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Maximize The Chance Victory of Basketball by Using Game

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Abstract
The use of mathematics in life is becoming more and more extensive, including in sports. The main purpose of this study is to help coaches and players by studying the application of game theory in the decision-making process of basketball games. This can be done by modeling the scenarios on the game as well as players' field goal percentage and the interaction between two team players. The best way for offensive team to make decision whether to choose a two-point shot or three-point shot to increase the winning rate can then be found. This study can help the coach or player to make the best strategy during a basketball match. With offensive team is two points behind defensive team, the team can choose to shoot a 2-point shot to make the game into overtime or a 3-point shot for an outright win. When offensive team is down by 3 points, fouls and free throws are also considered in the scenario. This study can be divided into a few phases. Firstly, gather the information about game theory model and example. Next, determine the best suitable game theory model for various scenarios. Lastly, determine the strategy that decides how the game should be played. Since there is no dominant pure strategy during the match, hence mixed strategy Nash equilibriums are obtained. Players with higher field goal percentages and the game with lower score gaps have more tactical selection and higher expected value to disrupt the opponents and improve the team's chances of winning.

Keywords: Game Theory; Mixed Strategy; Basketball Game

1. Introduction

Basketball is one of the world-famous sports. However, as the standard is getting higher, the outcome of some matches can only be determined at the last minute. Not only the physical skills of players, but coaching decisions also play very important role during this period in determining the outcome of the game. Therefore, in order to optimize the winning rate and enhance the performance of basketball teams, game theory can be used to analyze the player's actions and the team's strategy since it is a study of strategic decision making. Through game theory, the best response strategy to restrain the opponent's strategy could be formulated in advance.

Game theory is a study that aims to help the players understand situations and decide how to choose the best strategies in order to maximize the profits. The basis is to study the behaviour of a person or a company that will directly interact with the balance and also help decision makers select better decisions to some extent. Game theory is now widely used in different fields such as poker, economics, sport and even complicated politics competing for votes (Von & Oskar, 2007).

With the vigorous development of competitive and professional sports, game theory has more potential and opportunities to be applied in this field and plays an important role. The prediction of player's and team's action can be explained through the calculation of benefit for each player during the situation (Sindik, 2008).

The game theory that is usually applied in sport is zero-sum games. In zero-sum game, the total gain of a player will equal to the total loss of the other player and their sum will be zero. Basketball is one example of zero-sum game since win for one team means a loss of other team. This study will analyze the mixed strategy of zero-sum game by using some specific payoffs actions.

The rest of the paper is organized as follows. The next section covers the literature review, followed by Section 3 that covers the methodology. Results and discussion are presented in Section 4. The final section gives the conclusion of the study.

2. Literature Review

Game theory can be defined as a mathematical summary of strategic interaction between decision makers and use to study the direct interaction of decision maker behaviour. The decision of one of the players can be influenced by the choice of strategy for other players and also directly affect the other players and the equilibrium of the problem.

2.1 Type of games

A game without randomization is called pure strategy while mixed strategy is defined as at least one player play randomized strategy. The predicted payoff cannot be increased by switching to a different strategy. A player's success depends on his unpredictable behaviour during the strategic situations (Walker, 2008). The most famous example is the penalty kick in football games. The penalty taker needs to predict which side the goalkeeper is going to defend in order to decide the direction of his kick. At the same time, the goalkeeper is also doing the same prediction in determining his defend direction of the penalty kick. This situation is considered as a game with mixed strategy equilibrium.

Zero sum game is a special type of game theory where the gain of a player is equal to the loss of the other player with total sum equal to zero. Chess and other two player sport games are examples of zero-sum games. In contrast, non-zero-sum game is defined as the situation where the total sum of the gain and the loss are less than or more than zero.

Bayesian game is a strategic game with incomplete information. For example, participants do not know the actual payoff of the other participant but may have opinions about them. There are many situations in game theory where participants do not know the characteristics of their opponents. In football games, the goalkeeper does not know whether the player wishes to kick to left side or right side. Therefore, this can be defined as game with incomplete information.

