\beta)_f + (\phi)(\rho\beta)_s,$$
$$k_{nf} = k_f \frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)}, \tag{6}$$

where μ is the dynamic viscosity, g is the gravity acceleration, T is temperature of selected fluid, $k_0 > 0$ is the constant of the viscoelastic material which is Walter's Liquid-B model), ρ_{nf} and μ_{nf} are the density and dynamic viscosity of nanofluid, $(\beta)_{nf}$ is the thermal expansion of nanofluid, k_{nf} is the effective thermal conductivity of the nanofluid, and $(\rho Cp)_{nf}$ is the heat capacitance of nanofluid. Whereas ϕ is the nanoparticles volume fraction of nanofluid. k_f and k_s are the thermal conductivities of the fluid and of the solid fractions, respectively, μ_f is the viscosity of fluid fraction and α_{nf} is the thermal diffusivity of nanofluid.

The dimensionless variables based on Patil *et al.* [11] and Nazar *et al.* [12] are

$$x = \frac{\bar{x}}{a}, \qquad y = Re^{\frac{1}{2}}\left(\frac{\bar{y}}{a}\right), \qquad u = \frac{\bar{u}}{U_\infty}, \qquad v = Re^{\frac{1}{2}}\left(\frac{\bar{v}}{U_\infty}\right),$$
$$r(x) = \frac{\bar{r}(\bar{x})}{a}, \quad u_e(x) = \frac{\bar{u}_e(\bar{x})}{U_\infty}, \quad \theta = \frac{(T - T_\infty)}{(T_w - T_\infty)}, \tag{7}$$

where $Re = U_\infty \frac{a}{v}$ is a Reynold number. Substitute Equation (7) into Equations (1), (2), and (3), the equations become the dimensionless equations as below:

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial y}(rv) = 0. \tag{8}$$

$$\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right) = \left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)u_e\frac{\partial u_e}{\partial x}+\frac{1}{(1-\phi)^{2.5}}\frac{\partial^2 u}{\partial y^2}+K\left(\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right)+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2}\right) + \left((1-\phi)+(\phi)\frac{(\rho\beta)_s}{(\rho\beta)_f}\right)\lambda\theta\sin\,(x). \tag{9}$$

$$\left((1-\phi)+(\phi)\frac{(\rho Cp)_s}{(\rho Cp)_f}\right)\left(u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}\right)=\frac{1}{Pr}\frac{1}{(\rho Cp)_f}\left(\frac{\partial^2\theta}{\partial y^2}\right)-Ec\left(\left(\frac{\partial u}{\partial y}\right)^2+K\left(u\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}+v\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\right)\right), \tag{10}$$

and the boundary conditions becomes,

$$\begin{gathered} u=10, \quad v=10, \quad \theta=1, \quad \text{on } y=0, \quad x\geq 0, \\ u=u_e(x)=\frac{3}{2}\sin x, \quad \frac{\partial u}{\partial y}=0, \quad \theta=0, \quad \text{as } y\to\infty, \quad x\geq 0, \end{gathered} \tag{11}$$

where

$$\begin{gathered} Re=\frac{U_\infty a}{v}, \qquad Ec=\frac{{U_\infty}^2}{(\rho Cp)_f(T_w-T_\infty)}, \qquad K=\frac{k_0 U_\infty}{a(\rho Cp)_f v}, \\ Pr=\frac{v}{a}, \qquad \lambda=\frac{Gr}{Re^2}=\frac{g\beta_f(T_w-T_\infty)a}{{U_\infty}^2}, \qquad Gr=\frac{g\beta_f(T_w-T_\infty)a^3}{{v_f}^2}. \end{gathered} \tag{12}$$

Ec is the Eckert number, K is dimensionless viscoelastic parameter, Pr is the Prandtl number, and λ is the constant mixed convection parameter, Gr is known as Grashof number. It should be mentioned that the aiding flow (heated sphere) occurs when $\lambda > 0$, the opposing flow (cooling sphere) occurs when $\lambda > 0$, and the forced convection flow occurs when $\lambda = 0$. It is important to note that in the case of viscous (Newtonian) fluids, K = 0.

