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Dynamical Analysis of a Predator-Prey Model with Harvested and Infected Prey Species

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Abstract
Ecological systems commonly exhibit inconsistencies in population density, particular species rates of growth, harvesting rates, and predatory rates, along with other shifting biological variables. Accordingly, the development of an eco-epidemiological system that takes prey harvesting and disease in prey species is proposed and a analysis about of the system is carried out in this study. In constructing the model, hypotheses are put forward, and the Lotka-Volterra framework is subsequently altered to reflect the assumptions generated. An in-depth analysis of stability at equilibrium points is conducted applying the Jacobian matrix and Eigenvalues. The time series graphs of different scenarios are plotted, and phase portraits of the predator-prey model are then analyzed using the MATLAB software. The graphical analysis has shown that the effect of infection parameter gives a negligible effect, meanwhile the harvesting parameter generates a severe impact to the system. The overall results obtained show that uncontrolled harvesting activities with the presence of diseases in the ecosystem will result to the extinction of both populations as times elapsed.
Keywords: Predator; prey; harvesting; eigenvalue; stability.

Introduction

Bio-economic modelling has been widely applied and developed by a significant number of researchers and academicians throughout these decades. This approach employs the application of mathematics to portray the behaviors of complex biological processes and systems, including the fields of aquaculture, fisheries, and forestry, influenced by natural, ecological, economic, and conceptual variables (Llorente & Luna, 2015). Moussaoui and Auger (2021) assessed the stabilizing impact of marine reserves on fisheries dynamics using a bio-economic model of the fishery. In general, the dynamic mathematical models of harvested natural resources that have been applied in this study resemble the predominant Lotka-Volterra predator-prey models. This illustrates the significance of mathematical modelling and analysis in examining biological scenarios with numerous and fluctuating variables.

A mathematical formulation for the dynamics of a predator and prey population was originally devised by Alfred Lotka and Vito Volterra individually in the 1920s, and the Lotka-Volterra predator-prey model has since evolved into a prominent and defining model of mathematical biology (Cherniha & Davydovych, 2022). In succeeding years, the predator-prey model has revolved among the eco-epidemiology field of study and this problem has been the focus of much attention for many scientists' and scholars' discussion and studies. It was initially analyzed by Anderson and May in 1986 with the objective of evaluating the dynamics of this prey-predator model where the predator species came into contact with an infected prey population. It was subsequently improved and developed by a theoretical approach into multiple sorts of prey-predator models, including diseased predator species as well as infected populations for both predator and prey populations, all of which are still relevant and utilized in diverse research studies thus far.

Unpredictable and varying parameters caused by natural phenomena in ecological system should be taken into account, since these can become the biggest impediment and concurrently result in inaccuracies in an effort to propose and structure a mathematical model. The existence of toxins and various sources of infection can cause the ecosystem to suffer disease, which surely affects the predator-prey model. When the epidemiological and demographic components of a disease are combined into a single model, eco-epidemic research analyses how it spreads throughout interacting populations (Shaikh et al., 2017). The Susceptible-Infective-Removed (SIR) model, first developed by Kermack-McKendrick in 1927, illustrates how a disease develops and spreads through contact (Carvalho & Gonçalves, 2021). Since the prey and predator populations come into contact with each other in the ecosystem, disease in the prey species are predominantly transmitted to the predator species and the epidemics are anticipated to affect the predator species concurrently.

In particular, numerous species in the ecological system are indeed being mined and harvested, and this scenario cannot be disregarded since it results to several impacts on the predator-prey paradigm. The model of harvested predator-prey has also gained quite a lot of attention by researchers in this twentieth century. Mahata et al. (2021) conducted a study to examine the impact of harvesting effort disruption on the stable state of a collection of prey-predator models. They determine whether the disruption to the model is caused by selectively harvesting both predators and prey or by solely harvesting prey by examining several settings. A study by Ang et al. (2018) investigated the toxin emitted by both predator and prey species, and a prey-predator system of a fisheries model impacted by harvesting efforts has also been proposed. The study has concluded that, in comparison to the poison emitted, the effects of harvesting on the dynamical behaviors of the system are more visible and significant. The toxin parameters didn't seem to be as important or impactful as the harvesting settings. These highlighted that the harvesting efforts in both species are one of the vital variables to be taken into account in our mathematical model.

It takes a lot of work and time to conduct a research study with several varying parameters, especially when trying to study natural settings and occurrences. Nevertheless, with the advancement and improvement of mathematical analysis and its application in biological sciences, it has now become practically viable to explore these scenarios by making a few amendments to the conventional systems that have been developed and made available through prior studies based on the intended scenario. Considering that ecology has a significant beneficial impact on human essential nature as they provide natural sources, a study is required to analyze scenarios related so that the food chain in our ecosystem is not affected by extinction and further unwanted repercussions on natural resources can be avoided before they cause significant harm to the community.