2.2. Representation of games theory

An extensive-form game is a game theory representation in the form of decision tree that allows for the explicit representation of a number of key aspects such as the sequence of player movements, selection of decisions, the information each player has about the other player's movements when making the decision and their winnings rate and possible outcomes of the game.

On the other hand, normal form is a description of game in matrix form. In the other words, the game can be defined as normal form game if the payoff and strategies of the game can be presented in table form. This approach helps in identifying Nash equilibrium and dominated strategies but some information are lost compared to extensive form. The normal form games also include all perceptible and possible outcome and the corresponding payoffs for different players.

2.3 Related Works of Game Theory

This includes some simple two-player games or two-team games. The most classic example is the Rock-Paper-Scissors (RPS). Zhou (2015) studied the non-equilibrium statistical mechanics of non-cooperative strategic interactions which can be used as a starting point to enter the intersection field of statistical physics and game theory.

Next another epic example of two players game which has been analysed through game theory is poker. In game theory, there is at least one Nash equilibrium exists in all multiplayer games which has finite payment matrices (Li, 2018). Besides that, poker can be defined as zero sum game and it is well known that there are always an optimal strategy if mixed strategies are allowed in two player zero-sum games. By modelling the poker games, they found out that game theory can find a safe strategy by minimizing exploitability when playing with little knowledge on opponent's playing style.

2.4 Critical Analysis

Game theory has been applied widely in many areas such as economics, business, supply chain and many others. Although there are many references to sports, but there is still little application in basketball. Player's decision and the type of game will influence the best strategy played. Based on the literature review, the most suitable model in game theory for modelling various situations in a basketball game will be chosen and presented in extensive and normal forms.

3. Research Methodology

The operational framework and procedure will be presented in this section.

3.1 Data Collection

Data on the players' performance will be taken from various NBA games with similar situations as the studied situation.

3.2. Development of Game Theory Model and Solution

A few situations in a basketball games are modelled and solved. Game theory is used to analyze the basketball matches from the perspective of profit and risk namely the best choice of shooting, foul tactics and the strategy when leading or falling behind the opponent during the matches.

The calculation to obtain the optimal mixed strategies is given as follows.

Table 1 General Matrix Form

	S_1	S_2
D_1	a	b
D_2	c	d

Set the assumptions that a, b, c and d are distinct and $a - b - c + d \neq 0$. Give probability p to strategy S_1 and $1 - p$ to strategy S_2. Next, assign probability q to strategy D_1 and $1 - q$ to strategy D_2. There are four possibilities:1) player 1 chooses D_1 while player 2 chooses S_1, 2) player 1 chooses D_1 while player 2 chooses S_2, 3) player 1 chooses D_2 while player 2 chooses S_1 and 4) player 1 chooses D_2 while player 2 chooses S_2. Therefore there will be 4 expected payoff exists in this model. The expected payoff for each strategy will be as following:

$$E_{S1} = a(q) + c(1-q) = a(q) + c - c(q) = q(a-c) + c$$

$$E_{S2} = b(q) + d(1-q) = b(q) + d - d(q) = q(b-d) + d$$

$$E_{D1} = a(p) + b(1-p) = a(p) + b - b(p) = p(a-b) + b$$

$$E_{D2} = c(p) + d(1-p) = c(p) + d - d(p) = p(c-d) + d$$

$$\text{Hence, } p = \frac{d-b}{(a+d)-(b-c)}$$

$q = \frac{d - c}{(a+d) - (b - c)}$ and

$v = \frac{ad - bc}{(a+d) - (b+c)}$

Since we had calculated the value of p and q, hence we can also get the value of $1 - p$ and $1 - q$ and find the optimal mixed strategy for the players.

4. Result and Discussion

The confrontation between offensive team and defensive team is the main focus of this study. Through the derived strategy formulas, the optimal interests of the offensive and defensive teams of the basketball team are realized in order to maximize the expected payoff of both parties in the match.

4.2 Game Theory Model

There are 3 possible situations that will happen during a game that are the match become overtime, offensive team win the game or offensive team loss the game. Defensive team has two pure strategies which are defend two-point or defend three-point. Basketball match is divided into two situations. The first situation is when the team is behind by two points, and the second situation is when the team is behind by three points. Offensive team also has two pure strategies which are shoot two-point or shoot three-point.

