



A User Interface for a 3D Keller Box Method

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Abstract

The boundary layer flow problem is now frequently employed in real-world applications to solve a variety of problems. Most boundary layer flow problems will result in nonlinear equations that are difficult to solve. As a result, various numerical ways to solving nonlinear equations have been developed, including the Keller Box Method, Runge-Kutta-Fehlberg Method (RKF45), and the bvp4c method. Keller Box Method was chosen to solve nonlinear equations in this study. To begin, the Keller Box Method will be manually implemented by discretizing with the Finite Difference Method (FDM) and then linearizing with Newton's Method. The user interface will then be implemented using Microsoft Excel, which will be used for further research. Therefore, the goal of this research is to create a user interface for solving nonlinear equations in three dimensions using the Keller Box Method. Finally, the user interface template will be validated using various multidimensional nonlinear equations. It is found that the template created is in good agreement with the manual calculation of similar problems in the published papers.

Keywords: Keller Box Method; Boundary Layer Flow; Three-dimensional nonlinear equation.

1. Introduction

In Fluid Mechanics, Britannica [1] stated that flowing gas or liquid of a thin layer in contact with a surface that is subjected to shearing forces is defined as boundary layer flow. Nazir et al. [2] investigate the concentration and thermal conductivity of fluid passing through a hot surface by modelling the boundary value problem in the Cattaneo-Christov fluxes and Widodo et al [3] researched on solving boundary layer problem of unsteady 2-dimensional solid sphere using Euler implicit methods and Finite Difference method is used to discretize the equation. Then, Makanda et al [4] declared that Keller Box method is a common method that had been used in solving the boundary layer problem. Thus, the Keller Box Method is an appropriate approach for solving the model of nanofluids (NFs) flow with the effects of permeability material, viscous dissipative flow, Cattaneo-Christov heat flux, and heat radiate flowing involving boundary layer flow problem Jamshed et al [5]. Furthermore, the copper and alumina in water-based micropolar nanofluid of natural convection boundary layer flow over solid sphere can be studied using the Keller box approach Mohammed et al [6] and the boundary layer problem on magnetohydrodynamic (MHD) nanofluid can also be solved using the Keller Box Method because it is simple to implement Zeeshan et al [7]. Then, the boundary layer flow problem will generate nonlinear equations that consider all forces at the boundary layer flow. In 2019, Priyam et al [8] built Graphical User Interface for custom Basic Local Alignment Search Tool (BLAST) that designed to increase the productivity of biologist researchers to perform and interpret BLAST searches on custom data sets and bioinformaticians will set up laboratory of community databases. Therefore, the goal of this research is to solve numerically three-dimensional nonlinear equations using the Keller Box Method and to create a user-friendly interface using Microsoft Excel for future research.

2. Literature Review

2.1. Keller Box Method

This research will concentrate on creating a user interface for solving three-dimensional nonlinear equations using the Keller Box Method. The Keller Box Method consists of discretization using the Finite Difference Method and linearization using Newton's Method. The user interface for the Keller Box Method will be created in Microsoft Excel so that it can be easily implemented in future research. The results of the manual calculation and the user interface will be compared to ensure that they are in good agreement.

2.1.1. Finite Difference Method (FDM)

Before using the finite difference method, a three-dimensional nonlinear equation with a higher order partial differential equation will be reduced to a first order partial differential equation. As shown below, the nonlinear equation will be converted using the centred finite difference scheme. Since this study focus on three-dimensional nonlinear equations thus, each variable will be expanded in i , j , and k .

$$\left(\right)_{\frac{j}{2}} = \frac{\left(\right)_j + \left(\right)_{j-1}}{2}, \quad \left(\right)'_{\frac{j}{2}} = \frac{\left(\right)_j - \left(\right)_{j-1}}{b_i} \quad (2.1)$$

2.1.2. Newton's Method

After discretization process, the nonlinear equation will be linearize using Newton's Method. Only term $\left(\right)_i^{j,k}$ and $\left(\right)_{i-1}^{j,k}$ will be linearize by replacing it as below.

$$\left(\right)_i^{j,k} = \left(\right)_i^{j,k} + \delta \left(\right)_i, \quad \left(\right)_{i-1}^{j,k} = \left(\right)_{i-1}^{j,k} + \delta \left(\right)_{i-1} \quad (2.2)$$

2.2. User Interface using Microsoft Excel

In developing the Keller Box Method user interface, functions in Microsoft Excel such as 'IF', 'COUNTIF', 'OR', and 'AND' are used. Since Microsoft Excel is user friendly, it was chosen as a medium for developing user interfaces. Most people are familiar with Microsoft Excel. As a result, it is simple to implement for future research.

3. Results and discussion

3.1. Reduce Higher Order Partial Differential Equations (PDE) to the First Order PDE

The nonlinear equations from Widodo et al [3] study's will be used in solving three-dimensional nonlinear equations using Keller Box Method as follows.

Momentum Equations:

$$(1 + K)f'''' + Kh' + \frac{\eta}{2}f'' + Mt(1 - f') + \frac{2}{3}ast + \frac{3}{2}t(1 + ff' - (f')^2) = t \left(\frac{\partial f'}{\partial t} \right) \quad (3.1a)$$

Angular Momentum Equation:

$$\left(1 + \frac{K}{2} \right) h'' + \frac{\eta}{2} h' + \frac{h}{2} + \frac{3}{2} t (f h' - h f') = t \frac{\partial h}{\partial t} + K t (2h + f''). \quad (3.1b)$$

Energy Equation:

$$s'' + Pr \frac{\eta}{2} s' + Prt f \frac{3}{2} \cos x s' = Prt \left(\frac{\partial s}{\partial t} + \frac{3}{2} \sin x \left(\frac{\partial s}{\partial x} f' - \frac{\partial f}{\partial x} s' - \frac{f}{r} \frac{\partial r}{\partial x} s' \right) \right). \quad (3.1c)$$

with the initial and boundary condition as below

$$\begin{aligned} t = 0: f = f' = h = s = 0, \text{ for any } x, \eta, \\ t > 0: f = f' = 0, h = -\eta f'', s = 1 \text{ at } \eta = 0, \\ f' = 1, h = s = 0 \text{ as } \eta \rightarrow \infty. \end{aligned}$$

where,

$$\frac{\partial f}{\partial \eta} = f', \frac{\partial s}{\partial \eta} = s', \frac{\partial h}{\partial \eta} = h'. \quad (3.2)$$

Equation (3.1a), (3.1b), (3.1c) and (3.2) will be reduce to the first order PDE by introducing new variables. Before that, the equation needs to be set equal to zero in the right-hand side (RHS).