Model Formulation

The model of predator-prey with diseased and harvested prey population follows the transfer diagram of eco-epidemiological model as follows:

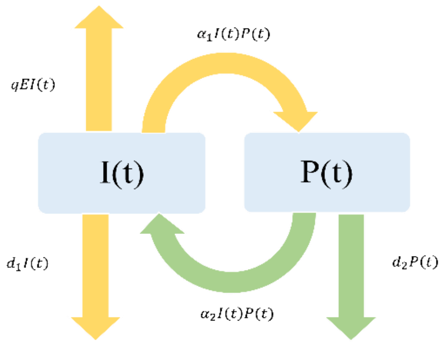

Figure 1 Transfer diagram of eco-epidemiological model for predator-prey model.

In this study, we address an eco-epidemiological prey-predator system incorporating infectious disease and harvested in prey population in the proposed framework. We presumed that there would be a predator and a mechanism to protect against predation. In this model, $I(t)$ represents the infected prey population density at time t with harvesting efforts is taken into consideration, while $P(t)$ represents the predator population density with respect to time t.

Furthermore, some of the following postulates and considerations are made in order to develop our eco-epidemiological model:

I. Infected prey population, $I(t)$ obeys extrinsic growth with death rate of d_1 with the assumption of the infected prey do not recover or become immune from the disease.

II. The death rate of predator population, d_2 is considered, where the death rate of the predator contains natural mortality due to infection from the infected prey, and

III. The infected prey species is harvested by a linear rate.

Based on above postulates, the following predator-prey model structure can be written as follows:

$$\boldsymbol{\frac{dI(t)}{dt} = A - \alpha_1 IP - d_1 I - qEI,}$$
$$\boldsymbol{\frac{dP(t)}{dt} = \beta + \alpha_2 IP - d_2 P} \tag{1}$$

$$I(0) > 0, \quad P(0) > 0, \quad t > 0$$

where

A : The constant recruitment rate of prey species.
β : The constant recruitment of predator species.
α_1 : Prey-predator interaction parameter of the prey species.
α_2 : Prey-predator interaction parameter of the predator species.
d_1 : Death rate of infected prey species.
d_2 : Death rate of predator species.
q : Catchability coefficient of infected prey species.
E : The harvesting effort.

Stability analysis

We examine and analyze the stability and equilibrium points of the system. One of the efficient approaches to determine the stability is by applying the Jacobian matrix, where the system is generalized to:

$$J = \begin{bmatrix} \frac{\partial F_1}{\partial I} & \frac{\partial F_1}{\partial P} \\ \frac{\partial F_2}{\partial I} & \frac{\partial F_2}{\partial P} \end{bmatrix},$$

where F_1 and F_2 were taken from system (1), such that

$$\boldsymbol{F_1 = A - \alpha_1 IP - d_1 I - qEI,}$$

$$\boldsymbol{F_2 = \beta + \alpha_2 IP - d_2 P,}$$

and the equilibrium points is obtained by solving both equations from system (1):

$$\frac{dI(t)}{dt} = 0 \text{ and } \frac{dP(t)}{dt} = 0.$$

Then, by using eigenvalue method, characteristic equations are obtained for the respective equilibrium points by substituting:

$$|J - \lambda I| = 0,$$

where λ is the eigenvalues.

The value of λ_i are calculated, and the stability for each equilibrium point is determined by referring to the following table of stability analysis.

Table 1 Stability Properties of Linear and Locally Linear Systems

Stability Properties of Linear Systems		
Eigenvalues	Type of Critical Point	Stability
$\boldsymbol{\lambda_1 > \lambda_2 > 0}$	Node	Unstable
$\boldsymbol{\lambda_1 < \lambda_2 < 0}$	Node	Asymptotically stable
$\boldsymbol{\lambda_1 > 0 > \lambda_2}$	Saddle point	Unstable
$\boldsymbol{\lambda_1 = \lambda_2 > 0}$	Proper or improper node	Unstable
$\boldsymbol{\lambda_1 = \lambda_2 < 0}$	Proper or improper node	Asymptotically stable
$\boldsymbol{\lambda_1, \lambda_2 = \mu + i\sigma}$		
$\boldsymbol{\mu > 0}$	Spiral point	Unstable
$\boldsymbol{\mu < 0}$	Spiral point	Asymptotically stable
$\boldsymbol{\mu = 0}$	Centre	Stable

Therefore, the equilibrium points obtained from the predator-prey model of system (1) are

$$I = \frac{A}{\boldsymbol{\alpha_1} P + d_1 + qE},$$

$$P = \frac{\boldsymbol{\alpha_1}\beta + \boldsymbol{\alpha_2}A - d_1 d_2 - d_2 qE \pm \sqrt{(\boldsymbol{\alpha_1}\beta + \boldsymbol{\alpha_2}A - d_1 d_2 - d_2 qE)^2 + 4(\boldsymbol{\alpha_1} d_2)(d_1\beta + \beta qE)}}{2\boldsymbol{\alpha_1} d_2}$$

