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The Application of Runge Kutta Fourth Order Method in SIR Model for simulation of COVID-19 Cases

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Abstract
The purpose of this study is to justify the application of the Runge-Kutta fourth (RK4) order method to the basic approach of the mathematical model implemented in the susceptible-infected-recovered (SIR model), which is based on the compartmental dynamics of infectious disease. A new disease called Coronavirus disease (COVID-19) has been spread worldwide. This COVID-19 is a contagious disease, which means this virus is readily spread by transmission of a pathogen from an infected person to another person, and it is caused by the severe acute respiratory syndrome Coronavirus 2 (SARS-CoV-2) enveloped RNA virus, which emerged in Wuhan, China. Many researchers are using Euler's Method in their research papers because they want to justify the application of the method in the SIR model. Therefore, this study aims to justify whether the RK4 order method are suitable to use in the SIR model same as Euler's method. This model has been developed based on the ordinary differential equations (ODE) system in SIR model. The computational aspect of this study was done by using MATLAB software. The values of β and γ parameters will study to see the correlation between both parameters whether the changes of the value will affect the result.

Keywords: SIR Model; ODE; Runge-Kutta 4th Order; Suitable; Significant

1. Introduction

The SIR Model is one of the mathematical models used in describe the epidemic pattern of an infectious disease. The number of Susceptible, S(t), the number of Infected, I(t), and the number of Removed, R(t), were called as SIR Model. This SIR Model is the classical model if we consider only susceptible, infected and recovered people but there are a few more model with more factor that related to SIR Model which is SEIR Model (the addition of factor is Exposed, E(t)). Nowadays, most researcher and scientist using the SIR Model and SEIR Model as the effective mathematical approach to predicting the epidemic spread which is trend and infectious rate of disease around the world especially COVID-19 [15].

In this perspective of disease, the mathematical models are needed to estimate disease transmission, recovery, fatalities, and other important factors individually for distinct countries or specific regions with high to low COVID-19 reported instances. Different governments have already implemented precise and distinct measures to help prevent the disease's spread. However, critical issues such as population density, insufficient evidence for distinct symptoms, transmission method, and the lack of a good vaccine continue to make dealing with such a highly contagious and fatal disease challenging, particularly in countries with concentration of people.

Therefore, the suitable mathematical model application is SIR and SEIR model by using Runge-Kutta 4th Order method for simulation and predicting the trend of COVID-19 cases in Malaysia. The Runge-Kutta method is an improved version of the Euler method that provides a higher level of precision. Carl Runge and Martin Wilhelm Kutta, two German mathematicians, devised the Runge-Kutta method. The Euler approach, which is near to the number at each point, is analogous to the Runge-Kutta method. However, the Runge-Kutta approach is more precise than the Euler method because it uses a larger number of slope weights at each time [8]. The Runge-Kutta formula is one of the most well-known and well-understood designs in numerical analysis [6]. Therefore, in this research, we will use Runge-Kutta Fourth Order method as a numerical approach for simulation of Covid-19 Cases in Malaysia.

2. Literature Review

2.1. Susceptible-Infected-Recovered Model (SIR Model)

The SIR model is a system of three coupled nonlinear ordinary differential equations established by Ronald Ross, William Hamer, and others in the early twentieth century [11]. These models attempt to forecast factors like how a disease spreads, the total number of people sick, and the length of an epidemic, as well as estimate epidemiological metrics like the reproductive number [1]. Models like these can indicate how alternative public health actions affect the epidemic's outcome, such as the most efficient method for distributing a limited supply of vaccines to a specific population [4].

The SIR model, named after its three main sections, is one of the most basic compartmental models (susceptible, infected, and recovered). S(t) stands for Susceptible, which refers to the amount of people who are exposed. When a susceptible person comes into "infectious contact" with an infectious person, the susceptible person catches the disease and moves to the infectious segment [9]. I(t) stands for Infected, which is the number of infected people. Individuals who have been infected and are capable of infecting susceptible individuals, and R(t), which stands for Removed or Recovered, is the number of individuals who have been removed (and are immune) or who have died [9]. The equation below shows the compartment in SIR model:

$$\frac{dS}{dt} = -\beta S(t)I(t) \quad (1)$$

$$\frac{dI}{dt} = \beta S(t)I(t) - \gamma I(t) \quad (2)$$

$$\frac{dR}{dt} = \gamma I(t) \quad (3)$$

2.2. Runge-Kutta Fourth Order Method

The Runge–Kutta methods are a series of implicit and explicit iterative methods used in numerical analysis. They include the well-known Euler Method, which is used in temporal discretization for approximate solutions to ordinary differential equations. Carl Runge and Wilhelm Kutta, two German mathematicians, devised these approaches around 1900 [11]. Differential equations are employed practically everywhere in today's environment and technology.