Assume

$$\begin{aligned} f' = u & & h' = \theta & & s' = \phi \\ f'' = u' = v & & h'' = \theta' & & s'' = \phi' \\ f''' = u'' = v' & & & & \end{aligned} \quad (3.3)$$

Momentum Equations:

$$(1 + K)v' + K\theta + \frac{\eta}{2}v + Mt - Mtu + \frac{2}{3}ast + \frac{3}{2}t + \frac{3}{2}tfu - \frac{3}{2}t(u)^2 - t \left(\frac{\partial u}{\partial t} \right) = 0. \quad (3.4a)$$

Angular Momentum Equation:

$$\left(1 + \frac{K}{2} \right) \theta' + \frac{\eta}{2} \theta + \frac{h}{2} + \frac{3}{2} t f \theta - \frac{3}{2} t h u - t \frac{\partial h}{\partial t} - 2Kth - Ktv = 0. \quad (3.4b)$$

Energy Equation:

$$\phi' + Pr \frac{\eta}{2} \phi + Prt f \frac{3}{2} \cos(x) \phi - Prt \frac{\partial s}{\partial t} - \frac{3}{2} Prtu \sin(x) \frac{\partial s}{\partial x} + \frac{3}{2} Prt \sin(x) \phi \frac{\partial f}{\partial x} + \frac{3}{2} Prt \sin(x) \frac{f}{r} \phi \frac{\partial r}{\partial x} = 0. \quad (3.4c)$$

with the initial and boundary condition as below

$$\begin{aligned} t = 0: f = u = h = s = 0, \text{ for any } x, \eta, \\ t > 0: f = u = 0, h = -\eta v, s = 1 \text{ at } \eta = 0, \\ u = 1, h = s = 0 \text{ as } \eta \rightarrow \infty. \end{aligned}$$

where,

$$\frac{\partial f}{\partial \eta} = f', \frac{\partial s}{\partial \eta} = s', \frac{\partial h}{\partial \eta} = h'. \quad (3.5)$$

3.2. Finite Difference Method (FDM)

Discretization of the reduced nonlinear equation will be expanded one by one from i, j , and k . The assumption below is based on the Euler Expansion.

$$\begin{aligned}
 f &= \frac{f_i + f_{i-1}}{2}, h = \frac{h_i + h_{i-1}}{2}, s = \frac{s_i + s_{i-1}}{2}, u = \frac{u_i + u_{i-1}}{2}, \\
 v &= \frac{v_i + v_{i-1}}{2}, v' = \frac{v_i - v_{i-1}}{b_i}, \theta = \frac{\theta_i + \theta_{i-1}}{2}, \theta' = \frac{\theta_i - \theta_{i-1}}{b_i}, \\
 \phi &= \frac{\phi_i + \phi_{i-1}}{2}, \phi' = \frac{\phi_i - \phi_{i-1}}{b_i}, \frac{\partial s}{\partial t} = \frac{s_j - s_{j-1}}{m_j}, \frac{\partial h}{\partial t} = \frac{h_j - h_{j-1}}{m_j}, \\
 \frac{\partial u}{\partial t} &= \frac{u_j - u_{j-1}}{m_j}, \frac{\partial f}{\partial x} = \frac{f_k - f_{k-1}}{q_k}, \frac{\partial s}{\partial x} = \frac{s_k - s_{k-1}}{q_k}.
 \end{aligned} \tag{3.6}$$

Here is the result for discretization of the momentum, angular momentum, and energy equation.

Momentum Equation,

$$\begin{aligned}
 &(1 + K) \left(\frac{v_i^{j,k} + v_i^{j,k-1}}{4b_i} + \frac{v_i^{j-1,k} + v_i^{j-1,k-1}}{4b_i} - \frac{v_{i-1}^{j,k} + v_{i-1}^{j,k-1}}{4b_i} - \frac{v_{i-1}^{j-1,k} + v_{i-1}^{j-1,k-1}}{4b_i} \right) \\
 &+ K \left(\frac{\theta_i^{j,k} + \theta_i^{j,k-1}}{8} + \frac{\theta_i^{j-1,k} + \theta_i^{j-1,k-1}}{8} + \frac{\theta_{i-1}^{j,k} + \theta_{i-1}^{j,k-1}}{8} + \frac{\theta_{i-1}^{j-1,k} + \theta_{i-1}^{j-1,k-1}}{8} \right) \\
 &+ \frac{\eta}{2} \left(\frac{v_i^{j,k} + v_i^{j,k-1}}{8} + \frac{v_i^{j-1,k} + v_i^{j-1,k-1}}{8} + \frac{v_{i-1}^{j,k} + v_{i-1}^{j,k-1}}{8} + \frac{v_{i-1}^{j-1,k} + v_{i-1}^{j-1,k-1}}{8} \right) + Mt \\
 &- Mt \left(\frac{u_i^{j,k} + u_i^{j,k-1}}{8} + \frac{u_i^{j-1,k} + u_i^{j-1,k-1}}{8} + \frac{u_{i-1}^{j,k} + u_{i-1}^{j,k-1}}{8} + \frac{u_{i-1}^{j-1,k} + u_{i-1}^{j-1,k-1}}{8} \right) \\
 &+ \frac{2}{3} \alpha t \left(\frac{s_i^{j,k} + s_i^{j,k-1}}{8} + \frac{s_i^{j-1,k} + s_i^{j-1,k-1}}{8} + \frac{s_{i-1}^{j,k} + s_{i-1}^{j,k-1}}{8} + \frac{s_{i-1}^{j-1,k} + s_{i-1}^{j-1,k-1}}{8} \right) + \frac{3}{2} t \\
 &+ \frac{3}{2} t \left(\frac{f_i^{j,k} + f_i^{j,k-1}}{8} + \frac{f_i^{j-1,k} + f_i^{j-1,k-1}}{8} + \frac{f_{i-1}^{j,k} + f_{i-1}^{j,k-1}}{8} + \frac{f_{i-1}^{j-1,k} + f_{i-1}^{j-1,k-1}}{8} \right) \\
 &\left(\frac{u_i^{j,k} + u_i^{j,k-1}}{8} + \frac{u_i^{j-1,k} + u_i^{j-1,k-1}}{8} + \frac{u_{i-1}^{j,k} + u_{i-1}^{j,k-1}}{8} + \frac{u_{i-1}^{j-1,k} + u_{i-1}^{j-1,k-1}}{8} \right) \\
 &- \frac{3}{2} t \left(\frac{u_i^{j,k} + u_i^{j,k-1}}{8} + \frac{u_i^{j-1,k} + u_i^{j-1,k-1}}{8} + \frac{u_{i-1}^{j,k} + u_{i-1}^{j,k-1}}{8} + \frac{u_{i-1}^{j-1,k} + u_{i-1}^{j-1,k-1}}{8} \right) \\
 &\left(\frac{u_i^{j,k} + u_i^{j,k-1}}{8} + \frac{u_i^{j-1,k} + u_i^{j-1,k-1}}{8} + \frac{u_{i-1}^{j,k} + u_{i-1}^{j,k-1}}{8} + \frac{u_{i-1}^{j-1,k} + u_{i-1}^{j-1,k-1}}{8} \right) \\
 &- t \left(\frac{u_i^{j,k} + u_i^{j,k-1}}{4m_j} - \frac{u_i^{j-1,k} + u_i^{j-1,k-1}}{4m_j} + \frac{u_{i-1}^{j,k} + u_{i-1}^{j,k-1}}{4m_j} - \frac{u_{i-1}^{j-1,k} + u_{i-1}^{j-1,k-1}}{4m_j} \right) = 0.
 \end{aligned} \tag{3.7a}$$