4.2.2 Payoff Matrix and Decision Tree

Situation 1: Offensive side less 2 points behind defensive side

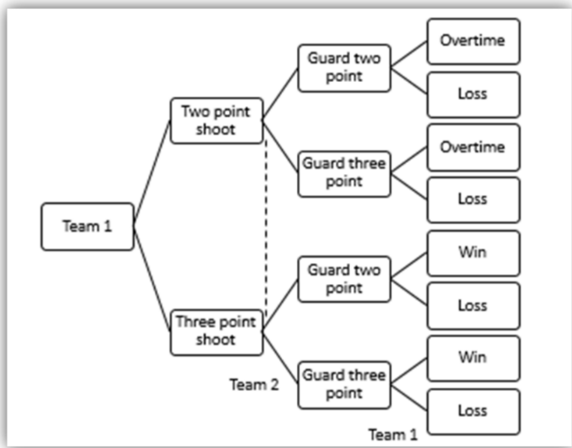


Figure 1 Decision Tree for Situation 1

Based on Figure 1, the offensive strategy space consists of two strategies (two-point shot and three point shot), and the defensive strategy consists of two strategies too (defensive two point and defensive three point). Since the game is played simultaneously, both team players will move at the same time with incomplete information of the opponent's move. The dotted line in the decision tree is the representation of incomplete knowledge since we do not know the other team strategy during the match.

Table 2 Probability Matric for Situation1

	Defending Team	
Offensive Team	Defend Two Point	Defend Three Point

Two Point Shot	P_{22}	P_{23}
Three Point Shot	P_{32}	P_{33}

The extensive form is shown in Table 2. The assumptions for the payoffs are given as follows: $P_{22} < P_{23}$, $P_{22} < P_{32}$, $P_{33} < P_{32}$ and $P_{33} < P_{23}$. The offensive team has more chance to win the match if the defensive team cannot anticipate the selection of shot while at the same time the defensive team is more likely to win the match if they correctly anticipate the shot. Therefore, there is a unique equilibrium in the strictly mixed strategies due to these assumptions. Let D_3 equal to the probability that the defensive team defend 3-points shot.

$$D_3 = \frac{P_{32} - P_{22}}{\left(P_{32} + P_{23} - P_{22} - P_{33}\right)} \text{ and } D_2 = 1 - D_3 \tag{1}$$

Let S_3 equal to the probability offensive team shoot 3-point shot.

$$S_3 = \frac{P_{23} - P_{22}}{\left(P_{32} + P_{23} - P_{33} - P_{22}\right)} \text{ and } S_2 = 1 - S_3 \tag{2}$$

If both team play optimally, the expected value for offensive team chance to win the game is given by

$$V = (P_{23}P_{32} - P_{22}P_{33}) / (P_{32} + P_{23} - P_{22} - P_{33}) \tag{3}$$

which gives the general solution for case 1. Each situation of the game will have a different payoff matrix, which requires determining each team's equilibrium strategies and offensive opportunities to win the game.

Situation 2: Offensive side 3 points behind the defensive side.

This situation is more complex compared to Situation 1 since it has larger score differential between match and associated growth in strategic options. As before, let look from the offensive side. Since it is more point behind compared to Situation 1 and there are only two ways for offensive team to get the chance for overtime, hence it is impossible to win directly. The offensive team had to choose shoot a three-point shot to get a tie and go to overtime or attempt a two-point shot and intend to foul the defending team. The decision tree of Situation 2 Part 1 in Figure 2 shows the situation before the free throws and the decision tree of Situation 2 Part 2 in Figure 3 shows the situation after the free throws.

