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Particle Swarm Optimization Approach for the Portfolio Optimization Problem

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Abstract
Portfolio optimization involves the allocation of resources among a set of available investment options to achieve a desired objective while considering various constraints. Traditional optimization methods often face challenges in handling the complex nature of portfolio optimization, including high dimensionality, non-linearity, and diverse objectives. In this study, we propose a PSO-based approach that utilizes the collective intelligence of a population of particles to efficiently search for optimal portfolios. The PSO algorithm simulates the behaviour of a swarm of particles, where each particle represents a potential solution (portfolio) and moves through the search space to find the best possible combination of investments. The particles dynamically adjust their positions and velocities based on their own experience and the collective knowledge of the swarm. To address the portfolio optimization problem, we define appropriate fitness functions that consider risk, return, and other relevant objectives. The PSO algorithm is then applied to optimize the portfolio weights, aiming to maximize returns while minimizing risk. Overall, the results and insights presented in this study have significant implications for investment management, enabling investors to make informed decisions in constructing optimal portfolios that balance risk and return while considering various constraints and objectives.

Keywords: Portfolio Optimization; Particle Swarm Optimization; Investment

1. Introduction

Portfolio optimization is the process of selecting the best portfolio (asset distribution), out of the set of all portfolios being considered, according to some objective. The objective typically maximizes factors such as expected return, and minimizes costs like financial risk. Factors being considered may range from tangible such as assets, liabilities, earnings or other fundamentals to intangible such as selective divestment. The concept of portfolio optimization has been an important tool in the development and understanding of financial markets. Portfolio optimization techniques can assist the search for the portfolio that best suits each investor's particular objective. As stated by the BusinessWeek, the single best weapon against risk is to form portfolios with uncorrelated or negatively correlated assets because when several such assets are combined together, the overall risk of the portfolio may be less than that of the individual asset [1]. Thus, finding a suitable combination of investments attracted attentions of investors and scholars.

The major breakthrough of portfolio optimization came in 1952 with the publication of Markowitz's theory of portfolio selection [2]. Markowitz quantified return and risk of a security using statistical measures of its expected return and standard deviation. Markowitz suggested that investors should consider return and risk together and determine the allocation of funds among investment alternatives on the basis of their return-risk trade-off. This theory is popularly referred to as the modern portfolio theory and it is also the theoretical basis for this work.

2. Algorithm for Portfolio Optimization

Chang, Yang [3] propose using GA to solve portfolio optimization problems with cardinality constraints efficiently. It involves generating an initial population of random solutions which are the chromosomes representing portfolio weights, evaluating their fitness using an objective function and apply selection, crossover, and mutation operations to evolve the population towards a better solution. According to the research done by Wihartiko, Wijayanti, the comparison of Particle Swarm Optimization (PSO) and Genetic Algorithm (GA) in solving the portfolio optimization problem shows that PSO is faster in terms of reaching the optimal solution and has a 100% accuracy rate while GA has a 0.17% probability of obtaining an optimal solution with an average accuracy of 99% [4].

Crama and Schyns presented the application of a simulated annealing approach to the solution of a complex portfolio selection model [5]. The model is a mixed integer quadratic programming problem which arises when Markowitz' classical mean-variance model is enriched with additional realistic constraints. From research done by Bagaram, the results showed that PSO slightly outperformed SA, but the discrepancy may be attributed to the specific implementation of SA with a high searching space or stopping criteria [6].

Tabu search (TS) is a metaheuristic that guides a local heuristic search procedure to explore the solution space beyond local optimality. Tabu search is based on the premise that problem solving, to qualify as intelligent, must incorporate adaptive memory and responsive exploration [7]. Aldaihani and Aldeehani introduces a tailored tabu search heuristic algorithm to address the portfolio optimization problem in emerging stock markets [8]. The objective is to find a balance between risk and return in constructing a stock portfolio. Baskar, Asokan [9] conducted research and the results indicate that the PSO algorithm always yields the best results when compared to the other methods. The study suggests that PSO should be used to solve the optimization problem of the milling operations, as it consistently yields better results than the other methods.

