Proceedings of Science and Mathematics
Faculty of Science, Universiti Teknologi Malaysia
Vol. 15, 2023, page 118-127

The Application of Root Finding Method to Determine the Volume of Petroleum in An Underground Storage Tank

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Abstract
While electronic devices have become prevalent for measuring petroleum in underground storage tanks at filling stations in many countries, dip gauge sticks are still used in certain regions due to their simplicity, cost-effectiveness, durability, and affordability. This study focuses on the application of root finding methods to accurately determine the volume of petroleum in underground storage tanks. The primary objective of the study is to design a straightforward volume tank chart that serves as a reliable reference for precise measurements. To achieve this, the study begins by deriving the governing equation that represents the tank's volume. The derivation employs a geometrical approach utilizing variables such as the tank's radius, length, and angle. Subsequently, the Newton-Raphson method is employed to numerically solve the dip gauge problem based on the derived governing equation. The method allows for efficient and accurate estimation of the tank's volume. Calibration charts specific to each storage tank are generated to provide essential details, including the angle, height, and volume of petroleum within the storage tank. Furthermore, the study conducts an analysis of the convergence rates of the solutions obtained through the numerical methods employed. This analysis provides insights into the efficiency and reliability of the applied root finding techniques. The study concludes that numerical methods play a critical role in accurately determining the volume of petroleum in underground storage tanks. This precise volume determination contributes to improved efficiency and optimization of oil storage processes, resulting in better resource management and operational benefits for oil storage facilities.
Keywords: Root finding; Dipstick problem; Newton-Raphson; Nonlinear equation; Numerical method

Introduction

Nowadays, every household in Malaysia has at least one car, which they use as their primary mode of transportation to travel anywhere they like. They are still able to reach their location even without a car thanks to several e-hailing services including MyCar, inDriver, Maxim, and Grab. Petrol serves as the main fuel source for each of these cars. There are numerous franchisers and businesses that provide the user's fuel source. There are Shell, Petron, Mobil, Caltex, Mobil, and Petronas, for instance. The primary provider, often known as the franchisor, is the business that explores, develops, and extracts crude oil and natural gas, such as PETRONAS, which has its headquarters in Malaysia.

In most petrol stations in Malaysia, the fuel level stored in their underground tank is often measured manually by having their employees measure the level of petroleum. In order to record the amount of petrol still in the tank, the employees will open the tank cover and insert a stick called a dipstick. It can take up to 30 minutes per day to appropriately dip tanks manually (Figure 1).

Figure 1 Manual measuring of underground storage tank fuel by using dipstick.

The oil pigment stops on the dipstick's volume inscription along its run indicates how much fuel is still in the tank. Dipstick or dip gauges are frequently used to determine the depth and volume of hydrocarbons still present in underground storage tanks. Despite the existence of numerous modern fluid measuring instruments based on ultrasonic sound waves and sophisticated electronics in use today, this manual method is still commonly seen in use in most gas stations in Malaysia. This is in part due to its dependability and simplicity compared to other complex electronic gadgets on the market, which dip sticks are relatively less expensive to design, sturdy, and cost-effective for end users.

Thus, during the construction of installing and filling station, station and company normally faced some difficulties on acquiring a reliable dip gauge for the buried tanks such the cost of removing the concrete floor, exhuming the tanks, and re-flooring them afterward, the disruption to sales and customer dissatisfaction, and finally, the damage to the anti- corrosion paint on the tank after exhuming from the ground. Therefore, alternative calibration techniques, which are frequently used in this situation, call for filling the buried tank incrementally with a known volume of hydrocarbon and noting the fluid's depth on the dip gauge.

This study describes the numerical modelling for fixed structure of two different size of cylindrical underground storage tanks. Figure 2 and Figure 3 demonstrate the common structure of cylindrical storage tank with dipstick that will be used in this study where L is the length of cylindrical storage tank, R is the radius of the cylindrical storage tank and h is the height of the oil level.