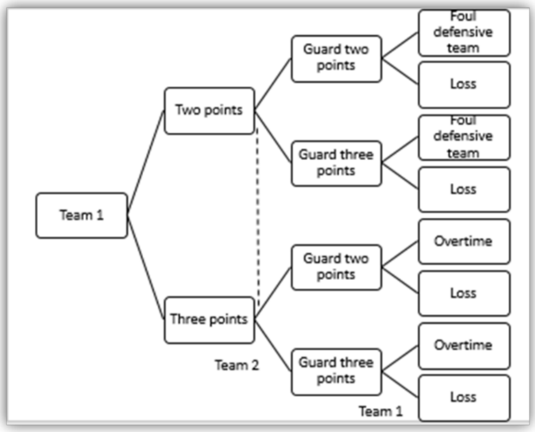


Figure 2 Decision Tree for Situation 2 Part 1

Based on Figure 2, there are two possibilities after two-point shot which are going to foul the defensive team or loss the game and two possibilities after three-point shot which are game go to overtime or loss the game. If both free throws are successful, team 1 will go back to where they started that are 3 point behind. If one of the two free throws is missed, then team 1 is just two points behind, If both serves are missed, team 1 is only 1 point behind team 2. Therefore, we can use the decision tree to divide the situation into 1 point behind, 2 points behind and 3 points behind as shown in Figure 3.

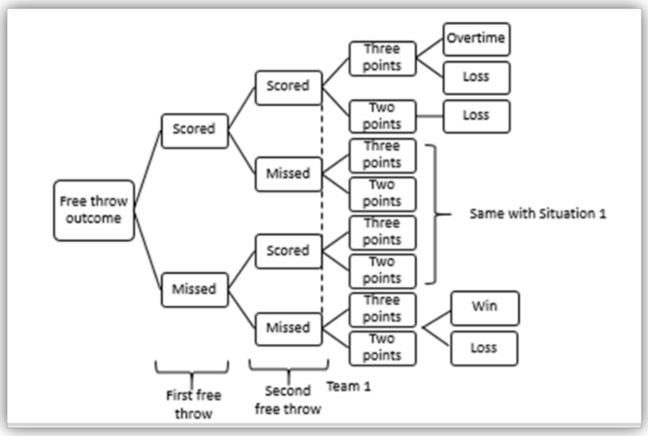


Figure 3 Decision Tree for Situation 2 Part 2

Based on the model in Figure 3, there are different strategies based on different situations. The strategies of being 2 points behind and 3 points behind are vastly different, although they are only one point different. The situation involving fouls is much more complicated than the situation that does not include fouls. Therefore, the impact of free throws on the game is also one of the strategy which can be considered by using game theory.

4.3 Numerical Example

Houston Rockets vs Trail Blazers
This is a game in 2014 NBA playoffs and Game 6 in Portland. Portland Trail Blazers lead Houston Rockets 3-2. With 28 second left, Rockets successfully scored a two point shoot and lead by 98-96. Only, 0.9 seconds left, Blazers was still two point behind. This is very much in line with Situation 1, time is running out, and the Blazers need to decide between a two-pointer and a three-pointer. Based on Table 3, it can be seen that the player's shooting rate has a great effect on the game.

Table 3 Trail Blazers and Rockets Quarterfinals Game 6 Technical Statistics

Player	2PT	3PT	FT	PF	PTS
N. Batum	3-4	1-6	0-0	4	9
L. Aldridge	10-26	0-0	10-10	4	30
R. Lopez	5-10	0-0	2-2	3	12
D. Lillard	2-4	6-10	3-3	2	25
W. Matthews	2-5	2-8	2-4	4	12
T. Robinson	3-5	0-0	2-2	0	8
D. Wright	0-1	0-0	0-0	0	0
M. Williams	1-3	0-0	1-1	3	3

With the selection, D.Lillard would be a better shooter for the final shot than L.Aldridge, since L.Aldridge lacked 3-point shooting ability. Therefore, D.Lillard 2-point and 3-point field goal throughout the season had been selected as data reference.

D.Lillard open 2-point field goal percentage during 13-14 session: 52.2%

D.Lillard open 3-point field goal percentage during 13-14 session: 43.4%
D.Lillard contested 2-point field goal percentage during session: 49.4%
D.Lillard contested 3-point field goal percentage during session: 25%

The game is analyzed by using Gambit software and formulas (1), (2) and (3).