Therefore, the equilibrium points $\mathrm{P_1}$ obtained from the model is

$$\mathrm{P_1} = \left(\frac{\mathrm{A}}{\boldsymbol{\alpha_1}\mathrm{m_1} + \mathrm{d_1} + \mathrm{qE}}, \mathrm{m_1}\right),$$

where

$$\mathrm{m_1} = \frac{\boldsymbol{\alpha_1}\beta + \boldsymbol{\alpha_2}\mathrm{A} - \mathrm{d_1 d_2} - \mathrm{d_2 qE} + \sqrt{(\boldsymbol{\alpha_1}\beta + \boldsymbol{\alpha_2}\mathrm{A} - \mathrm{d_1 d_2} - \mathrm{d_2 qE})^2 + 4(\boldsymbol{\alpha_1}\mathrm{d_2})(\mathrm{d_1}\beta + \beta\mathrm{qE})}}{2\boldsymbol{\alpha_1}\mathrm{d_2}}.$$

The equilibrium point $\mathrm{P_2}$ is given by

$$\mathrm{P_2} = \left(\frac{\mathrm{A}}{\boldsymbol{\alpha_1}\mathrm{m_2} + \mathrm{d_1} + \mathrm{qE}}, \mathrm{m_2}\right),$$

where

$$\mathrm{m_2} = \frac{\boldsymbol{\alpha_1}\beta + \boldsymbol{\alpha_2}\mathrm{A} - \mathrm{d_1 d_2} - \mathrm{d_2 qE} - \sqrt{(\boldsymbol{\alpha_1}\beta + \boldsymbol{\alpha_2}\mathrm{A} - \mathrm{d_1 d_2} - \mathrm{d_2 qE})^2 + 4(\boldsymbol{\alpha_1}\mathrm{d_2})(\mathrm{d_1}\beta + \beta\mathrm{qE})}}{2\boldsymbol{\alpha_1}\mathrm{d_2}}.$$

This system possess two coexistence equilibrium points, and both points are taken into consideration for the theoretical computation. After substituting possible values of all parameters in the next section of numerical simulation, only the positive equilibria where both species exist is referred to be the equilibrium point of the system. Hence, by using eigenvalue method, characteristic equations are obtained for the respective equilibrium points by substituting:

$$|\mathrm{J} - \lambda \mathrm{I}| = 0,$$

where the λ is the eigenvalues. Therefore,

$$\mathrm{J}_1 = \begin{bmatrix} -\alpha_1 \mathrm{m}_1 - \mathrm{d}_1 - \mathrm{qE} - \lambda & -\alpha_1 \left(\frac{\mathrm{A}}{\alpha_1 \mathrm{m}_1 + \mathrm{d}_1 + \mathrm{qE}}\right) \\ \alpha_2 \mathrm{m}_1 & \alpha_2 \left(\frac{\mathrm{A}}{\alpha_1 \mathrm{m}_1 + \mathrm{d}_1 + \mathrm{qE}}\right) - \mathrm{d}_2 - \lambda \end{bmatrix},$$

which yields the following characteristic equation:

$$\mathrm{j}\lambda^2 - (\alpha_2 \mathrm{A} - \mathrm{j}^2 - \mathrm{d}_2 \mathrm{j})\lambda + \mathrm{d}_2 \mathrm{j}^2 - \alpha_2 \mathrm{Aj} + \alpha_1 \alpha_2 \mathrm{Am}_1 = 0,$$

where

$$\mathrm{j} = \alpha_1 \mathrm{m}_1 + \mathrm{d}_1 + \mathrm{qE}.$$

As a result, the equilibrium P_1 generates the subsequent two eigenvalues:

$$\lambda_1 = \frac{\alpha_2 \mathrm{A} - \mathrm{j}^2 - \mathrm{d}_2 \mathrm{j} - \sqrt{(\alpha_2 \mathrm{A} - \mathrm{j}^2 - \mathrm{d}_2 \mathrm{j})^2 - 4\mathrm{j}(\mathrm{d}_2 \mathrm{j}^2 - \alpha_2 \mathrm{Aj} + \alpha_1 \alpha_2 \mathrm{Am}_1)}}{2\mathrm{j}},$$

$$\lambda_2 = \frac{\alpha_2 \mathrm{A} - \mathrm{j}^2 - \mathrm{d}_2 \mathrm{j} + \sqrt{(\alpha_2 \mathrm{A} - \mathrm{j}^2 - \mathrm{d}_2 \mathrm{j})^2 - 4\mathrm{j}(\mathrm{d}_2 \mathrm{j}^2 - \alpha_2 \mathrm{Aj} + \alpha_1 \alpha_2 \mathrm{Am}_1)}}{2\mathrm{j}}.$$