Angular Momentum Equation

$$\begin{aligned}
 & \left(1 + \frac{K}{2}\right) \left(\frac{\theta_i^{j,k} + \theta_i^{j,k-1}}{4b_i} + \frac{\theta_i^{j-1,k} + \theta_i^{j-1,k-1}}{4b_i} - \frac{\theta_{i-1}^{j,k} + \theta_{i-1}^{j,k-1}}{4b_i} - \frac{\theta_{i-1}^{j-1,k} + \theta_{i-1}^{j-1,k-1}}{4b_i} \right) \\
 & + \frac{\eta}{2} \left(\frac{\theta_i^{j,k} + \theta_i^{j,k-1}}{8} + \frac{\theta_i^{j-1,k} + \theta_i^{j-1,k-1}}{8} + \frac{\theta_{i-1}^{j,k} + \theta_{i-1}^{j,k-1}}{8} + \frac{\theta_{i-1}^{j-1,k} + \theta_{i-1}^{j-1,k-1}}{8} \right) \\
 & + \frac{1}{2} \left(\frac{h_i^{j,k} + h_i^{j,k-1}}{8} + \frac{h_i^{j-1,k} + h_i^{j-1,k-1}}{8} + \frac{h_{i-1}^{j,k} + h_{i-1}^{j,k-1}}{8} + \frac{h_{i-1}^{j-1,k} + h_{i-1}^{j-1,k-1}}{8} \right) \\
 & + \frac{3}{2} t \left(\frac{f_i^{j,k} + f_i^{j,k-1}}{8} + \frac{f_i^{j-1,k} + f_i^{j-1,k-1}}{8} + \frac{f_{i-1}^{j,k} + f_{i-1}^{j,k-1}}{8} + \frac{f_{i-1}^{j-1,k} + f_{i-1}^{j-1,k-1}}{8} \right) \\
 & \left(\frac{\theta_i^{j,k} + \theta_i^{j,k-1}}{8} + \frac{\theta_i^{j-1,k} + \theta_i^{j-1,k-1}}{8} + \frac{\theta_{i-1}^{j,k} + \theta_{i-1}^{j,k-1}}{8} + \frac{\theta_{i-1}^{j-1,k} + \theta_{i-1}^{j-1,k-1}}{8} \right) \\
 & - \frac{3}{2} t \left(\frac{h_i^{j,k} + h_i^{j,k-1}}{8} + \frac{h_i^{j-1,k} + h_i^{j-1,k-1}}{8} + \frac{h_{i-1}^{j,k} + h_{i-1}^{j,k-1}}{8} + \frac{h_{i-1}^{j-1,k} + h_{i-1}^{j-1,k-1}}{8} \right) \\
 & \left(\frac{u_i^{j,k} + u_i^{j,k-1}}{8} + \frac{u_i^{j-1,k} + u_i^{j-1,k-1}}{8} + \frac{u_{i-1}^{j,k} + u_{i-1}^{j,k-1}}{8} + \frac{u_{i-1}^{j-1,k} + u_{i-1}^{j-1,k-1}}{8} \right) \\
 & - t \left(\frac{h_i^{j,k} + h_i^{j,k-1}}{4m_j} - \frac{h_i^{j-1,k} + h_i^{j-1,k-1}}{4m_j} + \frac{h_{i-1}^{j,k} + h_{i-1}^{j,k-1}}{4m_j} - \frac{h_{i-1}^{j-1,k} + h_{i-1}^{j-1,k-1}}{4m_j} \right) \\
 & - 2Kt \left(\frac{h_i^{j,k} + h_i^{j,k-1}}{8} + \frac{h_i^{j-1,k} + h_i^{j-1,k-1}}{8} + \frac{h_{i-1}^{j,k} + h_{i-1}^{j,k-1}}{8} + \frac{h_{i-1}^{j-1,k} + h_{i-1}^{j-1,k-1}}{8} \right) \\
 & - Kt \left(\frac{v_i^{j,k} + v_i^{j,k-1}}{8} + \frac{v_i^{j-1,k} + v_i^{j-1,k-1}}{8} + \frac{v_{i-1}^{j,k} + v_{i-1}^{j,k-1}}{8} + \frac{v_{i-1}^{j-1,k} + v_{i-1}^{j-1,k-1}}{8} \right) = 0.
 \end{aligned}
 \tag{3.7b}$$