The following variables being assumed to solve Equation (8), Equation (9), and Equation (10) together with the boundary conditions (11):

$$\psi = xr(x)f(x,y), \qquad \theta=\theta(x,y), \tag{13}$$

where ψ is the stream function defined as follows:

$$u=\frac{1}{r}\frac{\partial\psi}{\partial y}, \qquad v=-\frac{1}{r}\frac{\partial\psi}{\partial x}. \tag{14}$$

Considering that

$$u_e(x)=\frac{\bar{u}_e(\bar{x})}{U_\infty}=\frac{3}{2}\sin x\,, \qquad r(x)=\sin x, \tag{15}$$

where $u_e(x)$ is the local free stream velocity outside the boundary layer and $r(x)$ is the radial distance from the symmetrical axis to the surface of the sphere. The stream function in Equation (13) is satisfy for continuity equation.

$$\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial^2\psi}{\partial x\partial y}=0. \tag{16}$$

$$\begin{aligned}
&\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)\left(x\frac{\partial^3 f}{\partial x\partial y^2}+\left(-\frac{\cos x}{\sin x}\right)\left(\frac{\partial^2 f}{\partial y^2}\right)+x\cos x\frac{\partial^2 f}{\partial y^2}\right)+\left(x\frac{\sin x}{\cos x}+1\right)\left(\frac{\partial^2 f}{\partial y^2}-\frac{\partial^2 f}{\partial y^2}f\right)\\
&=\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)\frac{9}{4}\frac{\sin x\cos x}{x}+\frac{1}{(1-\phi)^{2.5}}\left(\frac{\partial^3 f}{\partial y^3}\right)\\
&+K\left(x\frac{\partial^2 f}{\partial x\partial y}\frac{\partial^3 f}{\partial y^3}+2\left(1+x\frac{\cos x}{\sin x}\right)\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+x\frac{\partial^4 f}{\partial x\partial y^3}\frac{\partial f}{\partial y}-x\frac{\partial^4 f}{\partial y^4}\frac{\partial f}{\partial y}-x\frac{\partial^3 f}{\partial x\partial y^2}\frac{\partial f}{\partial y^2}\right.\\
&\left.-2\left(x\frac{\cos x}{\sin x}\right)\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}-\left(1+x\frac{\cos x}{\sin x}\right)\frac{\partial^4 f}{\partial y^4}\frac{\partial f}{\partial y}-\left(x\frac{\cos x}{\sin x}+1\right)\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial y^2}\right)\\
&+\left((1-\phi)+\phi\frac{(\rho\beta)_s}{(\rho\beta)_f}\right)\lambda\theta\frac{\sin x}{x}.
\end{aligned} \tag{17}$$

$$\begin{aligned}
&\left((1-\phi)+\phi\frac{(\rho Cp)_s}{(\rho Cp)_f}\right)\left(\left(x\frac{\partial f}{\partial y}\right)\frac{\partial\theta}{\partial x}-\left(x\frac{\partial f}{\partial x}+\left(1+x\frac{\cos x}{\sin x}\right)f\right)\frac{\partial\theta}{\partial y}\right)\\
&=\frac{1}{Pr}\frac{1}{(\rho Cp)_f}\frac{\partial^2\theta}{\partial y^2}-Ec\left(x^2\frac{\partial^2 f}{\partial y^2}\right)\frac{\partial^2 f}{\partial y^2}\\
&+K\left(\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+x\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial x\partial y^2}-x\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}-\frac{\partial^3 f}{\partial y^3}-\frac{\partial^3 f}{\partial y^3}f-\left(x\frac{\cos x}{\sin x}\frac{\partial^3 f}{\partial y^3}f\right)\right).
\end{aligned} \tag{18}$$

and the boundary conditions become,

$$\begin{aligned}
&f=0,\quad \frac{\partial f}{\partial y}=0,\quad \theta'=-1,\quad \text{on } y=0,\quad x\geq 0,\\
&\frac{\partial f}{\partial y}\to\frac{3}{2}\frac{\sin x}{x},\quad \frac{\partial^2 f}{\partial y^2}=0,\quad \theta\to 0,\quad \text{as } y\to\infty,\quad x\geq 0.
\end{aligned} \tag{19}$$

At lower stagnation point of the sphere, $x\approx 0$, Equations (17) and (18) reduce to the ordinary differential equations as follows:

$$\begin{aligned}
&\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)\left(2ff'-f'^2\right)+\frac{9}{4}\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)\\
&+\frac{1}{\upsilon\rho f}\left(\frac{\mu f}{(1-\phi)^{2.5}}+kf\frac{(k_s+2k_f)-2\phi(k_f-k_s)}{(k_s+2k_f)+\phi(k_f-k_s)}\right)(f''')+2K\left(f'f'''-ff''''-f''^2\right)\\
&+\lambda\left((1-\phi)\rho_f+\phi\rho_s\right)\left((1-\phi)+\phi\frac{(\rho\beta)_s}{(\rho\beta)_f}\right)=0.
\end{aligned} \tag{20}$$

$$\frac{1}{Pr}\frac{1}{(\rho Cp)_f}\theta''+2f\theta'\left((1-\phi)+\phi\frac{(\rho Cp)_s}{(\rho Cp)_f}\right)=0, \tag{21}$$

and the boundary conditions as follows:

$$f(0)=0,\quad f'(0)=0,\quad \theta(0)=1,\quad \text{on } y=0,\quad x\geq 0,$$
$$f'(y)\to\frac{3}{2},\quad f''(y)=0,\quad \theta\to 0,\quad \text{as } y\to\infty,\quad x\geq 0. \tag{22}$$