In this instance, we take into account two cases in which the parameter might vary. The cases that are considered are:

$$\boldsymbol{(\alpha_2 \mathrm{A} - \mathrm{j}^2 - \mathrm{d}_2 \mathrm{j})^2 - 4\mathrm{j}(\mathrm{d}_2 \mathrm{j}^2 - \alpha_2 \mathrm{Aj} + \alpha_1 \alpha_2 \mathrm{Am}_1) > 0}, \tag{i}$$

$$\boldsymbol{(\alpha_2 \mathrm{A} - \mathrm{j}^2 - \mathrm{d}_2 \mathrm{j})^2 - 4\mathrm{j}(\mathrm{d}_2 \mathrm{j}^2 - \alpha_2 \mathrm{Aj} + \alpha_1 \alpha_2 \mathrm{Am}_1) < 0}. \tag{ii}$$

The property of $\lambda_1 < 0 < \lambda_2$ is satisfied by the eigenvalues whenever the condition (i) is true, where in this case, the equilibrium point P_1 fulfils the characteristics of an unstable saddle point. Otherwise, the property of imaginary eigenvalues is fulfilled whenever case (ii) holds. As a corollary, in this case, we consider another three conditions for the real part of the eigenvalue. If the real part is positive, the equilibrium point P_1 fulfils the characteristics of an unstable spiral point. Contrarily, if the eigenvalue P_1 have a negative real part, then it is an asymptotically stable spiral point. Ultimately, when the real part of the eigenvalue P_1 is equals to zero, then it is a stable centre.

Furthermore, the eigenvalues for the equilibrium P_2 is determined, where $P_2 = \left(\frac{\mathrm{A}}{\alpha_1 \mathrm{m}_2 + \mathrm{d}_1 + \mathrm{qE}}, \mathrm{m}_2\right)$. Therefore, the Jacobian Matrix will become

$$\mathrm{J}_2 = \begin{bmatrix} -\alpha_1 \mathrm{m}_2 - \mathrm{d}_1 - \mathrm{qE} - \lambda & -\alpha_1 \left(\frac{\mathrm{A}}{\alpha_1 \mathrm{m}_2 + \mathrm{d}_1 + \mathrm{qE}}\right) \\ \alpha_2 \mathrm{m}_2 & \alpha_2 \left(\frac{\mathrm{A}}{\alpha_1 \mathrm{m}_2 + \mathrm{d}_1 + \mathrm{qE}}\right) - \mathrm{d}_2 - \lambda \end{bmatrix}.$$

The characteristic equation for P_2 is as follows

$$\mathrm{k}\lambda^2 - (\alpha_2 \mathrm{A} - \mathrm{k}^2 - \mathrm{d}_2 \mathrm{k})\lambda + \mathrm{d}_2 \mathrm{k}^2 - \alpha_2 \mathrm{Ak} + \alpha_1 \alpha_2 \mathrm{Am}_2 = 0,$$

where

$$\mathrm{k} = \alpha_1 \mathrm{m}_2 + \mathrm{d}_1 + \mathrm{qE}.$$

As a result, the equilibrium P_2 generates the subsequent two eigenvalues:

$$\lambda_1 = \frac{\alpha_2 A - k^2 - d_2 k - \sqrt{(\alpha_2 A - k^2 - d_2 k)^2 - 4k(d_2 k^2 - \alpha_2 A k + \alpha_1 \alpha_2 A m_2)}}{2k},$$
$$\lambda_2 = \frac{\alpha_2 A - k^2 - d_2 k + \sqrt{(\alpha_2 A - k^2 - d_2 k)^2 - 4k(d_2 k^2 - \alpha_2 A k + \alpha_1 \alpha_2 A m_2)}}{2k}.$$

In this circumstance, we are examining two possible situations in which the value of the parameter could vary. The following cases are taken into account:

$$(\boldsymbol{\alpha_2}\mathbf{A} - \mathbf{k^2} - \mathbf{d_2 k})^2 - \mathbf{4k}(\mathbf{d_2 k^2} - \boldsymbol{\alpha_2}\mathbf{Ak} + \boldsymbol{\alpha_1 \alpha_2}\mathbf{Am_2}) > \mathbf{0}, \tag{iii}$$
$$(\boldsymbol{\alpha_2}\mathbf{A} - \mathbf{k^2} - \mathbf{d_2 k})^2 - \mathbf{4k}(\mathbf{d_2 k^2} - \boldsymbol{\alpha_2}\mathbf{Ak} + \boldsymbol{\alpha_1 \alpha_2}\mathbf{Am_2}) < \mathbf{0}. \tag{iv}$$

When the requirement (iii) is met, the eigenvalues satisfy the property of $\lambda_1 < 0 < \lambda_2$, and in this particular case, the equilibrium point P_2 meets the criteria for an unstable saddle point. Otherwise, whenever circumstances (iv) is valid, the condition of imaginary eigenvalues is observed. In this instance, we analyze three additional specifications for the real part of the eigenvalue as a result of the analysis. The equilibrium point P_2 satisfies the criteria for an unstable spiral point if the eigenvalues have a positive real part. On the other hand, if the real part of the eigenvalue P_2 is negative, the point corresponds to the property of an asymptotically stable spiral. Following that, a stable center is attained when the real part of the eigenvalue P_2 equals zero.

