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Modelling One Dimensional Groundwater Flow in Confined and Homogeneous Aquifer

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Abstract
The purpose of this study is to investigate the application of a mathematical model to understand how hydraulic conductivity affects groundwater movement. Groundwater, a crucial underground resource, plays a vital role in maintaining surface water levels and supporting ecosystems. However, understanding groundwater flow is challenging due to factors like hydraulic gradient, hydraulic conductivity, and soil type. The research emphasizes the relationship between groundwater flow and hydraulic conductivity, where flow occurs from higher to lower hydraulic head, with velocity influenced by the hydraulic gradient and aquifer conductivity. Mathematical solutions and groundwater flow models are critical for groundwater management and remediation, despite the complexity caused by factors like hydraulic conductivity. The study aims to study hydraulic conductivity's influence on groundwater flow, develop a mathematical model using MODFLOW software for one-dimensional flow in confined aquifers, and utilize the finite difference approach to solve the flow equation. The study's significance lies in creating a hydrological model that accurately replicates groundwater behavior, aiding in improved understanding, management, and decision-making related to groundwater resources. Groundwater models serve as vital tools for forecasting and decision-making, facilitating quantitative groundwater investigations. By developing an accurate simulation of groundwater behavior, this research contributes to effective groundwater resource management.

Keywords groundwater flow; hydraulic conductivity; aquifer conductivity; MODFLOW software

1. Introduction

Groundwater, found underground, plays a vital role in replenishing surface water. Its movement is influenced by factors like hydraulic head, gradient, and conductivity. Hydraulic conductivity varies with soil properties and affects water transmission. Mathematical models are crucial for understanding groundwater flow, especially in varied soil types. This study aims to investigate hydraulic conductivity's impact on one-dimensional groundwater flow in confined aquifers using the finite difference method and MODFLOW software. The research focuses on developing an accurate hydrological model to forecast groundwater levels and soil water content, contributing to better resource management and decision-making.

2. Literature Review

It discusses various concepts related to groundwater flow and mathematical modeling in the field of hydrogeology. It begins by explaining the hydraulic gradient, which refers to the variation in hydraulic head along a groundwater flow path. The magnitude and direction of the hydraulic gradient, along with the aquifer's hydraulic conductivity, determine the speed of groundwater flow. Groundwater flow patterns often mimic those of surface waters, and the direction of groundwater flow can be determined by monitoring surface water drainage patterns. The placement and design of monitor wells are influenced by factors such as the suspected source of groundwater contamination.

Then introduces the basic principles and governing equations of groundwater flow, including Darcy's law and the groundwater flow equation. These equations describe the flow of groundwater through a porous medium and its changes over time.

Table 1 summarizes the main governing equations for groundwater flow.

Description of Process	**Governing Equations: Main Law**	**Parameter**
There are four equations usually used to describe water flow in groundwater **1.** Darcy's Law: related hydraulic gradient $\frac{\partial h}{\partial x}$ to discharge velocity	1. $U = -K\frac{\partial h}{\partial x} = -KI$	U: Darcy velocity K: Hydraulic conductivity
2. A 1D groundwater flow equation in homogeneous aquifer	2. $\frac{\partial}{\partial x}K_x\frac{\partial h}{\partial x} - S\frac{\partial h}{\partial t} + G$	S: Specific storage H: Head K_x: Hydraulic conductivity in x axes G: Source / sink term T: Transmissivity
3. The flow equation	3. $\frac{\partial}{\partial x}\left(T\frac{\partial h}{\partial x}\right) = Q + S_0\frac{\partial h}{\partial t}$	h(x,t): hydraulic head T(x): transmissivity S_0: storage coefficient Q: sink or source term t: time

Different numerical approaches, such as the finite difference method, finite element method, and finite volume method, are commonly used to solve these equations and simulate groundwater flow.

The MODFLOW software, developed by the US Geological Survey, is a widely used groundwater flow simulator based on the finite-difference method. It allows for the numerical resolution of the three-dimensional groundwater flow equation. However, it has limitations in modeling complex geology and discontinuous strata.

Mathematical modeling plays a crucial role in understanding and analyzing groundwater flow. It involves formulating partial differential equations and initial-boundary conditions to describe groundwater flow in aquifers. Hydraulic conductivity, also known as permeability, is a key factor in characterizing aquifers and determining the rate of water flow.