Ordinary differentials are often used to model population growth and overpopulation, overuse of natural resources leading to extinction of animal populations and resource depletion, genocide, and disease transmission. Solving these differential equations has been a challenging and interesting task. C Runge and M W Kutta, two mathematicians, discovered various strategies to find approximate solutions to ordinary differential equations around 1900. As a result, these techniques are known as Runge-Kutta methods. The Runge–Kutta technique is a popular and successful approach for solving differential equations' initial-value problems. The Runge–Kutta method can be used to build high-order accurate numerical methods from functions without the necessity for high-order derivatives and it is the most often used method for solving ordinary differential equations. Compared to Euler's Method, Runge-Kutta 4th Order method gives more precise value, it is necessary to use this method compared to the other method in SIR Model [13].

Runge-kutta methods are used widely in many researches mainly in fluid dynamics and mechanics for better solutions of the fluids. Other real life applications of Runge-Kutta method are simulation and games. In all present games, we find the motion of objects relatively and vary the position of different objects according to it. RK4 is an iterative method to find out the approximate solution of ODE (Ordinary Differential Equation). Starting with an initial given condition we calculate forward step by step using the

RK4 algorithm. In this research, we will be use Runge-Kutta 4th Order Method and see the application in SIR Model related to real life problem and situation [12].

2.3. Mathematical model in SIR for COVID-19

Coronavirus disease 2019 (COVID-19) is an ailment caused by a novel coronavirus currently known as severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) originally known as (2019-nCoV), which was initially detected in Wuhan City, Hubei Province, China, during an outbreak of respiratory sickness cases. It was first reported to the World Health Organization (WHO) on December 31, 2019. On January 30, 2020, the World Health Organization (WHO) declared a Public Health Emergency of International Concern, and on March 11, 2020, officially declared a pandemic. Since 2021, multiple viral variants have developed and grown dominant in countries all over the world, with the most virulent being the Alpha, Beta, Gamma, Delta, and Omicron variants. More than 262 million illnesses and 5.21 million deaths had been confirmed as of December 1, 2021, making the pandemic one of the deadliest in history.
The COVID-19 cases increasing rapidly around the world and it may cause by many factors such as people do a direct contact with an infectious person, people hilling a droplets air that contain the virus, people who are not getting any vaccination and many others factor [15]. The cases is most spreading in tourist spot in every country because the place has the saturated amount of people and most of them didn't know that they are infected because there is no symptoms shows.

COVID-19 cases have been reported in more than 188 countries worldwide. Some countries have been experiencing limited growth and spread of COVID-19 cases, while others are suffering widespread community transmission and fast, nearly exponential increase in the number of infections [14]. The total cases caused by CoronaVirus-2019 (COVID-19) around the world from the first cases reported on 12 December 2019 until 1 December 2021 is 262,963,333 with new cases reported on 1 December 2021 is 694,452. The total death rate around the world is 5,232,265 with new death cases on 1 December 2021 is 8993. However, there is a total new recovered case which is 235,696,816 with the new recovered cases is 648,254. In this research, we will look at the Malaysia data of cases in which the total cases is 2,632,782 with total death and recovery is 30,425 and 2,537,204 respectively.

3. Methodology

3.1. Research Data

This chapter discusses the mathematical methods development of this research methodology which include analysis of the Runge-Kutta fourth order method and its SIR model. This chapter also discusses the SIR epidemiological model and its basic reproduction number, R_0 and this R_0 will be calculated. The ODE's of SIR Model, equation of Euler's method, equation of Runge-Kutta fourth order method and its SIR model will be covered in this chapter. We will discover the appropriate assumption value and justifying the effectiveness of Runge-Kutta fourth order method whether it is suitable method use in SIR Model. The data will be calculated in Excel based on formula discuss in this chapter.