Energy Equation

$$\begin{aligned}
 & \left(\frac{\phi_i^{j,k} + \phi_i^{j,k-1}}{4b_i} + \frac{\phi_i^{j-1,k} + \phi_i^{j-1,k-1}}{4b_i} - \frac{\phi_{i-1}^{j,k} + \phi_{i-1}^{j,k-1}}{4b_i} - \frac{\phi_{i-1}^{j-1,k} + \phi_{i-1}^{j-1,k-1}}{4b_i} \right) \\
 & + Pr \frac{\eta}{2} \left(\frac{\phi_i^{j,k} + \phi_i^{j,k-1}}{8} + \frac{\phi_i^{j-1,k} + \phi_i^{j-1,k-1}}{8} + \frac{\phi_{i-1}^{j,k} + \phi_{i-1}^{j,k-1}}{8} + \frac{\phi_{i-1}^{j-1,k} + \phi_{i-1}^{j-1,k-1}}{8} \right) \\
 & + \frac{3}{2} Prt \cos(x) \left(\frac{f_i^{j,k} + f_i^{j,k-1}}{8} + \frac{f_i^{j-1,k} + f_i^{j-1,k-1}}{8} + \frac{f_{i-1}^{j,k} + f_{i-1}^{j,k-1}}{8} + \frac{f_{i-1}^{j-1,k} + f_{i-1}^{j-1,k-1}}{8} \right) \\
 & - Prt \left(\frac{s_i^{j,k} + s_i^{j,k-1}}{4m_j} - \frac{s_i^{j-1,k} + s_i^{j-1,k-1}}{4m_j} + \frac{s_{i-1}^{j,k} + s_{i-1}^{j,k-1}}{4m_j} - \frac{s_{i-1}^{j-1,k} + s_{i-1}^{j-1,k-1}}{4m_j} \right) \\
 & - \frac{3}{2} Prt \sin(x) \left(\frac{u_i^{j,k} + u_i^{j,k-1}}{8} + \frac{u_i^{j-1,k} + u_i^{j-1,k-1}}{8} + \frac{u_{i-1}^{j,k} + u_{i-1}^{j,k-1}}{8} + \frac{u_{i-1}^{j-1,k} + u_{i-1}^{j-1,k-1}}{8} \right) \\
 & \left(\frac{s_i^{j,k} - s_i^{j,k-1}}{4q_k} + \frac{s_i^{j-1,k} - s_i^{j-1,k-1}}{4q_k} + \frac{s_{i-1}^{j,k} - s_{i-1}^{j,k-1}}{4q_k} + \frac{s_{i-1}^{j-1,k} - s_{i-1}^{j-1,k-1}}{4q_k} \right) \\
 & \frac{3}{2} Prt \sin(x) \left(\frac{\phi_i^{j,k} + \phi_i^{j,k-1}}{8} + \frac{\phi_i^{j-1,k} + \phi_i^{j-1,k-1}}{8} + \frac{\phi_{i-1}^{j,k} + \phi_{i-1}^{j,k-1}}{8} + \frac{\phi_{i-1}^{j-1,k} + \phi_{i-1}^{j-1,k-1}}{8} \right) \\
 & \left(\frac{f_i^{j,k} - f_i^{j,k-1}}{4q_k} + \frac{f_i^{j-1,k} - f_i^{j-1,k-1}}{4q_k} + \frac{f_{i-1}^{j,k} - f_{i-1}^{j,k-1}}{4q_k} + \frac{f_{i-1}^{j-1,k} - f_{i-1}^{j-1,k-1}}{4q_k} \right) \\
 & \frac{3}{2} Prt \sin(x) \frac{1}{r} \frac{\partial r}{\partial x} \left(\frac{f_i^{j,k} + f_i^{j,k-1}}{8} + \frac{f_i^{j-1,k} + f_i^{j-1,k-1}}{8} + \frac{f_{i-1}^{j,k} + f_{i-1}^{j,k-1}}{8} + \frac{f_{i-1}^{j-1,k} + f_{i-1}^{j-1,k-1}}{8} \right)
 \end{aligned}$$

$$\left(\frac{\phi_i^{j,k} + \phi_i^{j,k-1}}{8} + \frac{\phi_i^{j-1,k} + \phi_i^{j-1,k-1}}{8} + \frac{\phi_{i-1}^{j,k} + \phi_{i-1}^{j,k-1}}{8} + \frac{\phi_{i-1}^{j-1,k} + \phi_{i-1}^{j-1,k-1}}{8} \right). \tag{3.7c}$$

3.3. Newton's Method

The full Manual calculation for this study is obtained by linearization of Newton's Method as follows.

For Momentum Equation

$$\begin{aligned} & (1 + K) \left(\frac{\delta v_i - \delta v_{i-1}}{4b_i} \right) + K \left(\frac{\delta \theta_i + \delta \theta_{i-1}}{8} \right) + \frac{\eta}{2} \left(\frac{\delta v_i + \delta v_{i-1}}{8} \right) + Mt \\ & - Mt \left(\frac{\delta u_i + \delta u_{i-1}}{8} \right) + \frac{2}{3} \alpha t \left(\frac{\delta s_i + \delta s_{i-1}}{8} \right) + \frac{3}{2} t \\ & + \frac{3}{2} t \left(\frac{(f_i^{j,k} + f_{i-1}^{j,k})(\delta u_i + \delta u_{i-1})}{8} \right) \left(\frac{(u_i^{j,k} + u_{i-1}^{j,k})(\delta f_i + \delta f_{i-1})}{8} \right) \\ & - \frac{3}{2} t \left(\frac{(u_i^{j,k} + u_{i-1}^{j,k})(\delta f_i + \delta f_{i-1})}{8} \right) \left(\frac{(u_i^{j,k} + u_{i-1}^{j,k})(\delta f_i + \delta f_{i-1})}{8} \right) \\ -t \left(\frac{\delta u_i + \delta u_{i-1}}{4m_j} \right) = & -(1 + K) \left(\frac{v_i^{j,k} + v_i^{j,k-1}}{4b_i} + \frac{v_i^{j-1,k} + v_i^{j-1,k-1}}{4b_i} - \frac{v_{i-1}^{j,k} + v_{i-1}^{j,k-1}}{4b_i} - \frac{v_{i-1}^{j-1,k} + v_{i-1}^{j-1,k-1}}{4b_i} \right) \\ & - K \left(\frac{\theta_i^{j,k} + \theta_i^{j,k-1}}{8} + \frac{\theta_i^{j-1,k} + \theta_i^{j-1,k-1}}{8} + \frac{\theta_{i-1}^{j,k} + \theta_{i-1}^{j,k-1}}{8} + \frac{\theta_{i-1}^{j-1,k} + \theta_{i-1}^{j-1,k-1}}{8} \right) \\ & - \frac{\eta}{2} \left(\frac{v_i^{j,k} + v_i^{j,k-1}}{8} + \frac{v_i^{j-1,k} + v_i^{j-1,k-1}}{8} + \frac{v_{i-1}^{j,k} + v_{i-1}^{j,k-1}}{8} + \frac{v_{i-1}^{j-1,k} + v_{i-1}^{j-1,k-1}}{8} \right) \\ & - Mt + Mt \left(\frac{u_i^{j,k} + u_i^{j,k-1}}{8} + \frac{u_i^{j-1,k} + u_i^{j-1,k-1}}{8} + \frac{u_{i-1}^{j,k} + u_{i-1}^{j,k-1}}{8} + \frac{u_{i-1}^{j-1,k} + u_{i-1}^{j-1,k-1}}{8} \right) \\ & - \frac{2}{3} \alpha t \left(\frac{s_i^{j,k} + s_i^{j,k-1}}{8} + \frac{s_i^{j-1,k} + s_i^{j-1,k-1}}{8} + \frac{s_{i-1}^{j,k} + s_{i-1}^{j,k-1}}{8} + \frac{s_{i-1}^{j-1,k} + s_{i-1}^{j-1,k-1}}{8} \right) \\ & - \frac{3}{2} t - \frac{3}{2} t \left(\frac{f_i^{j,k} + f_i^{j,k-1}}{8} + \frac{f_i^{j-1,k} + f_i^{j-1,k-1}}{8} + \frac{f_{i-1}^{j,k} + f_{i-1}^{j,k-1}}{8} + \frac{f_{i-1}^{j-1,k} + f_{i-1}^{j-1,k-1}}{8} \right) \\ & \left(\frac{u_i^{j,k} + u_i^{j,k-1}}{8} + \frac{u_i^{j-1,k} + u_i^{j-1,k-1}}{8} + \frac{u_{i-1}^{j,k} + u_{i-1}^{j,k-1}}{8} + \frac{u_{i-1}^{j-1,k} + u_{i-1}^{j-1,k-1}}{8} \right) \\ & + \frac{3}{2} t \left(\frac{u_i^{j,k} + u_i^{j,k-1}}{8} + \frac{u_i^{j-1,k} + u_i^{j-1,k-1}}{8} + \frac{u_{i-1}^{j,k} + u_{i-1}^{j,k-1}}{8} + \frac{u_{i-1}^{j-1,k} + u_{i-1}^{j-1,k-1}}{8} \right) \\ & \left(\frac{u_i^{j,k} + u_i^{j,k-1}}{8} + \frac{u_i^{j-1,k} + u_i^{j-1,k-1}}{8} + \frac{u_{i-1}^{j,k} + u_{i-1}^{j,k-1}}{8} + \frac{u_{i-1}^{j-1,k} + u_{i-1}^{j-1,k-1}}{8} \right) \\ & - t \left(\frac{u_i^{j,k} + u_i^{j,k-1}}{4m_j} - \frac{u_i^{j-1,k} + u_i^{j-1,k-1}}{4m_j} + \frac{u_{i-1}^{j,k} + u_{i-1}^{j,k-1}}{4m_j} - \frac{u_{i-1}^{j-1,k} + u_{i-1}^{j-1,k-1}}{4m_j} \right). \tag{3.8a} \end{aligned}$$