Overall, the text provides an overview of key concepts and methods used in the study of groundwater flow, including hydraulic gradient, governing equations, solution methods, mathematical modeling, and hydraulic conductivity.

3. Methodology

3.1 *Flow Equation*

The mathematical connection used to describe how groundwater moves through an aquifer in hydrogeology is known as the groundwater flow equation.
The formula use as follow

$$\frac{\partial}{\partial x}\left(T\frac{\partial h}{\partial x}\right) = Q + S_0\frac{\partial h}{\partial t} \quad (1)$$

subject to the following initial and boundary conditions:

$$h(x,0) = f_0(x), x \in \Omega \quad (2)$$

$$h(x,t) = f_0(x,t), x \in \partial\Omega_1 \quad (3)$$

$$T\frac{\partial h}{\partial n} = f_2(x,t), x \in \partial\Omega_2 \quad (4)$$

3.2 Finite Difference Method

The most general linear second order differential equation is in the form:

$$y''(x) + p(x)y'(x) + q(x)y'(x) + q(x)y(x) = r(x), a \leq x \leq b$$

We need to specify the value of the solution at two distinct points

$$y(a) = A \text{ and } y(b) = B$$

Thus, the step size, h, of each of the n subintervals is given by

$$h = \frac{b-a}{n}$$

3.2.1 *Finite Difference Approximate*

A finite difference approximation is an expression that uses the function at different points and gets close to an ordinary or partial derivative.
First derivative, forward FD:

$$f'(x) = \frac{f(x+h)-f(x)}{h} + O(h) \quad (5)$$

where O(h) is the limiting behavior of a function when the argument tends towards a particular value or infinity.

First derivative, backward FD:

$$f'(x) = \frac{f(x)-f(x-h)}{h} + O(h) \tag{6}$$

First derivative, central FD:

$$f'(x) = \frac{f(x+h)-f(x-h)}{2h} + O(h^2) \tag{7}$$

Second derivative, central FD:

$$f''(x) = \frac{f(x+h)-2f(x)+f(x-h)}{h^2} + O(h^2) \tag{8}$$

$y_1, y_2, \dots, y_{n-1}$ can be calculated as follow:

At any mesh point $x = x_i$ the finite-difference representation of the differential equation can be written as follows (based on central FD)

$$\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} + p_i \frac{y_{i+1} - y_{i-1}}{2h} q_i y_i = r_i, (i = 1,2, \dots, n-1)$$

$$2(y_{i+1} - 2y_i + y_{i-1}) + hp_i(y_{i+1} - y_{i-1}) + 2h^2 q_i y_i = 2h^2 r_i, (i = 1,2, \dots, n-1)$$

and arranging the equations with respect to $y_1 \cdots y_n$, we can obtain the system of linear equations.

$$(2 + hp_i)y_{i+1} - (2h^2 q_i - 4)y_i + (2 - hp_i)y_{i-1} = 2h^2 r_i, (i = 1,2, \dots, n-1)$$

The boundary conditions provide of the solution at the two ends of the grid: $y_0 = A$ and $y_n = B$. We can interpret y as a vector and write the equation formally as an algebraic matrix equation:

$$A_h Y_h = Rh$$

where

$$A = \begin{bmatrix} (2h^2q_1 - 4) & (2 + hp_1) & 0 & \cdots & \cdots & 0 \\ (2 - hp_2) & (2h^2q_2 - 4) & (2 + hp_2) & 0 & \cdots & 0 \\ 0 & (2 + hp_3) & (2h^2q_3 - 4) & (2 + hp_3) & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 \quad \cdots \quad 0 & (2 + hp_{n-2}) & & (2h^2q_{n-2} - 4) & (2 + hp_{n-2}) & \\ 0 \quad \cdots \quad \cdots & 0 & & (2 - hp_{n-1}) & (2h^2q_{n-1} - 4) & \end{bmatrix}$$

$$R = \begin{bmatrix} 2h^2r_1 - (2 + hp_1)A \\ 2h^2r_2 \\ 2h^2r_3 \\ \vdots \\ 2h^2r_{n-2} \\ 2h^2r_{n-1} - (2 + hp_{n-1})B \end{bmatrix}$$

$$y_h = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_{n-2} \\ y_{n-1} \end{bmatrix}$$

y_1 and y_{n-1} are the first and last element because the output $y[n]$ at every value of n depends only on the input $x[n]$ at the same value of n.