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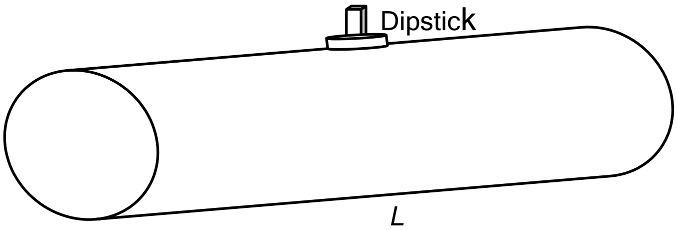


Figure 2 Diagram of cylindrical underground storage tank with dipstic

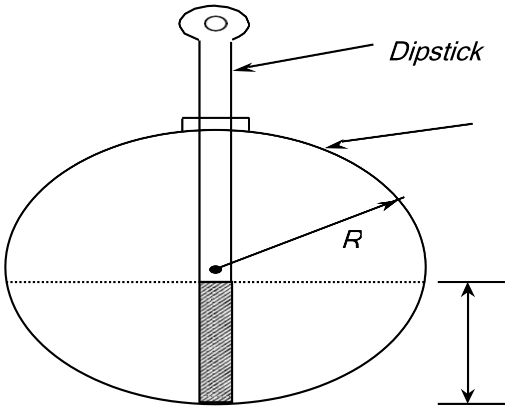


Figure 3 Oil level in cylindrical underground storage tank.

The information about example dimensions of the smaller and larger tanks is shown in Table 1 below.

Table 1 : Underground storage tank dimensions

Parameter	Smaller tank	Larger tank
Length, L (cm)	378.50	757.00
Radius, R (cm)	115.57	115.57
Volume of cylindrical tanks (cm²)	15,882,028.94	31,764,058.00

The Newton-Raphson iteration method is the focus of this study. Microsoft Excel software is used as a medium to sort the data based on differential calculus to determine the model gauge and analysis the relationship between several factors of certain underground tank volume. Numerical methods such as Newton-Raphson and secant method were also applied to develop a numerical model and numerical solution of the dip gauge problem.

Underground Storage Tank

According to United States federal regulations, underground storage tank (UST) defined as combination of tanks including any underground piping connected to the tank that is used to store regulated substances, and whose volume is at least 10 percent below the surface of the ground. The U.S. Environmental Protection Agency regulates USTs to stop the leakage of hazardous materials like petroleum and the subsequent contaminationof soil and groundwater.

Most USTs were constructed out of bare steel until the middle of the 1980s, which is likely to corrode with time and cause UST contents to seep into the environment. USTs may leak their contents into the environment due to improper installation, poor operating, or maintenance methods. The biggest risk from a leaky UST is that the petroleum or other hazardous material could seep into the earth and contaminate the groundwater, which provides about half of all Americans' drinking water. Other health and environmental dangers, such as the possibility of fire and explosion, can be brought on by a leaking UST. After U.S. Congress launched the Resource Conservation Recovery Act in 1985, there were over 2 million tanks in the nation, with more than 750,000 owners and operators. The program gave 90 employees to manage this duty. In 1988, the initial underground storage tank regulations were published by the Environment Protection Agency (EPA) in September, which included a 10-year phase-in period and required all operators to retrofit their USTs with spill prevention and leak detection technology.

Petroleum Storage Tank

During the late 1850s in North America petroleum field, new types of storage tanks were introduced. These were vertically displaced above-ground timber or metal-riveted tanks with a truncated cone shape that were constructed in a range of sizes.

One of them was called day tanks which is the smaller one. These were used to store petroleum that was collected daily or weekly, depending on the well's productivity, and were situated close to the producing well. Daily tanks were used to collect water and oil from one or more wells and to drain off water used during the extraction of petroleum. While larger tanks were typically constructed closer to transportation hubs, including railroads and river ports, stored oil as it was being transported to refineries. Up to the early 20th century, wood and mild steel tanks were employed in the oil fields, while metal tanks were mostly used in refineries.