In the next chapter, we will discuss the methodology and Equation (3.20) and Equation (3.21), along with boundary conditions (3.22) will be applied into MATLAB algorithm to determine the behaviour of the nanofluid by investigating the velocity and temperature profiles.

4. Research Methodology

The numerical computations are being solved by using BVP4C solver in MATLAB. The tree-stage Lobatto IIIa formula by Shampine *et al.* [29] implemented in BVP4C, a finite difference algorithm. Equations (20),(21) and (22) must be transformed into first-order system before using the BVP4C solver to solve ordinary differential equations and $sol = bvp4c\ (odefun, bcfun, solinit, options)$ used as basic syntax in BVP4C solver which combines an $odefun$ system of differential equations $y' = f(x, y)$ with $bcfun$, the boundary conditions, and $solinit$, the initial solution guess. The BVP4C solver's basic syntax includes an additional integration setting, which is an argument created with the $bvpset$ function. The equations become as follows:

$$y_1=f,\quad y_2=f',\quad y_3=f'',\quad y_4=f''',\quad {y'}_4=f'''',$$
$$y_5=\theta,\quad y_6=\theta',\quad {y'}_6=\theta''. \tag{23}$$

The momentum equation is being rearranged to equation below:

$${y'}_4=\frac{1}{y_1}\left((y_2y_4-{y_3}^2)+\frac{1}{2K}\left(\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)\left(2y_1y_3-{y_2}^2+\frac{9}{4}\right)+\frac{1}{(1-\phi)^{2.5}}y_4+\left((1-\phi)+\phi\frac{(\rho\beta)_s}{(\rho\beta)_f}\right)\lambda y_5\right)\right). \tag{24}$$

The energy equation being arranged as follows:

$${y'}_6=y_7=\Pr\left(-2(1-\phi)+\phi\frac{(\rho Cp)_s}{(\rho Cp)_f}\right)y_1y_6-\gamma y_5=0. \tag{25}$$

with boundary conditions,

$$y_1(0)=0,\quad y_2(0)=0,\quad y_6(0)=-1,$$
$$y_2(\infty)\to\frac{3}{2},\quad y_3(\infty)=0,\quad y_5(\infty)=0. \tag{26}$$

5. Results and Discussion

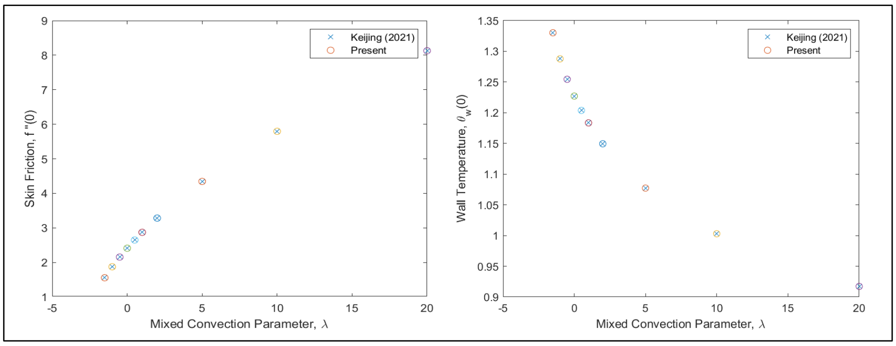

Figure 2: Skin friction, $f''(0)$ and Wall temperature, $\theta_w(0)$ for different λ when *K*=0, *Pr*=0.7 and γ=0.

Figure 2 shows the comparison result for skin friction, $f''(0)$ and wall temperature, $\theta_w(0)$ with the results from Keijing (2021) when viscous dissipation effect, $\gamma = 0$. The comparison shows that the numerical solutions found by the current study are well agreed to those obtained by Keijing (2021).