Nullclines

The nullclines for System (1) is generated by considering both equations

$$\frac{\mathbf{dI(t)}}{\mathbf{dt}} = \mathbf{0}, \tag{4}$$

$$\frac{\mathbf{dP(t)}}{\mathbf{dt}} = \mathbf{0}. \tag{5}$$

Solving equation both equation (4) and equation (5), the nullclines that have been obtained from System (1) are

$$I = \frac{A}{\alpha_1 P + d_1 + qE}, \qquad P = \frac{\beta}{d_2 - \alpha_2 I}.$$

Hence, the intersections between the I-nullcline and P-nullcline will give the equilibrium point for System (1).

The Presence of Periodic Solution

A method for determining upper bounds on the number of periodic solution or also known as limiting cycles in analytical differential systems is established by the Bendixson-Dulac theorem. Thus, consider the system (1) of ODEs where the first equation is assumed to be:

$$\begin{aligned} \frac{dI(t)}{dt} &= A - \alpha_1 IP - d_1 I - qEI \\ &= f(I, P), \end{aligned}$$

and the second equation of system (1) is considered to be:

$$\frac{\mathrm{dP(t)}}{\mathrm{dt}} = \beta + \alpha_2 \mathrm{IP} - \mathrm{d_2P}$$
$$= \mathrm{g(I,P)}.$$

Thus by considering the function of $\varphi(\mathrm{I,P}) = \frac{1}{\mathrm{IP}}$, thus the Dulac function is assessed to be:

$$\frac{\partial[\varphi \mathrm{f(I,P)}]}{\partial \mathrm{I}} + \frac{\partial[\varphi \mathrm{g(I,P)}]}{\partial \mathrm{P}} = \frac{\partial}{\partial \mathrm{I}}\left[\frac{\mathrm{A}}{\mathrm{IP}} - \alpha - \frac{\mathrm{d_1I + qEI}}{\mathrm{P}}\right] + \frac{\partial}{\partial \mathrm{P}}\left[\frac{\beta}{\mathrm{IP}} + \alpha - \frac{\mathrm{d_2}}{\mathrm{I}}\right]$$
$$= -\left(\frac{\mathrm{A}}{\mathrm{I^2P}} + \frac{\mathrm{d_1 + qE}}{\mathrm{P}} + \frac{\beta}{\mathrm{IP^2}}\right)$$
$$\neq 0.$$

Therefore, the system of ODEs of the predator-prey model proposed in this study possesses no periodic solution or limit cycle considering the fact that the Dulac's function will always be negative and will never be equal to zero on every circumstances that varies since all the parameters are assumed to be positive.

Numerical Simulations

For the purpose of investigating the dynamical behaviors of the predator-prey model of System (1), the numerical simulations are provided to substantiate the theoretical results obtained in the previous sections with the help of MATLAB software. For simplicity, we set the parameters of the model as follows:

Table 2 The parameters description of the predator-prey model.

Parameter	Value
$\boldsymbol{A}$	0.69
$\boldsymbol{\beta}$	0.12
$\boldsymbol{\alpha_1}$	0.241
$\boldsymbol{\alpha_2}$	0.196
$\boldsymbol{d_1}$	0.001
$\boldsymbol{d_2}$	0.34
$\boldsymbol{q}$	0.037
$\boldsymbol{E}$	1.905

In this section, we examine the effects of different parameters where the constant recruitment of both predator and prey populations are varied. In these plots, we assume that the initial conditions of both predator and prey populations is given by $I(0) = 1.05$ and $P(0) = 0.05$ to obtain the subsequent graphs of both populations. Firstly, when the parameters are set as $A = 0.69$ and $\beta = 0.12$, the time series plot of the predator-prey model of System (1) is given by the following Figure 2.

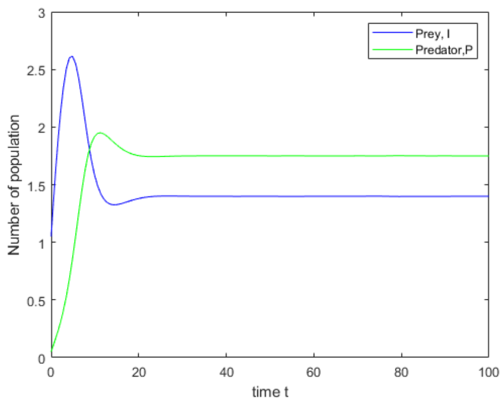

Figure 2 Time series plot of the predator-prey model with harvested prey species.