3.3 *The Mathematical Equations of MODFLOW*

The numerical model was created using the Processing MODFLOW software, computer programme uses central difference techniques. The key equation for groundwater flow in MODFLOW is:

$$\frac{\partial}{\partial x}\left[K_x \frac{\partial h}{\partial x}\right] = S\frac{\partial h}{\partial t}$$

where S is the specific storage, h is the head, K_x are the hydraulic conductivity in x directions. The basic parameters such as porosity, transmissivity and hydraulic conductivity can differ spatially during the simulation.

4 Result and Discussion

The process of creating an initial groundwater flow model using PMWIN involves several steps. First, a groundwater model is created, followed by selecting the grid size and defining the model's geometry and parameters such as porosity and hydraulic conductivity. MODFLOW is then used to run the flow simulation. The results can be viewed, water budgets can be determined for specific zones, and visual representations like head contours can be generated using PMWIN's modeling tools. To simulate transport procedures, finite difference transport models like MT3D or MOC3D can be used, and PMPATH can calculate and save path lines. An aquifer system with two stratigraphic units is considered, surrounded by no-flow boundaries to the north and south, and fixed-head boundaries in contact with rivers on the west and east sides. The hydraulic heads on the western and eastern boundaries are 9m and 8m above reference level, respectively. The aquifer system is isotropic, and the horizontal hydraulic conductivities of the stratigraphic units are specified. The surface area is effectively porous, and there is a contaminated area near the western boundary. The objective is to use a pumping well near the eastern boundary to isolate the contaminated area (as described in Figure 1).

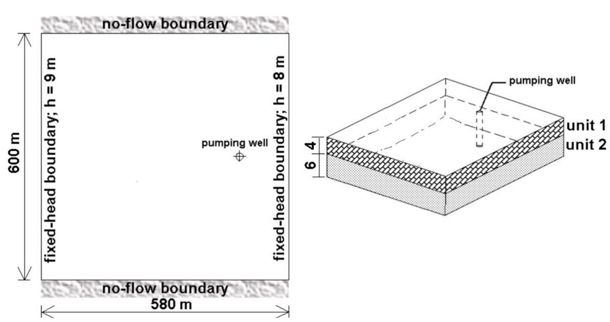


Figure 1 Configuration of the sample problem

Table 2 Parameters used in the model

Parameters	**Estimated Value**
Horizontal Hydraulic Conductivity	Layer 1= 0.0001 Layer 2= 0.0005 Layer 3= 0.0005
Vertical Hydraulic Conductivity	Layer 1= 0.00001, Layer 2= 0.00005 Layer 3= 0.00005
Constant Head	8 West 9 East
Effective Porosity	0.25
Recharge Rate	8E-9
Pumping Rate	Layer 1= -1E-10 Layer 2= -1E-10 Layer 3= -0.0012

Running a flow simulation in PMWIN involves creating a model grid, defining boundary conditions, and assigning model parameters using consistent units. Hydraulic conductivity is expressed in meters per second, pumping rates in cubic meters per second, and dispersivities in meters. Length is measured in meters, and time is measured in seconds.

The aquifer system in MODFLOW is represented by a mesh of cells and nodes where hydraulic heads are computed. The nodal grid serves as the framework for the numerical model, with one or more layers representing hydro stratigraphic units. The thickness of each cell and the width of columns and rows can vary. The location of cells is described using the index notation [J, I, K], where [2, 6, 1] refers to a cell in the second column, sixth row, and top layer.