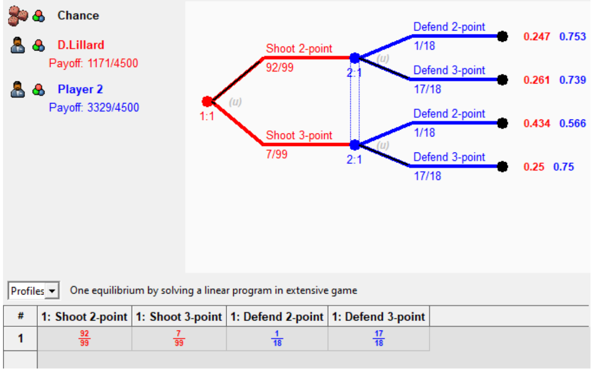


Figure 4 Decision Tree for Trail Blazers and Rockets basketball match

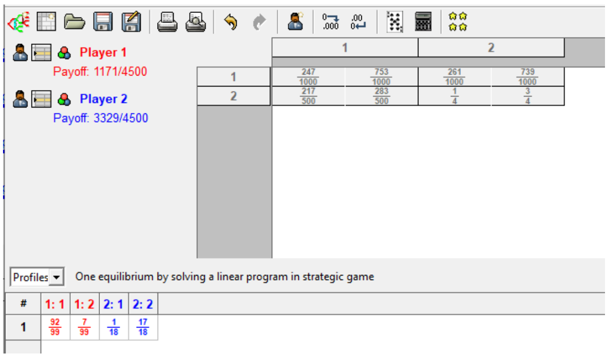


Figure 5 Matrix Form for Trail Blazers and Rockets basketball match

Based on the result, the equilibrium point under the assumption of representative of Trail Blazers to attempt a three-point shot is approximated to 7.07% and 92.93% to attempt a two-point shot. While, for defending team, the equilibrium point for them to defend three-point shot is approximated to 94.44% and 5.56% for defending two-point shot. However, the result of expected value based on the equilibrium point for Trail Blazers means that Trail Blazers will win the game 26.02% of the time, while Rocket will win the game 73.98% of the time.

San Antonio Spurs vs Miami Heat

This is a game in 2013 NBA Final playoffs and Game 6 in Miami. San Antonio Spurs lead Miami Heat 3-2. The score of this game had been very close. With 19.4 second left, the Spurs was leading for a long time and are still 3 points ahead. During this time, Miami Heat was expected to foul the opponent after scoring a shot and then decide on their next strategy. In order to get the expected value of the game, the performance of the Miami Heat players need to be considered throughout the game.

Table 4 San Antonio Spurs vs Miami Heat Finals Game 6 Technical Statistics

Player	2PT	3PT	FT	PF	PTS
C. Bosh	5-12	0-0	0-1	0	10
L. James	10-21	1-5	9-12	3	32
M. Miller	1-2	2-2	0-0	5	8
D. Wade	6-15	0-0	2-2	4	14
M. Chalmers	3-6	4-5	2-2	2	20
C. Anderson	0-1	0-0	1-2	4	1
S. Battier	0-0	3-4	0-0	3	9
R. Allen	2-5	1-3	2-2	5	9

L. James had been very consistent throughout the season and also received the Most Value Player (MVP) trophy for that season. So giving him the last ball would be the most appropriate choice. Based on the game throughout the 2013-2014 season,

L.James open 2-point field goal percentage during 13-14 session: 67%
L.James open 3-point field goal percentage during 13-14 session: 49.1%
L.James contested 2-point field goal percentage during session: 44.9%
L.James contested 3-point field goal percentage during session: 33%

In this case, L. James can only shoot a 3-point shot and drag the game into overtime or score a 2-point and foul the opponent for the next ball. In terms of free throws, it is necessary to consider whether the opponent successfully makes 2 free throws or 1 of 2 free throws, or does not score both. Since this is a more complex situation, so the match will be separated into different parts since the points behind will be different depending on how many free throws as shown in Figure 6 below.