Angular Momentum Equation:

$$\begin{aligned}
 & \left(1 + \frac{K}{2}\right) \left(\frac{\delta\theta_i - \delta\theta_{i-1}}{4b_i}\right) + \frac{\eta}{2} \left(\frac{\delta\theta_i + \delta\theta_{i-1}}{8}\right) + \frac{1}{2} \left(\frac{\delta h_i + \delta h_{i-1}}{8}\right) \\
 & + \frac{3}{2} t \left(\frac{(f_i^{j,k} + f_{i-1}^{j,k})(\delta\theta_i + \delta\theta_{i-1})}{8}\right) \left(\frac{(\theta_i^{j,k} + \theta_{i-1}^{j,k})(\delta f_i + \delta f_{i-1})}{8}\right) \\
 & - \frac{3}{2} t \left(\frac{(h_i^{j,k} + h_{i-1}^{j,k})(\delta u_i + \delta u_{i-1})}{8}\right) \left(\frac{(u_i^{j,k} + u_{i-1}^{j,k})(\delta h_i + \delta h_{i-1})}{8}\right) \\
 & - t \left(\frac{\delta h_i + \delta h_{i-1}}{4m_j}\right) - 2Kt \left(\frac{\delta h_i + \delta h_{i-1}}{8}\right) - Kt \left(\frac{\delta v_i + \delta v_{i-1}}{8}\right) \\
 = & - \left(1 + \frac{K}{2}\right) \left(\frac{\theta_i^{j,k} + \theta_i^{j,k-1}}{4b_i} + \frac{\theta_i^{j-1,k} + \theta_i^{j-1,k-1}}{4b_i} - \frac{\theta_{i-1}^{j,k} + \theta_{i-1}^{j,k-1}}{4b_i} - \frac{\theta_{i-1}^{j-1,k} + \theta_{i-1}^{j-1,k-1}}{4b_i}\right) \\
 & - \frac{\eta}{2} \left(\frac{\theta_i^{j,k} + \theta_i^{j,k-1}}{8} + \frac{\theta_i^{j-1,k} + \theta_i^{j-1,k-1}}{8} + \frac{\theta_{i-1}^{j,k} + \theta_{i-1}^{j,k-1}}{8} + \frac{\theta_{i-1}^{j-1,k} + \theta_{i-1}^{j-1,k-1}}{8}\right) \\
 & - \frac{1}{2} \left(\frac{h_i^{j,k} + h_i^{j,k-1}}{8} + \frac{h_i^{j-1,k} + h_i^{j-1,k-1}}{8} + \frac{h_{i-1}^{j,k} + h_{i-1}^{j,k-1}}{8} + \frac{h_{i-1}^{j-1,k} + h_{i-1}^{j-1,k-1}}{8}\right) \\
 & - \frac{3}{2} t \left(\frac{f_i^{j,k} + f_i^{j,k-1}}{8} + \frac{f_i^{j-1,k} + f_i^{j-1,k-1}}{8} + \frac{f_{i-1}^{j,k} + f_{i-1}^{j,k-1}}{8} + \frac{f_{i-1}^{j-1,k} + f_{i-1}^{j-1,k-1}}{8}\right) \\
 & \left(\frac{\theta_i^{j,k} + \theta_i^{j,k-1}}{8} + \frac{\theta_i^{j-1,k} + \theta_i^{j-1,k-1}}{8} + \frac{\theta_{i-1}^{j,k} + \theta_{i-1}^{j,k-1}}{8} + \frac{\theta_{i-1}^{j-1,k} + \theta_{i-1}^{j-1,k-1}}{8}\right) \\
 & + \frac{3}{2} t \left(\frac{h_i^{j,k} + h_i^{j,k-1}}{8} + \frac{h_i^{j-1,k} + h_i^{j-1,k-1}}{8} + \frac{h_{i-1}^{j,k} + h_{i-1}^{j,k-1}}{8} + \frac{h_{i-1}^{j-1,k} + h_{i-1}^{j-1,k-1}}{8}\right) \\
 & \left(\frac{u_i^{j,k} + u_i^{j,k-1}}{8} + \frac{u_i^{j-1,k} + u_i^{j-1,k-1}}{8} + \frac{u_{i-1}^{j,k} + u_{i-1}^{j,k-1}}{8} + \frac{u_{i-1}^{j-1,k} + u_{i-1}^{j-1,k-1}}{8}\right) \\
 & + t \left(\frac{h_i^{j,k} + h_i^{j,k-1}}{4m_j} - \frac{h_i^{j-1,k} + h_i^{j-1,k-1}}{4m_j} + \frac{h_{i-1}^{j,k} + h_{i-1}^{j,k-1}}{4m_j} - \frac{h_{i-1}^{j-1,k} + h_{i-1}^{j-1,k-1}}{4m_j}\right) \\
 & + 2Kt \left(\frac{h_i^{j,k} + h_i^{j,k-1}}{8} + \frac{h_i^{j-1,k} + h_i^{j-1,k-1}}{8} + \frac{h_{i-1}^{j,k} + h_{i-1}^{j,k-1}}{8} + \frac{h_{i-1}^{j-1,k} + h_{i-1}^{j-1,k-1}}{8}\right) \\
 & + Kt \left(\frac{v_i^{j,k} + v_i^{j,k-1}}{8} + \frac{v_i^{j-1,k} + v_i^{j-1,k-1}}{8} + \frac{v_{i-1}^{j,k} + v_{i-1}^{j,k-1}}{8} + \frac{v_{i-1}^{j-1,k} + v_{i-1}^{j-1,k-1}}{8}\right).
 \end{aligned} \tag{3.8b}$$