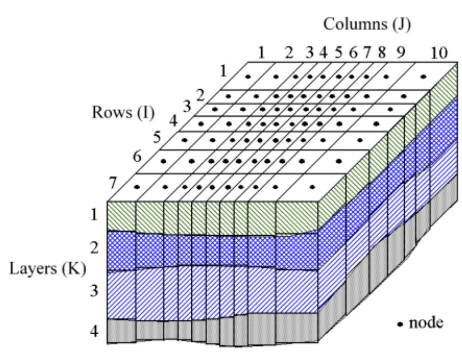


Figure 2 Spatial discretization of an aquifer system and the cell indices

For the model and each zone in each layer, the percent difference between the inflows and outflows is acceptable low. We repeat the process used to determine subregional water budgets in order to determine the precise flow rates to the well. This time, we only designate the cells [25, 15, 1], [25, 15, 2], and [25, 15, 3] as belonging to zones 1, 2, and 3, respectively. Zone 0 is assigned to all other cells. In Table 4.8.2, the water budget's output is displayed. The first layer is being extracted by the pumping well at a rate of 7.7992809E-05 m3/s, the second layer at a rate of 5.603538E-04 m3/s, and the third layer at a rate of 5.5766129E-04 m3/s. As would be expected given the aquifer's configuration, almost all of the water extracted originates from the second stratigraphic unit.

```
PMWBLF (SUBREGIONAL WATER BUDGET) RUN RECORD
FLOWS ARE CONSIDERED "IN" IF THEY ARE ENTERING A SUBREGION
THE UNIT OF THE FLOWS IS [L^3/T]

TIME STEP    1 OF STRESS PERIOD    1

=============================================================
 WATER BUDGET OF ZONES WITHIN EACH INDIVIDUAL LAYER
=============================================================

 ZONE   1  IN LAYER   1
----------------------------------------------
       FLOW TERM             IN             OUT          IN-OUT
         STORAGE  0.0000000E+00  0.0000000E+00  0.0000000E+00
   CONSTANT HEAD  0.0000000E+00  3.2555645E-03 -3.2555645E-03
 HORIZ. EXCHANGE  0.0000000E+00  0.0000000E+00  0.0000000E+00
EXCHANGE (UPPER)  0.0000000E+00  0.0000000E+00  0.0000000E+00
EXCHANGE (LOWER)  0.0000000E+00  1.4793761E-01 -1.4793761E-01
           WELLS  0.0000000E+00  1.0000000E-10 -1.0000000E-10
          DRAINS  0.0000000E+00  0.0000000E+00  0.0000000E+00
        RECHARGE  1.5120019E-01  0.0000000E+00  1.5120019E-01
              ET  0.0000000E+00  0.0000000E+00  0.0000000E+00
   RIVER LEAKAGE  0.0000000E+00  0.0000000E+00  0.0000000E+00
 HEAD DEP BOUNDS  0.0000000E+00  0.0000000E+00  0.0000000E+00
  STREAM LEAKAGE  0.0000000E+00  0.0000000E+00  0.0000000E+00
INTERBED STORAGE  0.0000000E+00  0.0000000E+00  0.0000000E+00
MULTI-AQIFR WELL  0.0000000E+00  0.0000000E+00  0.0000000E+00
SUM OF THE LAYER  1.5120019E-01  1.5119317E-01  7.0184469E-06
 DISCREPANCY [%]  0.00

 ZONE   1  DOES NOT EXIST IN LAYER   2
 ZONE   1  DOES NOT EXIST IN LAYER   3
--------------------------------------------------------------
```

Figure 3 Output from the Water Budget Calculator for Zone 1 in Layer 1

```
 ZONE   2  DOES NOT EXIST IN LAYER   1

 ZONE   2  IN LAYER   2
----------------------------------------------
       FLOW TERM             IN             OUT          IN-OUT
         STORAGE  0.0000000E+00  0.0000000E+00  0.0000000E+00
   CONSTANT HEAD  0.0000000E+00  7.3375754E-02 -7.3375754E-02
 HORIZ. EXCHANGE  0.0000000E+00  0.0000000E+00  0.0000000E+00
EXCHANGE (UPPER)  1.4793761E-01  0.0000000E+00  1.4793761E-01
EXCHANGE (LOWER)  0.0000000E+00  7.4462354E-02 -7.4462354E-02
           WELLS  0.0000000E+00  1.0000000E-10 -1.0000000E-10
          DRAINS  0.0000000E+00  0.0000000E+00  0.0000000E+00
        RECHARGE  0.0000000E+00  0.0000000E+00  0.0000000E+00
              ET  0.0000000E+00  0.0000000E+00  0.0000000E+00
   RIVER LEAKAGE  0.0000000E+00  0.0000000E+00  0.0000000E+00
 HEAD DEP BOUNDS  0.0000000E+00  0.0000000E+00  0.0000000E+00
  STREAM LEAKAGE  0.0000000E+00  0.0000000E+00  0.0000000E+00
INTERBED STORAGE  0.0000000E+00  0.0000000E+00  0.0000000E+00
MULTI-AQIFR WELL  0.0000000E+00  0.0000000E+00  0.0000000E+00
SUM OF THE LAYER  1.4793761E-01  1.4783812E-01  9.9495053E-05
 DISCREPANCY [%]  0.07

 ZONE   2  DOES NOT EXIST IN LAYER   3
--------------------------------------------------------------
```