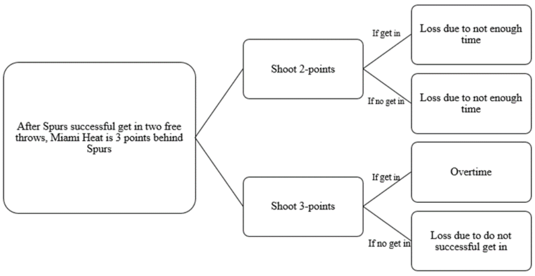


Figure 6 Decision Tree After Spurs Get Two Free Throws

The expected value for this situation become straight forward since we know that expected value for losing a game is 0 and expected value for overtime is 0.5. However, the opponents, Spurs is also famous tacticians in NBA. They must know that Miami heat only has 3-pointers as an option, so they will definitely try their best to prevent 3-pointers. Hence, the expected value of down by three after fouls can be directly calculated by using L.James contested 3-point field goal percentage that is 33%. The expected value of this situation is 0.33*0.5 = 0.175.

Next, if Spurs missed one of the free throws, the situation will become two points behind which expected value can be calculated by using Formulas (1), (2) and (3). This will be similar to Situation 1. L.James may shoot a two-point shot to drag the game into overtime or shoot a three-point shot to directly win the game. The solution is obtained by using Gambit sofware and Formulas (1), (2) and (3).

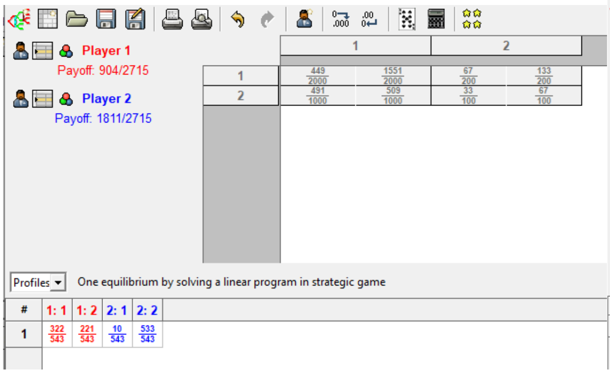


Figure 7 Matrix Form for Miami Heat and Spurs basketball match when two points behind

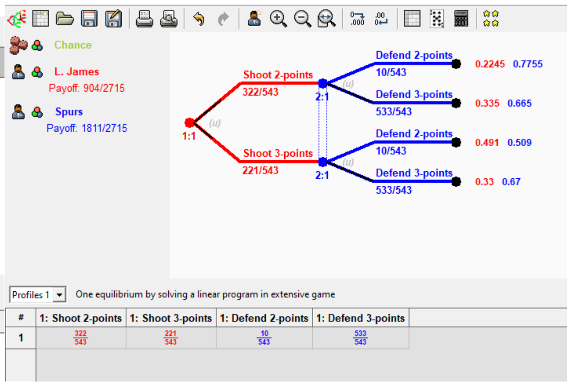
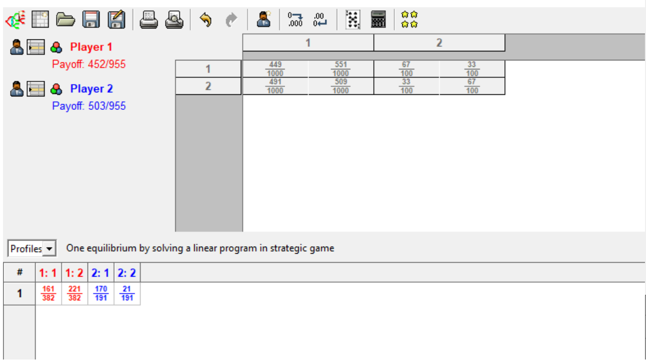


Figure 8 Decision Tree for Miami Heat and Spurs basketball match when two points behind

For the last situation, Spurs missed two out of two free throws. Therefore, the game become only 1point difference between the two team. Similar with situation 'down by two'. The game is analysed by using Gambit software and Formulas (1), (2) and (3).