In studying the behaviour of social animals with the artificial life theory, for how to construct the swarm artificial life systems with cooperative behaviour by computer, Millonas [10] proposed five basic principles: proximity, quality, diverse response, stability, adaptability. Zhu, Wang [11] proposed research discussing the importance of diversification in financial investments and focuses on solving the portfolio optimization problem in investment management. It introduces PSO method as a meta-heuristic algorithm for optimizing investment portfolios based on the Markowitz and Sharpe Ration models. After many years development, the optimization speed, quality and robustness of the PSO algorithm have been greatly improved. However, the current studies mostly focus on the algorithm's implementation, enhancement and applications, while relevant fundamental research is far behind the algorithm's development. Lacking of mathematical theory basis greatly limits the further generalization, improvement and application of the PSO algorithm [12].

2.1 Particle Swarm Optimization

The swarm in PSO consists of a population and each member of the population is called a particle, which represents a portfolio. When solving a portfolio optimization problem, a PSO heuristic method is introduced, which is one of the latest evolutionary optimization methods and is based on the metaphor of social interaction and communication such as bird flocking and fish schooling. According to Kennedy et al. [13], each particle remembers its best previous position and the best previous position visited by any particle in the whole swarm. In other words, a particle moves towards its best previous position and towards the best particle. In the algorithm of PSO, each solution is represented by a particle in the search space. Each particle has its position, velocity, and fitness value. At each iteration, every particle moves towards its personal best position and towards the best particle of the swarm found so far. A particle swarm's movement patterns are strongly influenced by the inertia weight and acceleration coefficients [14]. The particles roaming in high dimensions can be prevented by appropriate choice of acceleration coefficients which are c_1 , c_2 and inertia weight which is w. Trelea [15] categorized the movement patters into four groups: non-oscillatory, harmonic, zigzagging and harmonic-zigzagging.

2.2 Cardinality Constrained Mean-Variance (CCMV) model

This study basically employs the Markowitz mean–variance model. However, the standard model does not contain any cardinality or bounding constraints, which restrict the number of assets and, the upper and the lower bounds of proportion of each asset in the portfolio, respectively. Hence, the modified Markowitz model called a "cardinality constrained mean-variance (CCMV) model" is used and is solved by a PSO approach. In order to observe the different objective function values for varying R^* values, standard practice introduces a risk aversion parameter λ ϵ [0,1]. With this new parameter, the model can be described as:

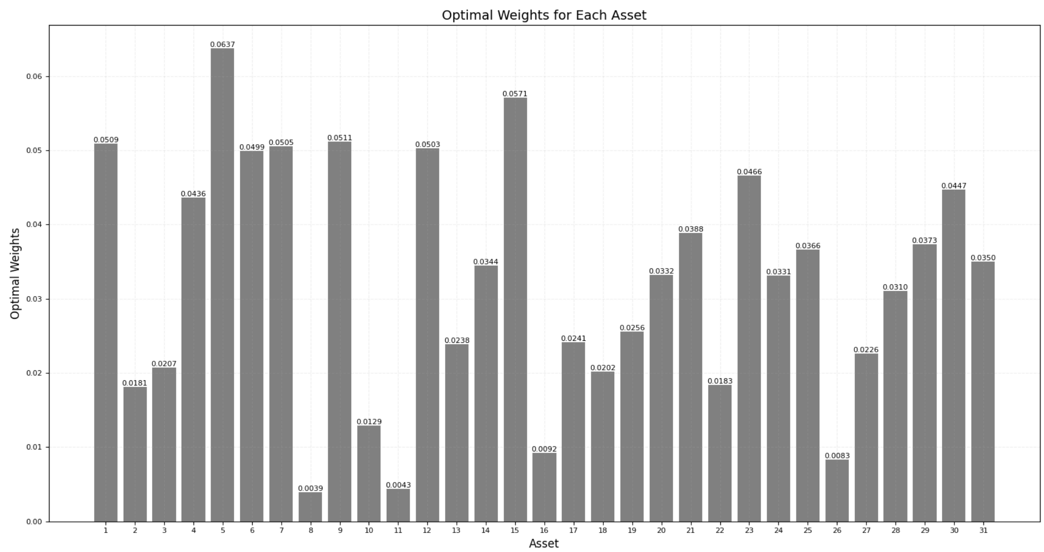
$$\min \lambda \sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \sigma_{ij} - (1-\lambda)\left[\sum_{i=1}^{N} x_i \mu_i\right]$$