Energy Equation:

$$\begin{aligned}
 & \left(\frac{\delta\phi_i - \delta\phi_{i-1}}{4b_i}\right) + Pr \frac{\eta}{2} \left(\frac{\delta\phi_i - \delta\phi_{i-1}}{4b_i}\right) + Prt \frac{3}{2} \cos(x) \left(\frac{(f_i^{j,k} + f_{i-1}^{j,k})(\delta\phi_i + \delta\phi_{i-1})}{8}\right) \\
 & \left(\frac{(\phi_i^{j,k} + \phi_{i-1}^{j,k})(\delta f_i + \delta f_{i-1})}{8}\right) - Prt \left(\frac{\delta s_i + \delta s_{i-1}}{4m_j}\right) - \frac{3}{2} Prt \sin(x) \left(\frac{(u_i^{j,k} + u_{i-1}^{j,k})(\delta s_i + \delta s_{i-1})}{8}\right)
 \end{aligned}$$

$$\begin{aligned}
 & \left(\frac{(s_i^{j,k} + s_{i-1}^{j,k})(\delta u_i + \delta u_{i-1})}{4q_k} \right) + \frac{3}{2} Prt \sin(x) \left(\frac{(\phi_i^{j,k} + \phi_{i-1}^{j,k})(\delta f_i + \delta f_{i-1})}{8} \right) \left(\frac{(f_i^{j,k} + f_{i-1}^{j,k})(\delta \phi_i + \delta \phi_{i-1})}{4q_k} \right) \\
 & + \frac{3}{2} Prt \sin(x) \frac{1}{r} \frac{\partial r}{\partial x} \left(\frac{(f_i^{j,k} + f_{i-1}^{j,k})(\delta \phi_i + \delta \phi_{i-1})}{8} \right) \left(\frac{(\phi_i^{j,k} + \phi_{i-1}^{j,k})(\delta f_i + \delta f_{i-1})}{8} \right) \\
 & Pr \frac{\eta}{2} \left(\frac{\phi_i^{j,k} + \phi_i^{j,k-1}}{4b_i} + \frac{\phi_i^{j-1,k} + \phi_i^{j-1,k-1}}{4b_i} + \frac{\phi_{i-1}^{j,k} + \phi_{i-1}^{j,k-1}}{4b_i} + \frac{\phi_{i-1}^{j-1,k} + \phi_{i-1}^{j-1,k-1}}{4b_i} \right) \\
 & - Prt \frac{3}{2} \cos(x) \left(\frac{f_i^{j,k} + f_i^{j,k-1}}{8} + \frac{f_i^{j-1,k} + f_i^{j-1,k-1}}{8} + \frac{f_{i-1}^{j,k} + f_{i-1}^{j,k-1}}{8} + \frac{f_{i-1}^{j-1,k} + f_{i-1}^{j-1,k-1}}{8} \right) \\
 & = - \left(\frac{\phi_i^{j,k} + \phi_i^{j,k-1}}{4b_i} + \frac{\phi_i^{j-1,k} + \phi_i^{j-1,k-1}}{4b_i} - \frac{\phi_{i-1}^{j,k} + \phi_{i-1}^{j,k-1}}{4b_i} - \frac{\phi_{i-1}^{j-1,k} + \phi_{i-1}^{j-1,k-1}}{4b_i} \right) \\
 & - \left(\frac{\phi_i^{j,k} + \phi_i^{j,k-1}}{8} + \frac{\phi_i^{j-1,k} + \phi_i^{j-1,k-1}}{8} + \frac{\phi_{i-1}^{j,k} + \phi_{i-1}^{j,k-1}}{8} + \frac{\phi_{i-1}^{j-1,k} + \phi_{i-1}^{j-1,k-1}}{8} \right) \\
 & + Prt \left(\frac{s_i^{j,k} + s_i^{j,k-1}}{4m_j} + \frac{s_i^{j-1,k} + s_i^{j-1,k-1}}{4m_j} + \frac{s_{i-1}^{j,k} + s_{i-1}^{j,k-1}}{4m_j} + \frac{s_{i-1}^{j-1,k} + s_{i-1}^{j-1,k-1}}{4m_j} \right) \\
 & + \frac{3}{2} Prt \sin(x) \left(\frac{u_i^{j,k} + u_i^{j,k-1}}{8} + \frac{u_i^{j-1,k} + u_i^{j-1,k-1}}{8} + \frac{u_{i-1}^{j,k} + u_{i-1}^{j,k-1}}{8} + \frac{u_{i-1}^{j-1,k} + u_{i-1}^{j-1,k-1}}{8} \right) \\
 & \left(\frac{s_i^{j,k} - s_i^{j,k-1}}{4q_k} + \frac{s_i^{j-1,k} - s_i^{j-1,k-1}}{4q_k} + \frac{s_{i-1}^{j,k} - s_{i-1}^{j,k-1}}{4q_k} + \frac{s_{i-1}^{j-1,k} - s_{i-1}^{j-1,k-1}}{4q_k} \right) \\
 & - \frac{3}{2} Prt \sin(x) \left(\frac{\phi_i^{j,k} + \phi_i^{j,k-1}}{8} + \frac{\phi_i^{j-1,k} + \phi_i^{j-1,k-1}}{8} + \frac{\phi_{i-1}^{j,k} + \phi_{i-1}^{j,k-1}}{8} + \frac{\phi_{i-1}^{j-1,k} + \phi_{i-1}^{j-1,k-1}}{8} \right) \\
 & \left(\frac{f_i^{j,k} - f_i^{j,k-1}}{4q_k} + \frac{f_i^{j-1,k} - f_i^{j-1,k-1}}{4q_k} + \frac{f_{i-1}^{j,k} - f_{i-1}^{j,k-1}}{4q_k} + \frac{f_{i-1}^{j-1,k} - f_{i-1}^{j-1,k-1}}{4q_k} \right) \\
 & - \frac{3}{2} Prt \sin(x) \frac{1}{r} \frac{\partial r}{\partial x} \left(\frac{f_i^{j,k} + f_i^{j,k-1}}{8} + \frac{f_i^{j-1,k} + f_i^{j-1,k-1}}{8} + \frac{f_{i-1}^{j,k} + f_{i-1}^{j,k-1}}{8} + \frac{f_{i-1}^{j-1,k} + f_{i-1}^{j-1,k-1}}{8} \right) \\
 & \left(\frac{\phi_i^{j,k} + \phi_i^{j,k-1}}{8} + \frac{\phi_i^{j-1,k} + \phi_i^{j-1,k-1}}{8} + \frac{\phi_{i-1}^{j,k} + \phi_{i-1}^{j,k-1}}{8} + \frac{\phi_{i-1}^{j-1,k} + \phi_{i-1}^{j-1,k-1}}{8} \right).
 \end{aligned}$$