3.2. SIR Model

The SIR model is given by a system of three ordinary differential equations (ODEs) which can be implemented to gain better understanding of how the COVID-19 virus transmission spreads within communities of variable populations in time including the possibility of surges in the susceptible populations.

Major assumptions of the SIR model are:

1. Total populations remain constant.
2. The rate of infectives are propotional to susceptible.
3. The rate of infectious who recovered or die are constant.

Individuals who have recovered from the virus are assumed to be immune.

$$\frac{dS}{dt} = -\beta S(t)I(t) \tag{4}$$

$$\frac{dI}{dt} = \beta S(t)I(t) - \gamma I(t) \tag{5}$$

$$\frac{dR}{dt} = \gamma I(t) \tag{6}$$

3.3. Runge-Kutta 4th order method

Consider the following initial value problem of ODE

$$\frac{dy}{dt} = f(t, y) \tag{7}$$

$$y(t_0) = y_0 \tag{8}$$

where $y(t)$ is the unknown function (scalar or vector) which I would like toapproximate.

The iterative formula of RK4 method for solving ODE is as follows:

$$y_{n+1} = y_n + \frac{\Delta t}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{9}$$

$$k_1 = f(t_n, y_n) \tag{10}$$

$$k_2 = f\left(t_n + \frac{\Delta t}{2}, y_n + \frac{k_1 \Delta t}{2}\right) \tag{11}$$

$$k_3 = f\left(t_n + \frac{\Delta t}{2}, y_n + \frac{k_2 \Delta t}{2}\right) \tag{12}$$

$$k_4 = f(t_n + \Delta t, y_n + k_3 \Delta t) \tag{13}$$

$$t_n + 1 = t_n + \Delta t, \qquad n = 0,1,2,3,\cdots \tag{14}$$

Runge-Kutta Fourth Order Method in SIR Model

According to the generative formula in Runge-Kutta 4th Order Method, for S(t), I(t) and R(t) of SIR Model can be written as:

$$S_{n+1} = S_n + \frac{\Delta t}{6}(k_1^S + 2k_2^S + 2k_3^S + k_4^S) \tag{15}$$

$$k_1^S = f(t_n, S_n, I_n) = -\beta S_n I_n \tag{16}$$

$$k_2^S = f(t_n + \frac{\Delta t}{2}, S_n + \frac{k_1^S \Delta t}{2}, I_n + \frac{k_1^I \Delta t}{2}) = -\beta(S_n + \frac{k_1^S \Delta t}{2})(I_n + \frac{k_1^I \Delta t}{2}) \quad (17)$$

$$k_3^S = f(t_n + \frac{\Delta t}{2}, S_n + \frac{k_2^S \Delta t}{2}, I_n + \frac{k_2^I \Delta t}{2}) = -\beta(S_n + \frac{k_2^S \Delta t}{2})(I_n + \frac{k_2^I \Delta t}{2}) \quad (18)$$

$$k_4^S = f(t_n + \Delta t, S_n + k_3^S \Delta t, I_n + k_3^I \Delta t) = -\beta(S_n + k_3^S \Delta t)(I_n + k_3^S \Delta t) \quad (19)$$

$$I_{n+1} = I_n + \frac{\Delta t}{6}(k_1^I + 2k_2^I + 2k_3^I + k_4^I) \quad (20)$$

$$k_1^I = f(t_n, S_n, I_n) = \beta S_n I_n - \gamma I_n \quad (21)$$

$$k_2^I = f\left(t_n + \frac{\Delta t}{2}, S_n + \frac{k_1^S \Delta t}{2}, I_n + \frac{k_1^I \Delta t}{2}\right) \quad (22)$$
$$= \beta\left(S_n + \frac{k_1^S \Delta t}{2}\right)\left(I_n + \frac{k_1^I \Delta t}{2}\right) - \gamma\left(I_n + \frac{k_1^I \Delta t}{2}\right)$$

$$k_3^I = f\left(t_n + \frac{\Delta t}{2}, S_n + \frac{k_2^S \Delta t}{2}, I_n + \frac{k_2^I \Delta t}{2}\right) \quad (23)$$
$$= \beta\left(S_n + \frac{k_2^S \Delta t}{2}\right)\left(I_n + \frac{k_2^I \Delta t}{2}\right) - \gamma\left(I_n + \frac{k_2^I \Delta t}{2}\right)$$