Despite being served collect petroleum to separate from water, tanks also used to eliminate gases evaporating of petroleum. The first separators or devices were initiatedbetween 1863 and 1865 in the United States and Canada. These devices were built to maintain atmospheric pressure. Separators were highly useful in preventing unintentional fires, which were frequently started by abrupt gas leaks and caught and supplied cheap gas on-site to feed the boilers used to move the rigs and pump the well. In 1904, the first pressurized separator, able to function at a pressure of 10 bars, was introduced in 1904. It wasquickly discovered that if the separator was positioned upstream of the storage tank rather than directing the well stream directly into the tank, the volume of produced oil would be greater.

Dip Stick

The dip gauge stick is used to determine the liquid's depth in an underground tank. A pole that is inserted into a liquid fuel storage tank to determine how much product is inside of it, typically made of varnished hardwood to minimize the creeping of fuel above the actual fuel level in the tank. Starting with zero at the bottom end, the tank must be marked or notched to 1/8 inch. After inserting the stick into the tank, the height of the liquid mark on calibrator number will being recorded. The number on calibrator indicated the number of gallons remaining in the tank.

Tank Chart

Tank chart is a table that determines the volume of liquid within a specific shape and structure of tank by converting measured units of depth to units of volume. You can find out how much liquid is in your tank at a certain level by looking at a tank chart. You may quickly and effectively convert these levels of measurement into volumes using a tank strapping chart. This is especially helpful for tanks that are not linear.

Newton-Raphson Method

The Newton-Raphson method is such an important to figure out the best answer to several issues in a variety of fields, including statistics, applied mathematics, numerical analysis, finance, economics, and marketing. The Newton-Raphson method is also well-known of its iterative technique for determining the roots of an objective function.

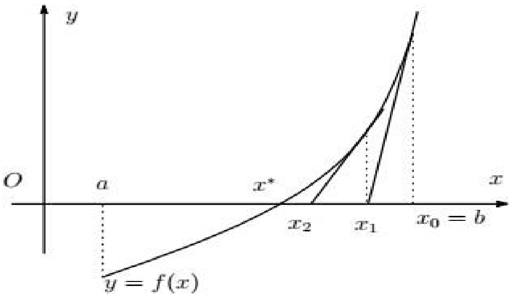


Figure 4 Geometrical illustration of the Newton-Raphson method.

The Newton Method should not be understood solely in terms of tangent lines. The Newton method is used to locate complex roots of polynomials and the roots of systems of equations involving several variables, when the geometry is less obvious, but a linear approximation still makes sense.

Derivation of the Formula for the Hydrocarbon Volume in the Horizontal Cylindrical Tank

Additional variables were added to the structure as shown in Figure 5. For examples, *O* represent the center of cylinder flat surface, *A* and *A'* represent the segment point of oil level, □ is the angle between segment points, and *H* is the height difference between segment line and center of cylinder flat surface.

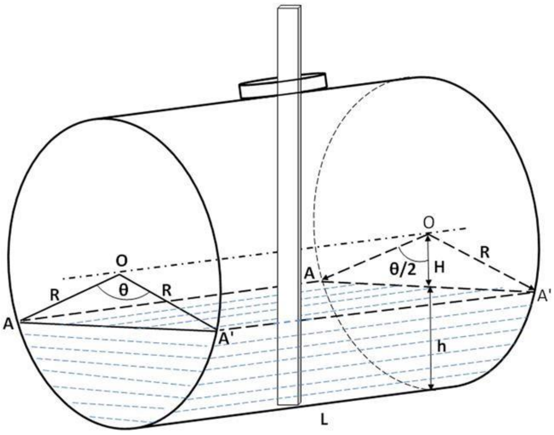


Figure 5 Schematic diagram of cylindrical underground storage tank

There are two scenarios that are possible for the hydrocarbon in the tank. It is either partially half-full or more than half-full.