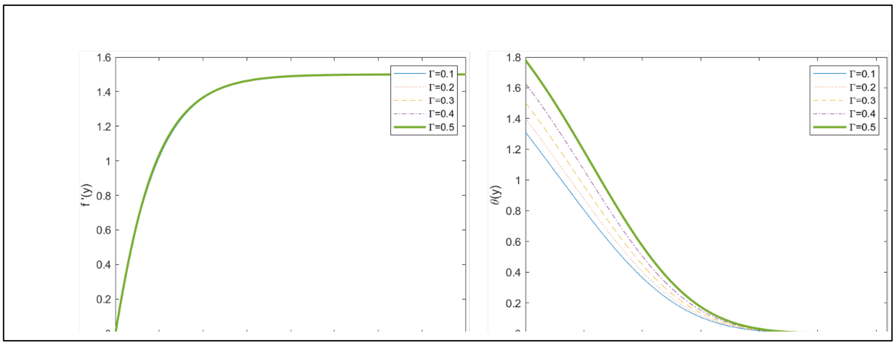

Figure 3: Velocity profile, $f'(y)$ and Temperature, $\theta(y)$ when *Pr*=1, *K*=1, λ=1, and ϕ=0.1

Figure 3 shows the existence of the effects for viscous dissipation, γ to the nanofluid velocity and temperature profiles. The results show when γ increases, the velocity profile is slightly increases and also in Figure 5.4, the temperature profile, $\theta(y)$ is increases when the values of γ increases. Generally, the increment of γ leds to the higher value of velocity and temperature profile.

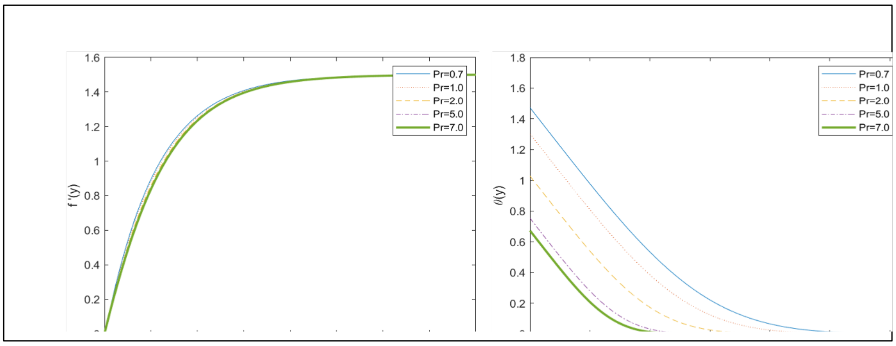

Figure 4: Velocity profile, $f'(y)$ and Temperature, $\theta(y)$ when γ=0.1, *K*=1, λ=1, and ϕ=0.1

Figure 4 illustrates the velocity and temperature profiles for different values of Prandtl number, *Pr* when viscoelastic, *K* = 1 and viscous dissipation effect, $\gamma = 0.1$. It shows when the value of *Pr* increases, both the velocity and temperature profile decrease. It is found that the increment of *Pr* value influenced its thermal diffusivity, which reduces the energy ability and thermal boundary layer thickness.

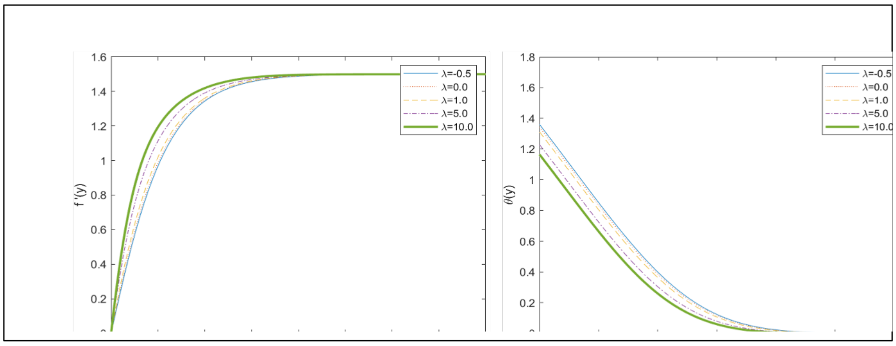

Figure 5: Velocity profile, $f'(y)$ and Temperature, $\theta(y)$ when γ=0.1, *K*=1, Pr=0.7, and ϕ=0.1

The results for Figure 5 show the velocity profile respectively for different values of mixed convection parameter, λ in presence of viscous dissipation effect, $\gamma = 0.1$. The velocity profile increases when the value of λ increases while the temperature profile decreases when the value of λ increases. Based on graph, thermal boundary layer thickness decreases when λ increases because the thermal diffusivity decreases which reduces the energy ability and the thickness of the thermal boundary layer.