$$k_4^I = f(t_n + \Delta t, S_n + k_3^S \Delta t, I_n + k_3^I \Delta t) \quad (24)$$
$$= \beta(S_n + k_3^S \Delta t)(I_n + k_3^S \Delta t) - \gamma(I_n + k_3^I \Delta t)$$

$$R_{n+1} = R_n + \frac{\Delta t}{6}(k_1^R + 2k_2^R + 2k_3^R + k_4^R) \quad (25)$$

$$k_1^R = f(t_n, I_n) = \gamma I_n \quad (26)$$

$$k_2^R = f(t_n + \frac{\Delta t}{2}, I_n + \frac{k_1^I \Delta t}{2}) = \gamma(I_n + \frac{k_1^I \Delta t}{2}) \quad (27)$$

$$k_3^R = f(t_n + \frac{\Delta t}{2}, I_n + \frac{k_2^I \Delta t}{2}) = \gamma(I_n + \frac{k_2^I \Delta t}{2}) \quad (28)$$

$$k_4^R = f(t_n + \Delta t, I_n + k_3^I \Delta t) = \gamma(I_n + k_3^I \Delta t) \quad (29)$$

Note that since the population $N = S(t) + I(t) + R(t)$ is constant, there will have $\frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt} = 0$. Therefore, only two of the three ODEs are independent and sufficient to solve the ODEs. Here, only iterative formulas for $S(t)$ and $I(t)$ are used and $R(t)$ is calculated by $R(t) = N - S(t) - I(t)$

4. Results and discussion

From Figure 4.3 and Figure 4.4, the number of individuals who were infected are more than 200000 which had peaked at day 20. Then, the curve started to decrease steadily. This is most likely due to the government's protective steps, which require everyone to isolate themselves at home and practise *social distancing. Based on Figure 4.5 and Figure 4.6, the infected individuals who recovered from COVID-19 increases. The number of recovered individuals is around 400000. This could be due to a greater number of infected individuals undergo isolation and various treatment from quarantine centre. Figure 4.7 and Figure 4.8 depicts the graph of all three classes of SIR model with initial parameters of β= 1.63{10}^{-7},γ = 0.125, indicating that the vaccine has not yet been used in this simulation. The basic reproduction number, R_0 for the simulation model in Figure 4.7 and Figure 4.8 are below 1. Hence, we can conclude that the output calculation is perfect and fitted very well for Euler's method with SIR model.*

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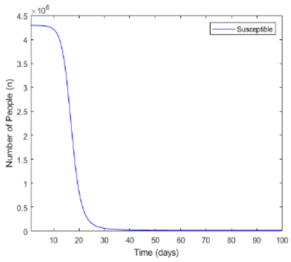

Figure 4. 1 Simulation of Susceptible (S) in SIR calculation of population

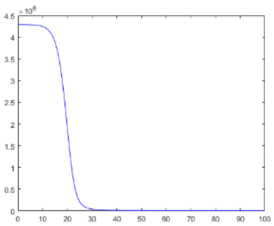

Figure 4. 2 Simulation of Susceptible (S) in Euler's calculation of population

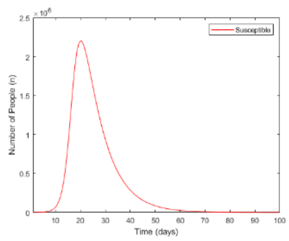

Figure 4. 3 Simulation of Infected (I) in SIR calculation of population

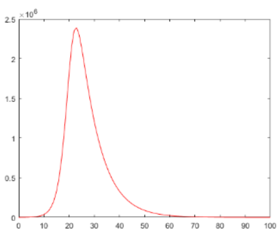

Figure 4. 4 Simulation of Infected (I) in Euler's calculation of population

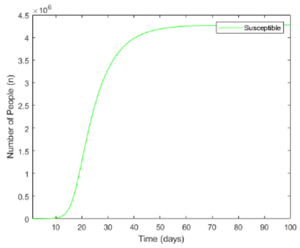

Figure 4. 5 Simulation of Recovered (I) in SIR calculation of population

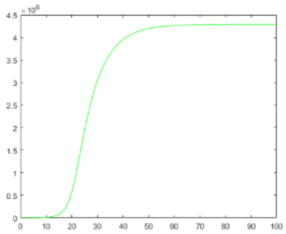

Figure 4. 6 Simulation of Recovered (I) in Euler's calculation of population

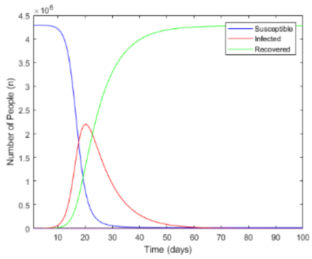

Figure 4. 7 Simulation of the Susceptible (S), Infected (I), Recovered (R) in SIR