Case 1: $h \leq R$

If h is the height of the hydrocarbon while R is the task radius, then, mathematically this case implies $h \leq R$. The lateral cross section for this case can be illustrated in Figure 5

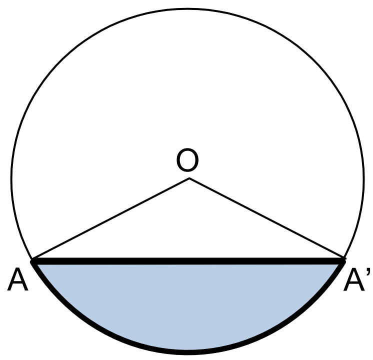


Figure 7 *Lateral cross section for less than half-full tank.*

$$\text{Area of sector AOA'} = \frac{1}{2}R^2\theta \qquad (1)$$

Area of triangle AOA' = $\frac{1}{2}R^2\sin\theta$ (2)

Area of minor segment AA' = $\frac{1}{2}R^2\theta - \frac{1}{2}R^2\sin\theta$ (3)

Area of the triangle = $(R-h)\sqrt{2Rh-h^2}$ (4)

It is also known that,

$$\cos\left(\frac{\theta}{2}\right) = \frac{R-h}{R}$$ which yields

$$\theta = 2\cos^{-1}\left(1-\frac{h}{R}\right) \quad (5)$$

Therefore,

Area of minor segment AA' = $R^2\cos^{-1}\left(1-\frac{h}{R}\right) - (R-h)\sqrt{2Rh-h^2}$

The volume of the hydrocarbon in the tank is then,

$$V = L\left(R^2\cos^{-1}\left(1-\frac{h}{R}\right) - (R-h)\sqrt{2Rh-h^2}\right) \quad (6)$$

Case 1: h > R

Case *h* > *R* implies the hydrocarbon in the tank is more than half-full. The lateral cross-section for this case can be illustrated in Figure 8

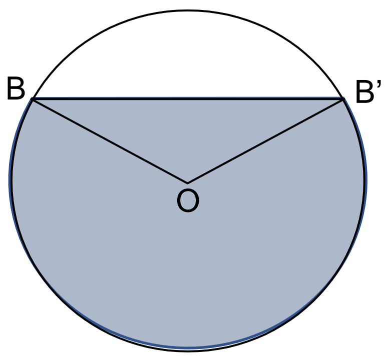


Figure 8 Lateral cross-section for more than half-full tank

Area of minor segment BB' = $R^2\cos^{-1}\left(1-\frac{h}{R}\right) - (R-h)\sqrt{2Rh-h^2}$

Area of major segment BB'= $\pi R^2 - R^2\cos^{-1}\left(1-\frac{h}{R}\right) + (R-h)\sqrt{2Rh-h^2}$

Volume of hydrocarbon in the tank,

$$V = L\left(\pi R^2 - R^2\cos^{-1}\left(1-\frac{h}{R}\right)+\left(R-h\right)\sqrt{2Rh-h^2}\right) \tag{7}$$

Newton-Raphson Iteration Formula Corresponding to Volume and Height of Hydrocarbon

Based on equation (6) on case $h \le R$, the nonlinear equation*, f(h)=0* can be written as

$$f(h) = V - L\left(R^2\cos^{-1}\left(1-\frac{h}{R}\right)-\left(R-h\right)\sqrt{2Rh-h^2}\right) \tag{8}$$

$$f'(h) = -L\left[\left(\frac{R}{\sqrt{1-\left(1-\frac{h}{R}\right)^2}}\right)+\sqrt{2Rh-h^2}-\frac{(R-h)^2}{\sqrt{2Rh-h^2}}\right] \tag{9}$$