Referring to Figure 2, it is illustrated that the number of populations for the prey species is smaller than it is for the predator populations considering the fact that the prey species encounter from prey-predation interactions and simultaneously suffer from infectious disease as the time t increases. In addition, it is portrayed in the time series graph that the number of populations for the predator species is slightly higher to some extent compared to the prey species. This is due to the certainty that, after the prey-predation interactions between both species, the predator populations will benefit more since they are at the top of the food chain. In spite of that, a noticeable decrease in the predator populations is shown in the graph owing to the fact that they were affected by the toxin released by the prey populations and the death rate themselves.

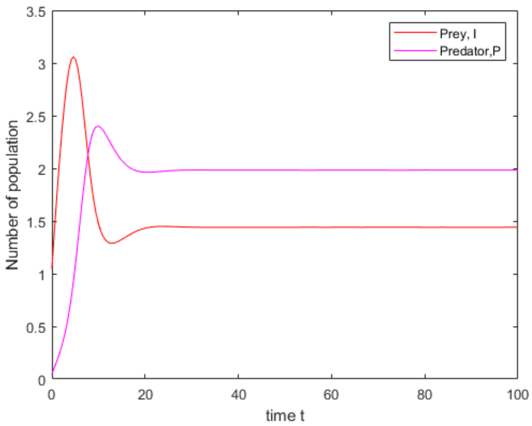

Figure 3 Time series plot of the predator-prey model with no harvesting efforts.

The time series for the proposed predator-prey model, ignoring the harvesting efforts where the harvesting parameter is set to be zero, where $E = 0$ is illustrated in Figure 3. Notice that the plots for the number of populations for both predator and prey populations are slightly different from the time series plot with harvested prey species with the harvesting parameter $q = 1.095$ which was illustrated in Figure 2.

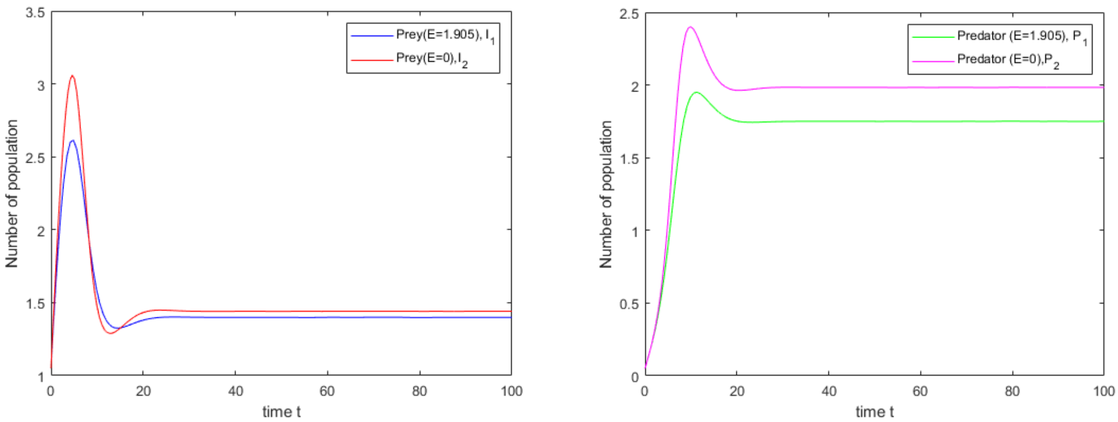

Figure 4 Comparison of time series plots between the predator-prey model with harvesting and

no harvesting efforts.

Assume I_1 and P_1 represents the prey and predator population with harvesting parameter $q = 0.195$ respectively; while I_2 and P_2 represents the prey and predator population with non-harvested prey populations where $q = 0$ respectively. Collating both prey populations, it is shown that I_2 is slightly higher than I_1 at the beginning of the time series plot. However, it is decreasing and slightly lower than I_1 as the time t increases. This is logically true considering the fact that the prey populations affected by the harvesting activities as the time passed. As the prey populations decreases, there are unnoticeable impact of harvesting activities for the predator populations due to the harvesting activities is still under control. Hence, the source of food for the predator populations is still enough, thus the number of populations does not decline. In spite of that, the slope between prey and predator populations with harvesting parameters is smaller compared to the non-harvested plots. This is because it is certainly true that when the harvesting activities occur, the prey populations, without any shadow of doubt, will continuously decline and harvested, while in contrast, the number of predator population is still high considering that the harvesting activities is under control and not affecting the food chain in the ecosystem.