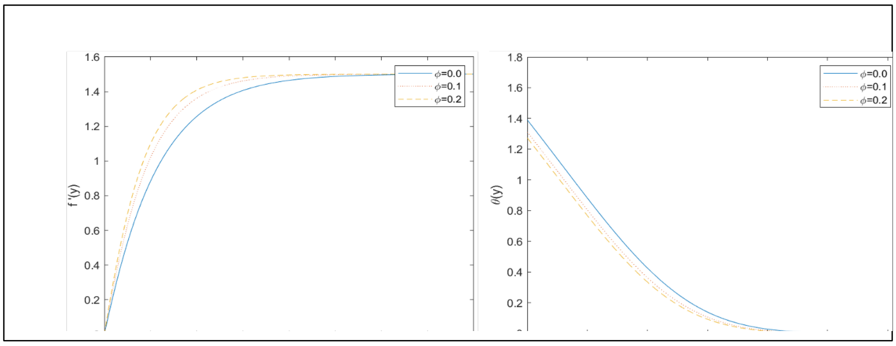

Figure 6: Velocity profile, $f'(y)$ and Temperature profile, $\theta(y)$ when γ=0.1, *K*=1, Pr=0.7, and λ=1

Figure 6 illustrates the effect of nanoparticles volume fraction, ϕ on velocity and temperature surface of a sphere in presence of viscous dissipation effect, γ. The results show when the value of ϕ increases, velocity profiles of the sphere increases but the temperature profile decreases. Therefore, when the value of ϕ increases, the thermal conductivity increases but reduce the thermal boundary layer thickness.

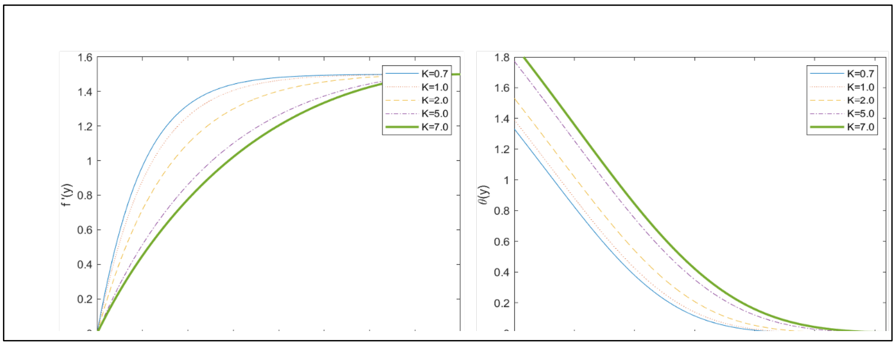

Figure 7: Velocity profile, $f'(y)$ and Temperature profile, $\theta(y)$ when γ=0.1, *K*=1, Pr=0.7, and λ=1

Figure 7 shows the existence of the effects for viscoelastic parameter, K to velocity and temperature profiles in presence of viscous dissipation effect, γ. The velocity profile decreases by increasing the value of *K* whereas the temperature profile increases when the values of *K* increase. It can be stated that the values of these profiles are lower for viscoelastic fluid compared to Newtonian fluid (K=0). As a result, the velocity boundary layer thickness for a viscoelastic fluid is greater than Newtonian fluid.

6. Conclusion

The problem of mixed convection boundary layer flow of viscoelastic nanofluid in presence of viscous dissipation effect solved by using MATLAB BV4PC solver. It was shown how the viscoelastic parameter, *K,* Prandtl number, *Pr*, viscous dissipation, γ, mixed convection parameter, λ and nanoparticles volume fraction, ϕ influenced the nanofluid flow characteristics in terms of velocity of the flow and the temperature on the surface of the sphere. The results of this study are listed as below:

- As an increase of viscous dissipation effect, γ, led to an increase of velocity of the nanofluid and the temperature distribution.
- Therefore, it can be concluded that when viscous dissipation effect increases, the velocity and thermal boundary layer thickness increases.
- The velocity of the nanofluid flow and temperature on the surface of the sphere are decreasing when Prandtl number, *Pr,* increasing.
- The velocity of nanofluid increases when mixed convection parameter, λ increases, the velocity of nanofluid increases but temperature distribution decreases.
- The velocity distribution shows an increment with the increase of nanoparticles volume fraction, ϕ. However, the temperature distribution shows a decrement.
- As the viscoelastic parameter, *K* increases, the velocity distribution decreases while the temperature distribution decreases.

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