The Newton-Raphson formula that will be used as root finding is then

$$h_{i+1} = h_i - \frac{f(h_i)}{f'(h_i)} \ , \tag{10}$$

Similarly, based on equation (3.7), for the case *h > R,*

$$f(h) = V - L\left(\pi R^2 - R^2\cos^{-1}\left(1-\frac{h}{R}\right)+\left(R-h\right)\sqrt{2Rh-h^2}\right) \tag{11}$$

$$f'(h) = -L\left[\pi R^2 - R^2\left(\frac{1}{R\sqrt{1-\left(1-\frac{h}{R}\right)^2}}\right)-\sqrt{2Rh-h^2}+\frac{(R-h)^2}{\sqrt{2Rh-h^2}}\right] \tag{12}$$

$$h_{i+1} = h_i - \frac{f(h_i)}{f'(h_i)}$$

Experimental Setting

Based on the non-linear equation (8) and (9), Newton-Raphson iteration method is implemented to obtain the value of *h,* for a fixed value of *V.* The ranges of volume measurable in the tank is 200-15000 litres for smaller tank, and 400-30000 litres for larger tank. Hence, *V*=200 litres (200 000 cm^3) is used in equation (8). The other parameter values are as follows:

R = 115.57 cm
L = 378.50 cm (smaller tank)
L = 757.00 cm (larger tank)
Initial guess, h_0 = 8 cm
Tolerance error, $\varepsilon = 1\times10^{-8}$

By Newton-Raphson formula in equation (10), the nonlinear equation (8) is solved (*h* is obtained). To mark the scale for other volumes, we need to substitute the value for the volume in equation (8) and solved for *h*. The process is repeated for any desired *V* value.

All experiments are conducted based on equation (11). However, since there are two possible scenarios which are $h \leq R$ and $h>R$, the following procedures are considered:

i. Based on equation (8), apply Newton-Raphson method to obtain *h.*
ii. If the obtained *h* value is greater than *R*, the experiment will be repeated based on equation (11) will be accepted.
iii. If the result, *h* is also greater than *R*, then the value of *h* based on equation (11) will be accepted.

Experimental Result

The volume of the hydrocarbon in the tank, *V* and the corresponding height, *h* determined by the Newton-Raphson method is presented in Table 4.1.