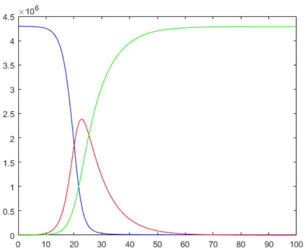

Figure 4. 8 Simulation of the Susceptible (S), Infected (I), Recovered (R) in Euler's

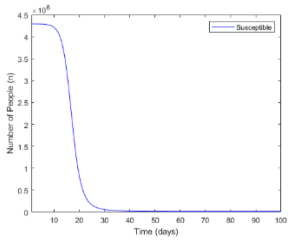

Figure 4. 9 Simulation of Recovered (I) in Runge Kutta fourth order calculation of

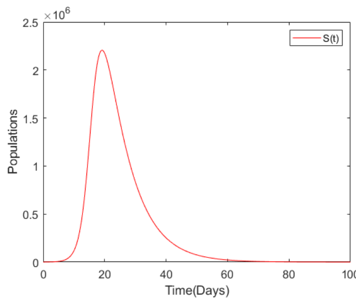

Figure 4. 10 Simulation of Recovered (I) in Runge Kutta fourth order calculation of

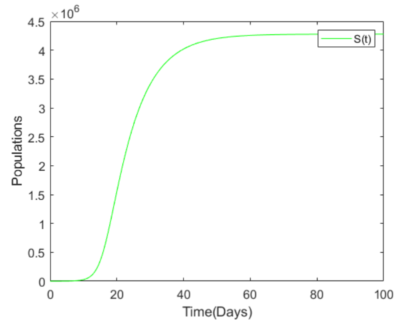

Figure 4. 1 Simulation of Recovered (I) in Runge Kutta fourth order calculation of population

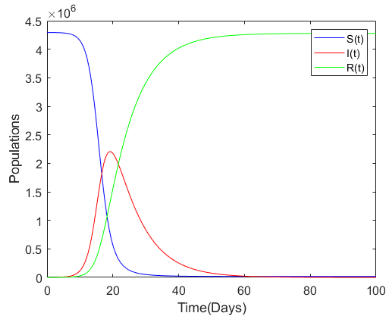

Figure 4. 12 Simulation of the Susceptible (S), Infected (I), Recovered (R) in Runge

Conclusion

Based on the result obtain in the previous subtopic, we can say that the RK4 order method output is also same with the output of Euler's method and SIR method. Therefore, we conclude here that the RK4 order method can be used in SIR model. This study also provides calculations for simple reproduction numbers, Ro in chapter 3, and stability analysis for the SIR model. Ro is the basic reproduction number. The Ro for the SIR model was calculated based on beta and gamma value but in the study is taken in the article. Based on the calculation on Chapter 4, we conclude that the infection has a low chance of spreading if $R0 < 1$. As a result, certain preventive steps like wearing a surgical face mask and practice social distancing must be considered to prevent the disease from spreading and eventually eliminate the infection from the population. Next, in Chapter 4, the numerical simulations are carried out in this study. The impact of the same the values of the parameters in this SIR model for all method use is observed and investigated. The built-in MATLAB software stated in the appendices in ODE45 was used to simulate graphs with a few different values for each of the parameters. According to the simulation results, the infection rate, β, is one parameter that must be considered in preventing COVID-19 transmission. The rate of infection can be decreased by educating people about the transmission. Aside from that, the recovery rate, γ, should also be increased. Following government protective measures such as wearing a face mask, avoiding crowded areas, and practicing social distancing and minimizing close contact with infected people can improve recovery rates. The study's findings concluded that the RK4 order method in SIR model can be used as a reference model other than Euler's method for COVID-19 spread. Analysis of the model provides an overview of global stability in the spread of COVID-19.

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