(3.8c)

3.4. User Interface – Keller Box Method

This user interface is developed by using the function ‘IF’, ‘OR’, ‘AND’, and ‘COUNTIF’ in Microsoft Excel. The Keller Box – Excel User Interface is designed as below.

coefficient	INPUT														
	var f1	prime1	power1	prime 2	power 2	prime 3	power 3	var f2	prime 1	power 1	prime 2	power 2	prime 3	power 3	
1+K		no		1 no		1 no		f		3	1 no		1 no		1
K		no		1 no		1 no		f			1 no		1 no		1
(eta/2)		no		1 no		1 no		f		2	1 no		1 no		1
Mt		no		1 no		1 no		f			1 no		1 no		1
(-Mt)		no		1 no		1 no		f		1	1 no		1 no		1
(2/3)(alpha)(t)*		no		1 no		1 no		f			1 no		1 no		1
(3/2)(t)		no		1 no		1 no		f			1 no		1 no		1
(3/2)(t)	f		0	1 no		1 no		f		1	1 no		1 no		1
(-3/2)(t)		no		1 no		1 no		f		1	2 no		1 no		1
(-t) *		no		1 no		1 no		f		1	1		1 no		1
1+(K/2)		no		1 no		1 no		f			1 no		1 no		1
eta/2		no		1 no		1 no		f			1 no		1 no		1
(1/2)		no		1 no		1 no		f			1 no		1 no		1
(3/2)(t)		no		1 no		1 no		f		0	1 no		1 no		1
(-3/2)(t)		no		1 no		1 no		f		1	1 no		1 no		1
(-t) *		no		1 no		1 no		f			1 no		1 no		1
(-2Kt)		no		1 no		1 no		f			1 no		1 no		1
(-Kt)		no		1 no		1 no		f		2	1 no		1 no		1
(Pr)(eta/2)	1			1 no		1 no		f			1 no		1 no		1
(Prt)(3/2)(cos(x))		no		1 no		1 no		f		0	1 no		1 no		1
(-Prt)		no		1 no		1 no		f			1 no		1 no		1
(Prt)(-3/2)(sin(x))		no		1 no		1 no		f		1	1 no		1 no		1
(Prt)(3/2)(sin(x))		no		1 no		1 no		f		0	1 no		1		1
(Prt)(3/2r)(sin(x))(r_x)		no		1 no		1 no		f		0	1 no		1 no		1

Figure 1 Input Section of User Interface

var h	prime 1	power 1	prime 2	power 2	prime 3	power 3	var s	prime 1	power 1	prime 2	power 2	prime 3	power 3	
h		no	1 no		1 no		1	no		1 no		1 no		1
h		1					1	no		1 no		1 no		1
h		no	1 no		1 no		1	no		1 no		1 no		1
h		no	1 no		1 no		1	no		1 no		1 no		1
h		no	1 no		1 no		1	no		1 no		1 no		1
h		no	1 no		1 no		1	s	0	1 no		1 no		1
h		no	1 no		1 no		1	no		1 no		1 no		1
h		no	1 no		1 no		1	no		1 no		1 no		1
h		no	1 no		1 no		1	no		1 no		1 no		1
h		no	1 no		1 no		1	no		1 no		1 no		1
h		2					1	no		1 no		1 no		1
h		1	1 no		1 no		1	no		1 no		1 no		1
h		0	1 no		1 no		1	no		1 no		1 no		1
h		1	1 no		1 no		1	no		1 no		1 no		1
h		0	1 no		1 no		1	no		1 no		1 no		1
h		0	1	1	1 no		1	no		1 no		1 no		1
h		0	1 no		1 no		1	no		1 no		1 no		1
h		no	1 no		1 no		1	no		1 no		1 no		1
h		no	1 no		1 no		1	s	2	1 no		1 no		1
h		no	1 no		1 no		1	s	1	1 no		1 no		1
h		no	1 no		1 no		1	s	1	1 no		1 no		1
h		no	1 no		1 no		1	s	0	1	1	1 no		1
h		no	1 no		1 no		1	s	0	1 no		1	1	1
h		no	1 no		1 no		1	s	1	1 no		1 no		1
h		no	1 no		1 no		1	s	1	1 no		1 no		1

Figure 2 Continuation of Figure 1

coefficient	f1 eta	power1	f1 t	f1 x	f2 eta	power1	f2 t	f2 x
1+K			1		v		1	
K			1				1	
(eta/2)			1		v		1	
Mt			1				1	
(-Mt)			1		u		1	
(2/3)(alpha)(t)*			1				1	
(3/2)(t)			1				1	
(3/2)(t)	f		1		u		1	
(-3/2)(t)			1		u		2	
(-t) *			1		u		1 t	
1+(K/2)			1				1	
eta/2			1				1	
(1/2)			1				1	
(3/2)(t)			1		f		1	
(-3/2)(t)			1		u		1	
(-t) *			1				1	
(-2Kt)			1				1	
(-Kt)			1		v		1	
(Pr)(eta/2)	1		1				1	
(Prt)(3/2)(cos(x))			1		f		1	
(-Prt)			1				1	
(Prt)(-3/2)(sin(x))			1		u		1	
(Prt)(3/2)(sin(x))			1		f		1	
(Prt)(3/2r)(sin(x))(r_x)			1		f		1	x

Figure 3 Calculation Section of User Interface

CALCULATION							
h_eta	power h_eta	h_t	h_x	s_eta	power s_eta	s_t	s_x
theta	1				1		
	1				1		
	1				1		
	1				1		
	1			s	1		
	1				1		
	1				1		
	1				1		
	1				1		
	1				1		
theta'	1				1		
theta	1				1		
h	1				1		
theta	1				1		
h	1				1		
h	1	t			1		
h	1				1		
	1				1		
	1			phi	1		
	1			phi	1		
	1			phi	1		
	1			s	1	t	
	1			s	1		x
	1			phi	1		
	1			phi	1		