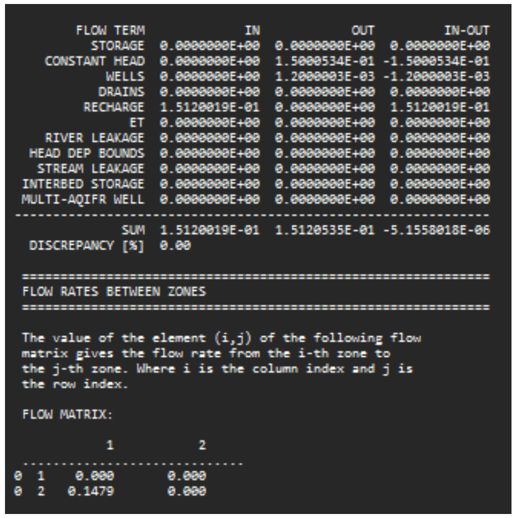
Figure 4 Output from the Water Budget Calculator for Zone 2 in Layer 2

```
=============================================================
WATER BUDGET OF ZONES OVER THE ENTIRE MODEL
=============================================================

ZONE: 1
----------------------------------------------
                             IN             OUT          IN-OUT
         STORAGE  0.0000000E+00  0.0000000E+00  0.0000000E+00
   CONSTANT HEAD  0.0000000E+00  3.2555645E-03 -3.2555645E-03
 HORIZ. EXCHANGE  0.0000000E+00  0.0000000E+00  0.0000000E+00
EXCHANGE (UPPER)  0.0000000E+00  0.0000000E+00  0.0000000E+00
EXCHANGE (LOWER)  0.0000000E+00  1.4793761E-01 -1.4793761E-01
           WELLS  0.0000000E+00  1.0000000E-10 -1.0000000E-10
          DRAINS  0.0000000E+00  0.0000000E+00  0.0000000E+00
        RECHARGE  1.5120019E-01  0.0000000E+00  1.5120019E-01
              ET  0.0000000E+00  0.0000000E+00  0.0000000E+00
   RIVER LEAKAGE  0.0000000E+00  0.0000000E+00  0.0000000E+00
 HEAD DEP BOUNDS  0.0000000E+00  0.0000000E+00  0.0000000E+00
  STREAM LEAKAGE  0.0000000E+00  0.0000000E+00  0.0000000E+00
INTERBED STORAGE  0.0000000E+00  0.0000000E+00  0.0000000E+00
MULTI-AQIFR WELL  0.0000000E+00  0.0000000E+00  0.0000000E+00
SUM OF ZONE(  1)  1.5120019E-01  1.5119317E-01  7.0184469E-06
 DISCREPANCY [%]  0.00

ZONE: 2
----------------------------------------------
```

Figure 5 Output from the Water Budget Calculator for Zone 1

```
                             IN             OUT          IN-OUT
         STORAGE  0.0000000E+00  0.0000000E+00  0.0000000E+00
   CONSTANT HEAD  0.0000000E+00  7.3375754E-02 -7.3375754E-02
 HORIZ. EXCHANGE  0.0000000E+00  0.0000000E+00  0.0000000E+00
EXCHANGE (UPPER)  1.4793761E-01  0.0000000E+00  1.4793761E-01
EXCHANGE (LOWER)  0.0000000E+00  7.4462354E-02 -7.4462354E-02
           WELLS  0.0000000E+00  1.0000000E-10 -1.0000000E-10
          DRAINS  0.0000000E+00  0.0000000E+00  0.0000000E+00
        RECHARGE  0.0000000E+00  0.0000000E+00  0.0000000E+00
              ET  0.0000000E+00  0.0000000E+00  0.0000000E+00
   RIVER LEAKAGE  0.0000000E+00  0.0000000E+00  0.0000000E+00
 HEAD DEP BOUNDS  0.0000000E+00  0.0000000E+00  0.0000000E+00
  STREAM LEAKAGE  0.0000000E+00  0.0000000E+00  0.0000000E+00
INTERBED STORAGE  0.0000000E+00  0.0000000E+00  0.0000000E+00
MULTI-AQIFR WELL  0.0000000E+00  0.0000000E+00  0.0000000E+00
SUM OF ZONE(  2)  1.4793761E-01  1.4783812E-01  9.9495053E-05
 DISCREPANCY [%]  0.07


=============================================================
WATER BUDGET OF THE WHOLE MODEL DOMAIN:
=============================================================
```