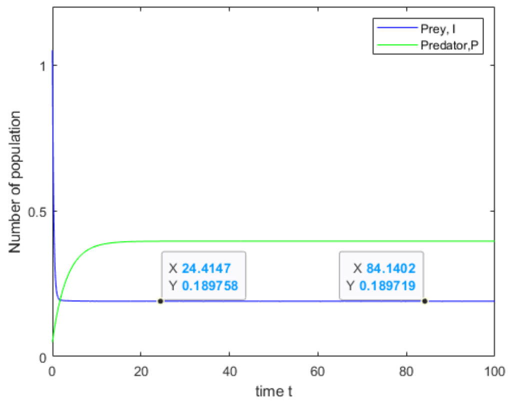

Figure 5 Time series plot of the predator-prey model with uncontrolled harvesting activity for the prey species.

As time goes to infinity, it is demonstrated in the graph from Figure 5 that the prey species will continue to decline and is expected to be endangered or threaten to extinction as the time passed. In this plot, we set the harvesting parameters $q = 9.570$ to represent the uncontrolled harvesting activities. There will be a rapid decline in the number of populations for the prey species after some time t due to the prey populations experiencing uncontrolled harvesting activities, prey-predator interactions and also infected simultaneously. Concurrently, the predator populations is also experiencing aforementioned characteristic as the prey populations in this scenario considering that their source of food is decreasing in the ecosystem which influencing their populations as well.

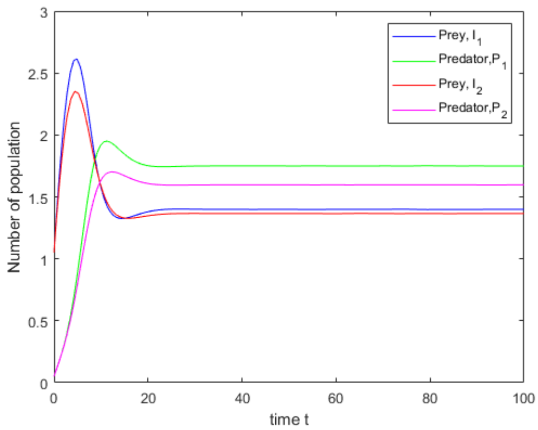

Figure 6 Comparison of time series plot when death rate of prey population is higher.

On top of that, we further examined the effect of infection on the prey populations of the predator-prey system. This scenario is assessed by varying the death rate parameter of the prey species, where we set $d_1 = 0.39$. Assume I_1 and P_1 represent the prey and predator population with $d_1 = 0.001$

respectively; while I_2 and P_2 represent the prey and predator populations with infected prey populations where $d_1 = 0.39$ respectively. There is no obvious difference between both plots, thus leading us to the conclusion that the infection in the prey species does not yield any severe impacts to the ecological system and will only affect a smaller number of populations for both predator and prey species in the ecosystem.

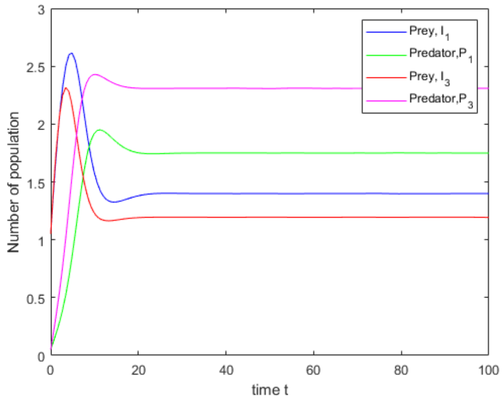

Figure 7 Time series plots with varying parameters of constant recruitment for predator and prey populations.

In addition to all the above analysis, we further analyze the system by altering the parameters for constant recruitment of predator and prey populations. Therefore, we first assume I_1 and P_1 represents the prey and predator population with $A = 0.69$ and $B = 0.12$ respectively; while I_3 and P_3 represents the prey and predator population where $A = 0.0.75$ and $B = 0.25$ respectively while other parameters keep the same value as in Table 2. It is clearly portrayed that higher value of parameters for both populations will result to a higher number of populations for predator species. When the constant recruitment of prey species increases, food supply for predator species also increases, and thus producing a higher number of populations for the predator species. Contradictorily on the flip side, the number of population for the prey species P_3 is moderately lower than P_1. This is believed to be due to the increment of predator-prey interactions and more prey species is infected to the disease, generating a declined trend for the prey populations.

A dynamical system's paths in a phase plane are represented geometrically by a phase portraits. A distinct curve or point is used to represent each set of beginning circumstances. Consequently, a phase portraits of the system of ODEs proposed in this study was plotted utilizing the *pplane8* function in the MATLAB software. The phase portraits gives an illustration of the stability at equilibrium points of the System (1). The trajectories in each quadrant of the system is represented by arrows in the phase portraits and further analysis is evaluated. The phase portraits is illustrated in the following Figure 8.