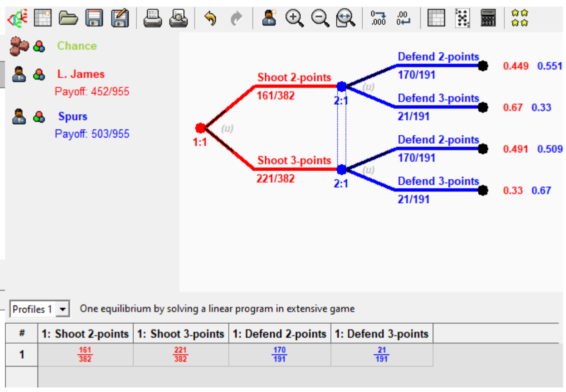


Figure 9 Matrix Form for Miami Heat and Spurs basketball match when one point behind

Figure 10 Decision Tree for Miami Heat and Spurs basketball match when one point behind

With all the expected value calculated, the overall expected value of foul scenario are obtained. The expected value for foul scenario is equal to (0.175*0.791 + 0.333*0.209)*0.791 + 0.4733*0.209 = 0.2635. The expected value to complete the situation of 3-points behind can then be calculated by using Gambit software.

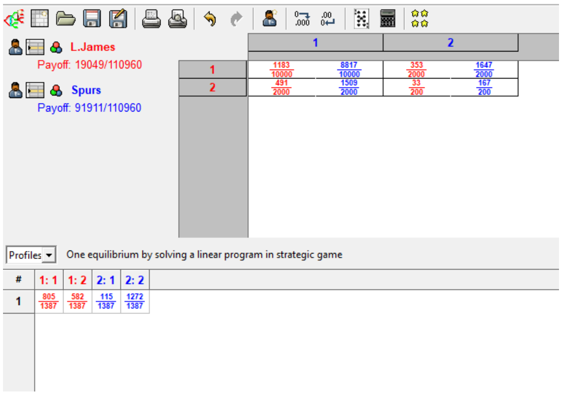


Figure 11 Payoff Matrix between Miami Heat and Spurs

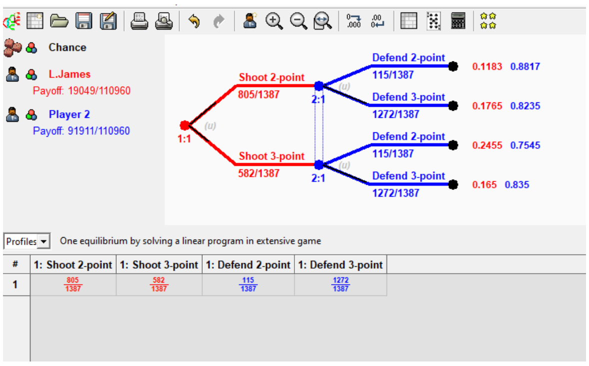


Figure 12 Decision Tree for Miami Heat and Spurs

From Figures 11 and 12, it can be seen that L.James shooting the three pointer 41.96% of the time and the Spurs defending the three 91.71% of the time.

5. Conclusion

In this study, the interaction between basketball teams had been modelled and the strategy for offensive team and defensive team had been determined. For the case study, the best interests for team Trail Blazers will always be shoot three-point shot all the time as long as team Rockets defend three-point is less than 94.4% instead of taking risk to overtime. However, defensive team is also clear about the fact. Therefore, the most common outcome for the last moment of game will be close and tight defend on three point while the offensive team will try to drag the game into overtime with higher field goal percentage. Without taking the others factors, the offensive team should choose a higher-percentage 2-point to drag the game into overtime, or try to get a foul on a 2-point shot and take a free throw to improve the game's win rate unless the shot percentage deviates significantly for example an open 3-point shot. For Situation 2, L.James has 41.96% to shoot three-point shot to drag the game into overtime. However, the Defensive teams seem to prefer framing the game into complex situations that could allow the offensive team to win the game outright. Due to the consistency of team free throw percentage, the defensive able to make three-point shot become the only choice for offensive team and focus their tight and close defend more at the three-point shot. Therefore, when the team is behind by three points, the offensive team has a smaller chance of winning. Even the team had the Most Value Player (MVP), their expected value is still much lower than the defensive team.

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