Figure 6 Output from the Water Budget Calculator for Zone 2



```
         FLOW TERM            IN            OUT         IN-OUT
           STORAGE  0.0000000E+00  0.0000000E+00  0.0000000E+00
     CONSTANT HEAD  0.0000000E+00  1.5000534E-01 -1.5000534E-01
             WELLS  0.0000000E+00  1.2000003E-03 -1.2000003E-03
            DRAINS  0.0000000E+00  0.0000000E+00  0.0000000E+00
          RECHARGE  1.5120019E-01  0.0000000E+00  1.5120019E-01
                ET  0.0000000E+00  0.0000000E+00  0.0000000E+00
     RIVER LEAKAGE  0.0000000E+00  0.0000000E+00  0.0000000E+00
   HEAD DEP BOUNDS  0.0000000E+00  0.0000000E+00  0.0000000E+00
    STREAM LEAKAGE  0.0000000E+00  0.0000000E+00  0.0000000E+00
  INTERBED STORAGE  0.0000000E+00  0.0000000E+00  0.0000000E+00
 MULTI-AQIFR WELL  0.0000000E+00  0.0000000E+00  0.0000000E+00
-------------------------------------------------------------------
               SUM  1.5120019E-01  1.5120535E-01 -5.1558018E-06
   DISCREPANCY [%]  0.00

 ==================================================================
 FLOW RATES BETWEEN ZONES
 ==================================================================

 The value of the element (i,j) of the following flow
 matrix gives the flow rate from the i-th zone to
 the j-th zone. Where i is the column index and j is
 the row index.

 FLOW MATRIX:

            1           2
 ----------------------------
0  1    0.000       0.000
0  2   0.1479       0.000
```

Figure 7 Output from the Water Budget Calculator for the whole model domain

Contour maps can be created using input data, simulation results, or ASCII Matrix files by following the described procedure. They can display starting heads, concentration distribution, or fields generated by the Field Interpolator or Field Generator.

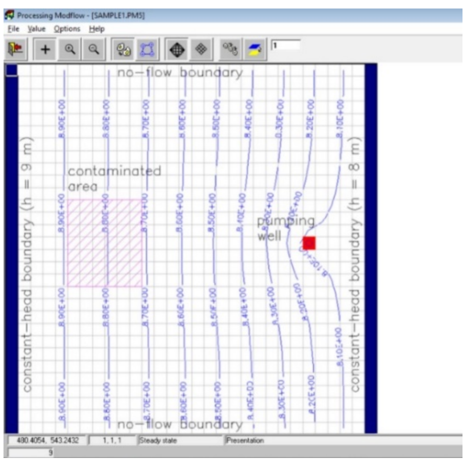


Figure 8 A contour map of the hydraulic in the first layer

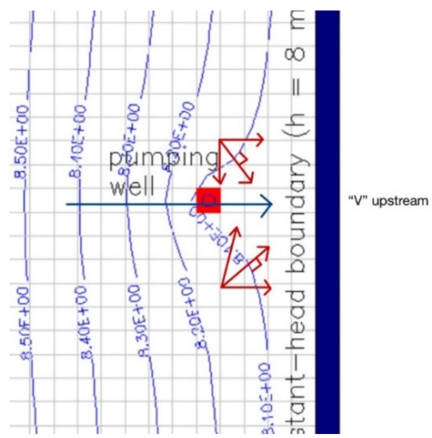


Figure 9 A contour map of the hydraulic heads with the perpendicular line

If we look at some contours that cross streams where the stream is flowing in the right direction in this case. what we can do is infer that the flow is in that direction, it is perpendicular to the contours, and it goes from high to low, that indicates that there is a component of flow in that direction and a component in that direction, so there is a component of the groundwater flow in the direction that the stream is flowing. there's a component towards the stream and similarly on that side we have flow vectors with components like that. So, this kind of the V-pattern where the groundwater or the hydraulic head contours are crossing the stream and the contours are pointing upstream that indicates that there's groundwater flow towards the stream and that means that the stream is gaining.