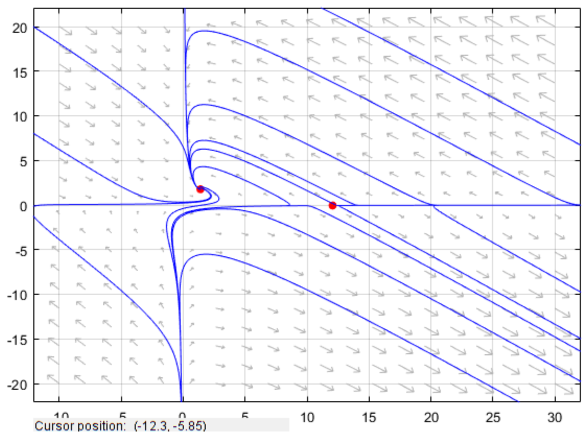

Figure 8 Phase portraits of the predator-prey model with harvested prey species.

The value of equilibrium P_1 based on the parameters in Table 2 is (1.388, 1.7661). Thus, the positive equilibrium P_1 in this predator-prey system is considered to be asymptotically stable spiral point since it yields imaginary eigenvalues and have the real parts of both eigenvalues less than zero. Specifically, the value of eigenvalues for equilibrium P_1 is $-0.2825 + 0.2641i$ and $-0.2825 - 0.2641i$ where the Jacobian Matrix yields

$$J = \begin{bmatrix} -0.4971 & -0.3345 \\ 0.3462 & -0.0679 \end{bmatrix}.$$

This is on par with the theoretical computation that has been evaluated in previous section by substituting the value of parameters as stated in Table 2.

It is clearly portrays in the phase portraits that the plane curve consisting of points continuously moves towards the fixed equilibrium point while revolving around it demonstrating a spiral formation. Thus, the numerical solutions obtained using the MATLAB software has proven that the theoretical computations in the previous sections is true.

On the other hand, the other equilibrium point specifically point P_2 which holds the value of $(12.0631, -0.0593)$ is ignored since it possess negative values of the predator populations. The plot of nullclines for the predator-prey system in this study is shown in the following Figure 10.

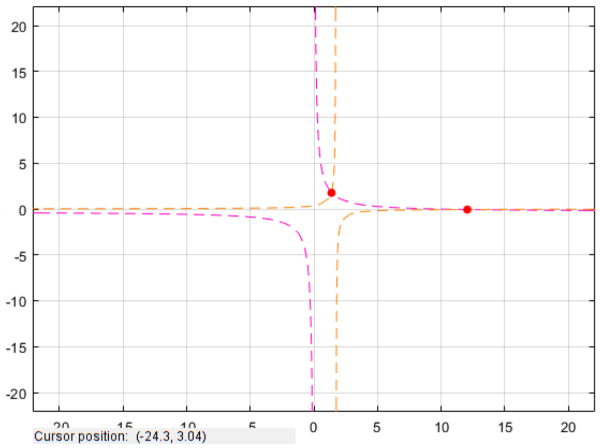

Figure 10 The plot of nullclines for predator-prey model of System (1).

Therefore, the only equilibrium point that represent the coexistence of both species in this study is equilibrium point P_1 which possess the value of (1.388, 1.7661) and attain the properties of an asymptotically stable spiral point.

Conclusion

In the previous section, a predator-prey system with harvested and infected prey species was proposed based on postulates that had been put forward beforehand. The equilibrium points of the system were obtained, and stability analysis was investigated using the Jacobian matrix and the eigenvalue method. The nullclines of the system have been analyzed and the non-existence of limit cycle have been proved by the Bendixson-Dulac theorem. Then, numerical simulations have been conducted with the aid of the MATLAB software, distinct time series plots were plotted, the phase portrait of the predator-prey system that have been proposed in this study was obtained, and thorough analyses have been discussed.

The effect of harvesting efforts and infection on the prey species has been studied based on a variety of time series plots that have been obtained in this study. The comparison between the initial time series plot and the plot of infected prey populations yields a negligible difference, indicating that the infection does not have any severe consequences in the predator-prey system. Variation in the harvesting coefficient, on the other hand, has an immense influence on the overall system. A high harvesting parameter value resulted in a visible shift in the graph, illustrating that unrestrained harvesting measures on prey populations will lead to the extinction of both species as evolution proceeds.

Therefore, it can be concluded that the harvesting parameters are more influential than the infection parameters in the ecosystem. Hence, law enforcement of harvesting activities should be administered seriously, and severe sentences should be imposed on irresponsible parties with the objective of curbing illicit harvesting. With these measure, the biological diversity will be preserved, extinction can be prevented from happening, and the wealth of marine resources will immensely benefit society as a whole.

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