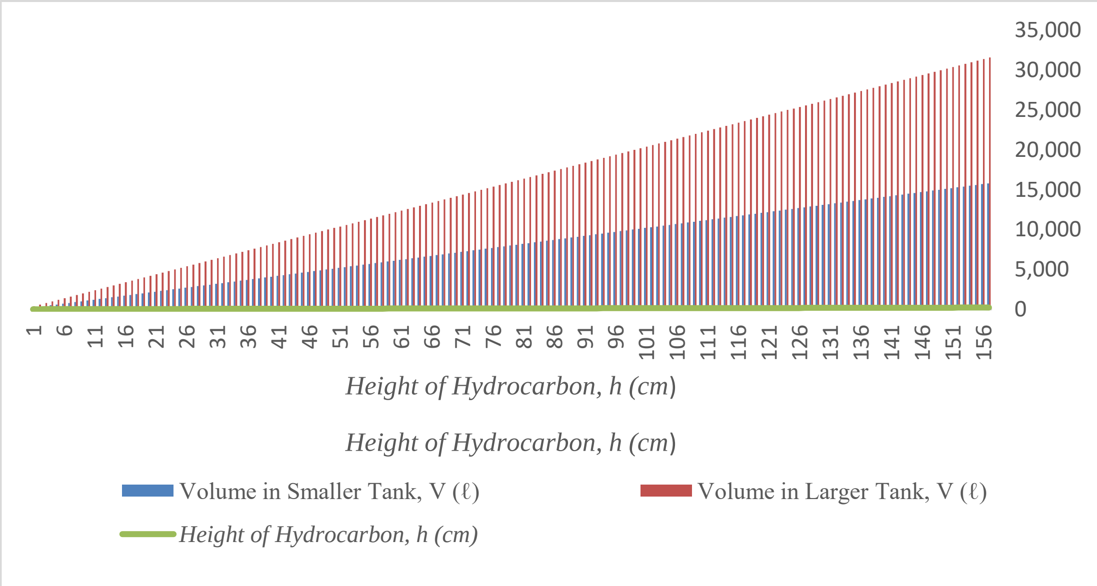
Table 2 : Tank Chart

Volume in Smaller Tank, V (ℓ)	*Volume in Larger Tank, V (ℓ)*	*Height of Hydrocarbon, h (cm)*
200	400	8.8599
300	600	11.6386
400	800	14.1307
500	1000	16.4314
600	1200	18.5916
700	1400	20.6426
800	1600	22.6054
900	1800	24.4949
1000	2000	26.3221
1100	2200	28.0956
1200	2400	29.8221
1300	2600	31.5070
1400	2800	33.1548
1500	3000	34.7691
1600	3200	36.3532
1700	3400	37.9096
1800	3600	39.4408
1900	3800	40.9487
2000	4000	42.4351
2100	4200	43.9016
2200	4400	45.3496
2300	4600	46.7803
2400	4800	48.1949
2500	5000	49.5943
2600	5200	50.9795
2700	5400	52.3514
2800	5600	53.7106
2900	5800	55.0580
3000	6000	56.3941
3100	6200	57.7196
3200	6400	59.0350
3300	6600	60.3409

3400	6800	61.6376
3500	7000	62.9258
3600	7200	64.2057

Based on the obtained data from Table 4.1 above, the relation between tank volume and height can be illustrated in the form of graph as depicted in Figure 9

Figure 9 Tank Graph



For discussion, let us consider the Newton-Raphson method for V = 500 litre in the smaller tank. The approximation value of h at each iteration is shown in Table 3.

Table 3 : Newton-Raphson iteration results :

Iteration, i	h_i	$\| h_i - h_{i-1} \|$
0	8	-
1	18.2615	10.2615
2	16.4743	1.7872
3	16.4315	0.0428
4	16.4314	0.0001
5	16.4314	0.0000

Since $| h_i - h_{i-1} | < \varepsilon$, therefore $h_5 = 16.4314$ is the root of *h.* The error of each iteration, *i* is presented in Figure 10.

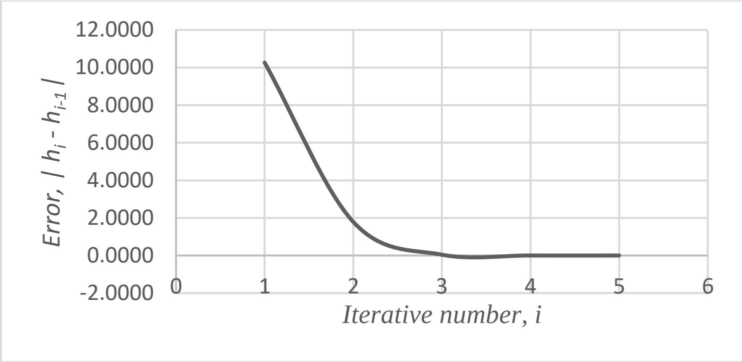


Figure 10 Graphical Relation of Error and Iterations

Based on Figure 4.2, it can be seen that the error in each i^{th} iteration converges positively and remain positive with fast speed and accuracy throughout the iteration.

Conclusion

During the study on numerical modelling of underground storage tank chart, it can be concluded that the model calibrator was designed from mathematical analysis using differential calculus, Newton-Raphson iteration methods and MS-Excel as the alternative computation software. Excel is a popular software that is accessible and simple to use. Calculation is simple once the algorithm is understood. Meanwhile, the study of the numerical methods, which is Newton-Raphson clearly has led to a deeper understanding of finding the roots of a given function. The Newton-Raphson approach makes use of derivative information and tends to converge more quickly, but it requires an accurate starting guess and there is a possibility that it could struggle to converge in certain circumstances. In conclusion, numerical methods play a crucial role in calculating the volume of an oil storage tank accurately and efficiently especially when dealing with complex tank forms, irregular profiles, real-time monitoring requirements, and the need for flexibility in various operational scenarios. By leveraging numerical methods, it is possible to achieve precise volume calculations, allowing for efficient management and optimization of oil storage operations.

Acknowledgement

I wish to express my sincere gratitude to all who have contributed throughout the course of this work.

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