The vertical scale exaggeration in Cross Sections can be increased using Environment Options to enhance the detail of pathline projections. Figure 5 displays pathlines with a vertical exaggeration of 10. In two-dimensional models like ASM for Windows, FINEM, or MOC, there is no vertical velocity term, making it impossible to trace pathlines back to the groundwater surface. PMPATH can be used to create pumping well capture zones. Figure 6 shows a 100-day capture zone, with the first layer having a smaller zone due to lower hydraulic conductivity and flow velocity.

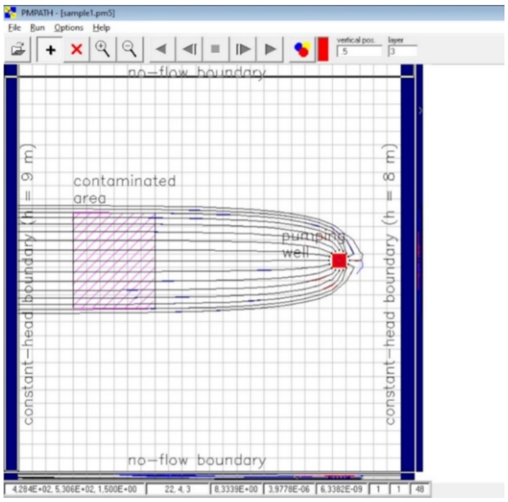


Figure 10 The capture zone of the pumping well (with vertical exaggeration=1)

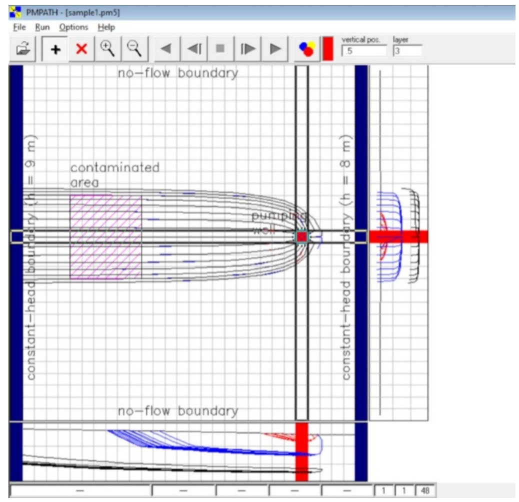


Figure 11 The capture zone of the pumping well (with the vertical exaggeration=10)

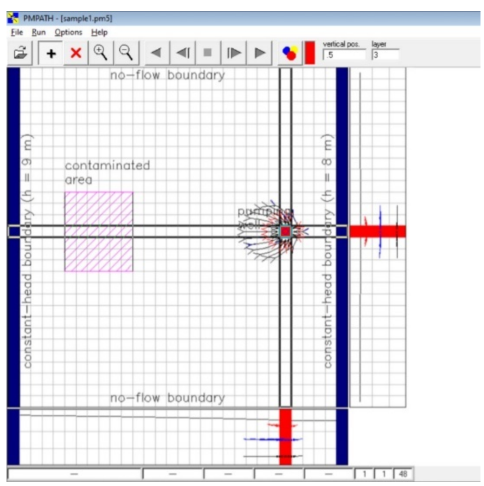


Figure 12 100-days-capture zone calculated by PMPATH

The developed one-dimensional groundwater flow model using PMWIN proved to be a useful tool for simulating flow behavior in the confined and homogeneous aquifer. The model's performance, calibration, sensitivity analysis, and practical implications highlight its potential for informing groundwater management strategies. However, further research is recommended to address the model's limitations and incorporate additional complexities for improved accuracy.

Conclusion

The study covers three comprehensive chapters that provide an overview of groundwater flow in confined and homogeneous aquifers. Chapter 1 introduces the importance of hydraulic conductivity and its impact on groundwater flow, addressing the relationship between surface water drainage, pumping wells, and changes in surface water levels. Chapter 2 discusses the concepts of one-dimensional groundwater flow, numerical techniques, and the introduction of the MODFLOW software. Chapter 3 focuses on the finite difference method and its application in solving groundwater flow equations, incorporating boundary conditions and highlighting the MODFLOW software. Chapter 4 presents analytical solutions for groundwater flow in confined aquifers, specifically addressing leakage and distributed recharge. Overall, these chapters contribute to our understanding of groundwater flow mechanisms, modeling approaches, and analytical solutions, providing valuable insights for studying and managing groundwater resources in confined and homogeneous aquifers.

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