$$subject\ to \sum_{i=1}^{N} x_i = 1\ ,$$

$$0 \le x_i \le 1, \qquad i = 1, \dots, N$$

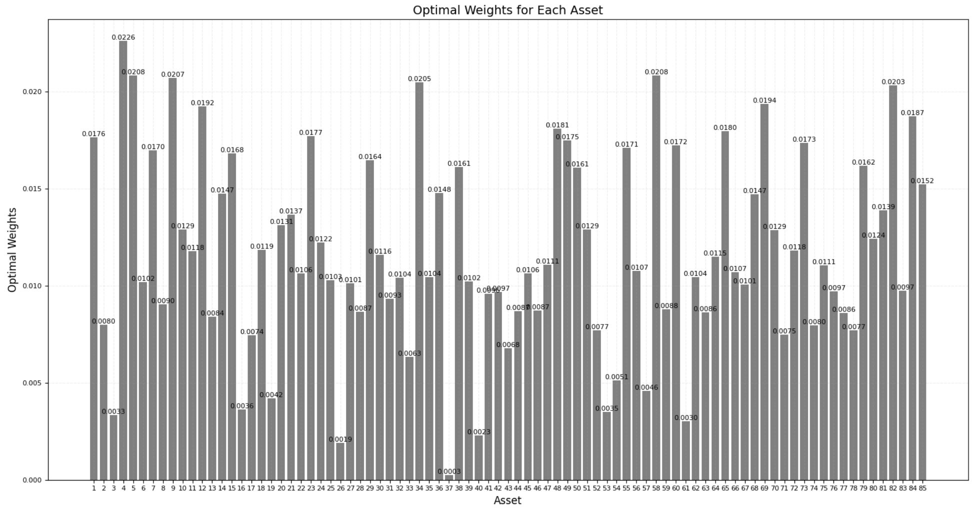
2.3 Computational Results

The PSO approach of this research is used to determine the optimal weight of each asset. The test data correspond to the weekly prices between March 1992 and September 1997 from the indices: Hang Seng in Hong Kong and DAX 100 in Germany. Python coding is used to solve the algorithm and the coding is shown in the appendix. The following results have been computed using the values $\lambda = 0.02$, swarm size = 500. We initialize random positions for the particles in the swarm and also random velocities for the particles in the swarm.



The optimal weights for each assets presented in the form of bar chart to make the comparism between each assets visually easier. The optimal weight to be invested for each asset provides a detailed allocation strategy for constructing an investment portfolio. In this particular case, the optimal weights are represented by a list of values ranging from 0.0039 to 0.0637. Each value corresponds to the proportion of the portfolio's total value that should be allocated to a specific asset. Based on the obtained optimal weights, we examine a few assets and potential reasons for investing in them. For

Asset 5, having the highest weight of 0.0637 could be a high-growth asset with the potential for substantial returns. The underlying factors might suggest strong growth prospects, making it a prominent component of the portfolio. The relatively low weight assigned to Asset 8 with the weight of 0.0039 suggests a limited allocation in the portfolio. This could be due to factors such as uncertainty in its performance, or potential risks associated with the asset. The low weight may indicate that it is considered a higher-risk investment.



Based on the obtained optimal weights, we examine a few assets and potential reasons for investing in them. For Asset 4 which has relatively higher weight, suggesting a significant allocation in the portfolio. This could be because Asset 4 exhibits strong historical performance, growth potential, or low correlation with other assets. It might be considered a strategic investment to enhance portfolio diversification or capture specific market opportunities. While Asset 37 has a very low weight, indicating a minimal allocation in the portfolio. This could be attributed to factors like high risk, lack of historical performance data, or uncertainty surrounding the asset. It might be considered a speculative investment or an asset with limited market liquidity.

To determine these optimal weights, a computational optimization process is employed. The process takes into account various factors and considerations. The computational algorithm used explores different combinations of asset weights to find the optimal solution. The algorithm evaluates the objective function, which aims to maximize returns or minimize risk. The resulting optimal weights represents the recommended allocation of investment across the different assets in the portfolio. Each weight indicates the proportion of the portfolio's total value that should be invested in a specific asset. By following these optimal weights, investors can create a well-diversified portfolio that seeks to maximize returns while managing risk.

Conclusion

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