Figure 4 Continuation of Figure 3

OUTPUT														
coefficient	var 1	prime	t	x	di	di-1	delta	var 2	prime	t	x	di	di-1	delta
1+K	v	YES	NO	NO	(1/4b)	(-1/4b)	v		NO	NO	NO			
K	theta	NO	NO	NO	(1/8)	(1/8)	theta		NO	NO	NO			
(eta/2)	v	NO	NO	NO	(1/8)	(1/8)	v		NO	NO	NO			
Mt		NO	NO	NO					NO	NO	NO			
(-Mt)	u	NO	NO	NO	(1/8)	(1/8)	u		NO	NO	NO			
(2/3)(alpha)(t)	s	NO	NO	NO	(1/8)	(1/8)	s		NO	NO	NO			
(3/2)(t)		NO	NO	NO					NO	NO	NO			
(3/2)(t)	f	NO	NO	NO	(1/8)(i-1/2)	(1/8)(i-1/2)	u	u	NO	NO	NO	(1/8)(i-1/2)	(1/8)(i-1/2)	f
(-3/2)(t)	u	NO	NO	NO	(1/8)(i-1/2)	(1/8)(i-1/2)	u	u	NO	NO	NO	(1/8)(i-1/2)	(1/8)(i-1/2)	u
(-t)	u	NO	YES	NO	(1/4m)	(1/4m)	u		NO	NO	NO			
1+(K/2)	theta'	YES	NO	NO	(1/4b)	(-1/4b)	theta		NO	NO	NO			
eta/2	theta	NO	NO	NO	(1/8)	(1/8)	theta		NO	NO	NO			
(1/2)	h	NO	NO	NO	(1/8)	(1/8)	h		NO	NO	NO			
(3/2)(t)	f	NO	NO	NO	(1/8)(i-1/2)	(1/8)(i-1/2)	theta	theta	NO	NO	NO	(1/8)(i-1/2)	(1/8)(i-1/2)	f
(-3/2)(t)	u	NO	NO	NO	(1/8)(i-1/2)	(1/8)(i-1/2)	h	h	NO	NO	NO	(1/8)(i-1/2)	(1/8)(i-1/2)	u
(-t)	h_t	NO	YES	NO	(1/4m)	(1/4m)	h		NO	NO	NO			
(-2k)	h	NO	NO	NO	(1/8)	(1/8)	h		NO	NO	NO			
(-k)	v	NO	NO	NO	(1/8)	(1/8)	v		NO	NO	NO			
(Pr)	phi'	YES	NO	NO	(1/4b)	(-1/4b)	phi		NO	NO	NO			
(Pr)(eta/2)	phi	NO	NO	NO	(1/8)	(1/8)	phi		NO	NO	NO			
(Prt)(3/2)(cos(x))	f	NO	NO	NO	(1/8)(i-1/2)	(1/8)(i-1/2)	phi	phi	NO	NO	NO	(1/8)(i-1/2)	(1/8)(i-1/2)	f
(-Prt)	s_t	NO	YES	NO	(1/4m)	(1/4m)	s		NO	NO	NO			
(Prt)(-3/2)(sin(x))	u	NO	NO	NO	(1/8)(i-1/2)	(1/8)(i-1/2)	s_x	s_x	NO	NO	YES	(1/4qk)(i-1/2)	(1/4qk)(i-1/2)(plus)	u
(Prt)(3/2)(sin(x))	f_x	NO	NO	YES	(1/4qk)(i-1/2)	(1/4qk)(i-1/2)	phi	phi	NO	NO	NO	(1/8)(i-1/2)	(1/8)(i-1/2)	f_x
(Prt)(3/2r)(sin(x))(r_x)	f	NO	NO	NO	(1/8)(i-1/2)	(1/8)(i-1/2)	phi	phi	NO	NO	NO	(1/8)(i-1/2)	(1/8)(i-1/2)	f

Figure 5 Output Section of User Interface

4. Conclusion

Overall, the Keller Box Method user interface is created by using Microsoft Excel to solve three-dimensional nonlinear equations. Keller Box Method employs discretization via Finite Difference Method and linearization via Newton's Method. The results of the manual calculation and the user interface Keller Box Method are found to be in good agreement.

References

- [1] Britannica, The Editors of Encyclopaedia. "Boundary layer". *Encyclopedia Britannica*, 6 Mar. 2020, Retrieved from <https://www.britannica.com/science/boundary-layer>.
- [2] Nazir, U., Sohali, M., Ali, U., Sherif, E-S, M., Park, C., Lee, R. J., Selim, M., & Thounthong, P. (2021). Application of Cattaneo-Christov fluxes on modelling the boundary value problem of Parandtl fluid comprising variable properties. *Scientific Reports*, 11 (1), Retrieved from <https://www.nature.com/articles/s41598-021-97420-2#citeas>
- [3] Widodo, B., Fauziyah, M., & Adzkiya, D. (2019). Implementation Euler Scheme of unsteady magnetohydrodynamic micropolar fluid pass a sphere affected by mixed convection. *International Conference on Physics and Its Application*, 1153(1), 1-6, doi: 10.1088/1742-6596/1153/1/012062, Retrived from https://www.researchgate.net/publication/331348586_Implementation_Euler_implicit_scheme_of_unsteady_magnetohydrodynamic_micropolar_fluid_pass_a_sphere_affected_by_mixed_convection

- [4] Makanda, G., Magagula, V. M., Sibanda, P., & Motsa, S. S. (2020). Numerical analysis of free convection from a spinning cone with variable wall temperature and pressure work effect using MD-BSQLM. *Open Physics*, 19(1),179-187, Retrieved from <https://www.degruyter.com/document/doi/10.1515/phys-2020-0189/html>
- [5] Jamshed, W., Nisar, K. S., Ibrahim, R. W., Mukhtar, T., Vijayakumar, V., & Ahmad, F. (2021). Computational framework of Cattaneo-Christov heat flux effects on Engine Oil based Williamson hybrid nanofluids: A thermal case study. *Case Studies in Thermal Engineering*, 26(2021), 1-16, Retrieved from <https://www.sciencedirect.com.ezproxy.utm.my/science/article/pii/S2214157X21003427>
- [6] Mohammed, Z. S., Hamzah, T. A., Abid, H., & Mustafa, M. (2018). Heat transfer flow of Cu-water and Al₂O₃-water micropolar nanofluids about a solid sphere in the presence of natural convection using Keller-box method. *Results in Physics*, 9(2018),717-724, <https://doi.org/10.1016/j.rinp.2018.03.033>
- [7] Zeeshan, A., Majeed, A., Muhammad, J. A., & Faris, A. (2021) Numerical investigation of MHD radiative heat and mass transfer of nanofluid flow towards a vertical wavy surface with viscous dissipation and Joule heating effects using Keller-box method. *Mathematics and Computers in Simulation*, 190(2021),1080-1109. Retrieved from <https://login.ezproxy.utm.my/login?qurl=https://www.sciencedirect.com%2fscience%2farticle%2fpii%2fS0378475421002524>
- [8] Priyam, A., Woodcroft, B. J., Rai, V., Moghul, I., Munagala, A., Ter, F., Chowdhary, H., Pieniak, I., Maynard, L. J., Gibbins, M. A., Moon, H., Davis-Rischarson, A., Uludag, M., Rafique, K., Anwar, M. I., Misiran, M., Khan, I., Seikh, A. H., Sherif, E-S. M., & Nisar, K. S. (2019). Keller-Box Simulation for te Buogiorno Mathematical Model of Micropolar Nanofluid Flow over a Nonlinear Inclined Surface. *Advanced Digital and Other Processes*, 7(12), 926, Retrieved from <https://www.mdpi.com/2227-9717/7/12/926/htm>