Proceedings of Science and Mathematics
Faculty of Science, Universiti Teknologi Malaysia

Vol. 16, 2023, page 11 – 16

Mathematical Modelling Newton's Law of Cooling for Home Heating Systems

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Abstract
The purpose of this study is to learn more about temperature management, which is the process of maintaining a region's temperature within a certain range by using various forms of augmentation and insulation. By being able to extract the equation for both dimensions space and solve using the provided conditions and ambient temperature, it is possible to compare models that use 2 and 3 system of equations. By taking a real-life example of thermal coefficient and assumption of ambient temperature that act as a period, the mathematical modelling for home heating by using Newton's law of cooling is created by applying eigenvalue method for both equations. Each final particular equation is being tested and the graph of 2 and 3 system of equation being compared to find the relationship between ambient temperature and the temperature inside the house also the raising of temperature when the heater is turned on at certain degrees. From this research, we can explore the solutions under various heat source operating settings for different types of houses and select the heating mode that is both comfortable and financially viable.

Keywords Newton's Law of Cooling; Home heating; Mathematical modeling of home heating systems; Eigenvalue method of two and three systems.

1. Introduction

Mathematical modeling is a conceptual and specific study surrounding the model that simulates actual things that we want to discover. The model consists of dependent variables, independent variables and also the input and output variables. The three important things to bear in mind while modeling are the input , output and neglected variables[9] as it will affect the efficiency of our model. If the wrong variables are neglected, the output might not be accurate and the model would be considered as unsuccessful.

We are also familiar with the term 'heat' in everyday life. But in fact the term was used inconsistent with its real definition. What is heat? Heat is a form of energy which can actively move from one place to another because of the high energy level. Heat transfer in general is in which the internal energy from one material is transferred to another due to the temperature difference between two conditions. The transmission of energy through conduction is essentially tied to the interactions of these energy carriers, with more energetic (high temperature) energy carriers emit energy to less energetic (low temperature) surfaces [10].

In relation to heat study, Newton's law of cooling which is responsible for the heat transfer theory founded by Isaac Newton in the 18th century . He had studied different kinds of rigid forms of cooling. Though Isaac Newton was the first to find out about the cooling law, the Russian scientist was the one who immensely influenced the modern interpretation of the law of cooling which is why in Russia, the newton's law of cooling is named as Newton-Richmann law of cooling as per the Russian scientist name, Georg Wilhelm Richmann [2].

2. Understanding derivation of Newton's Law and theoretical assumptions

Newton's law of cooling is used in daily life even as simple as to find out how much time is needed to cool down a hot coffee. Therefore, we want to study the application of Newton's law of Cooling in home heating systems. There are few basic assumptions before we able to further our study on the mathematical modelling of Newton's law of cooling First, we need to assume that the house parameters are constant and each layer is built with homogenous materials so that it would not be a variable for our model later. Next, we assume that the floor was covered by thermal insulation thus the rate of heat loss is constant and very few. We also need to make an assumption that there is no other heat sources inside the house like another heater or we did not switch on the stove during the simulation so that there is no other factor that influence the model.

Based on the previous research study, they proved that the basic Newton's law of cooling can be solved by using ordinary differential equations method [3]. Seperable equation method is used and to be continued by using eigenvalue method. This method is familiar with us but the condition given of the model might not similar with other research study. The model prediction for this study research is to find the accurate model based on mathematical modelling of two storey house with one heating sources and try to fit in the house with basement and attic and have two heating sources with assumptions that the heating source is provide a constant heat source throughout the day.

For two storey house with one heat sources on the ground floor and the area of the house assumed the model is being created as below:

$$\frac{dx}{dt} = k_1\left(T_g - x\right) + k_2\,(y - x) + k_3\,(T_\alpha - x) + f(t)$$

$$= -(k_1 + k_2 + k_3)x + k_1 T_g + k_2\, y + k_3\, T_\alpha + f(t) \tag{1}$$

$$\frac{dy}{dt} = k_2\,(x - y) + k_4\,(T_\alpha - y)$$

$$= k_2\, x + k_4\, T_\alpha - (k_2 + k_4)\mathrm{y} \tag{2}$$

For house with basement and attic with two heat sources located in the basement and on the main living area and the area of the house assumed the model is being created as below:

$$\frac{dx}{dt} = k_1\left(T_g - x\right) + k_2\left(T_g - y - x\right) + f(t)$$

$$= (k_1 + k_2)T_g - (k_1 + k_2)x - k_2\, y + f(t)$$

$$\frac{dy}{dt} = k_2\left(T_g - y - x\right) + k_3\,(T_\alpha - y) + k_4\,(z - y) + f(t)$$

$$= -k_2\, x - (k_2 + k_3 + k_4)y + k_4\, z + k_3\, T_\alpha + k_2 T_g + f(t)$$

$$\frac{dz}{dt} = k_4\,(z-y) + k_5(T_\alpha - y)$$

$$= -(k_4+k_5)y + k_4 z + k_5 T_\alpha$$

3. Results and discussion

The transition process is described by the first two terms, which are damped. The third phrase has to do with heat transfer with the surroundings. The law of the change in ambient temperature was selected for this equation. The final element in the formula describes how the constant heat source behaves. The parameter and source power are inversely proportional. The temperature (on the ground floor) rises by degrees once the heat source operates for an hour.

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = (18.0712 - 0.1960\mathrm{f}_0)e^{-0.7043\,t}\begin{bmatrix} 0.5530 \\ 1 \end{bmatrix}$$

$$+$$

$$(-16.2052 - 1.4193\mathrm{f}_0)e^{-0.2677\,t}\begin{bmatrix} 1 \\ 0.5534 \end{bmatrix}$$

$$+$$

$$\begin{bmatrix} 6.5741 + 11.6773\sin\left(\frac{\pi}{12}t\right) + 4.9516\cos\left(\frac{\pi}{12}t\right) \\ -1.1481 + 0.6087\sin\left(\frac{\pi}{12}t\right) + 2.04491\cos\left(\frac{\pi}{12}t\right) \end{bmatrix} + \begin{bmatrix} 1.9627\,f_0 \\ \frac{\mathrm{f}_0}{1.019} \end{bmatrix}.$$

The transition process is described by the first two terms, which are damped. The third phrase has to do with heat transfer with the surroundings which involving the existence of heat sources in the basement and also the main living floor. The law of the change in ambient temperature was selected for this equation. The final element in the formula describes how the constant heat source behaves. The parameter and source power are inversely proportional. The temperature (on the ground floor) rises by degrees once the heat source operates for an hour.

$$\begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix} = -25.8341 - 1.1957\mathrm{f}_0\,e^{-2.4056\,t}\begin{bmatrix} 1 \\ 1.7010 \\ 0.7237 \end{bmatrix} +$$

$$48.5164 - 0.1306\mathrm{f}_0\,e^{0.2857t}\begin{bmatrix} -0.1149 \\ 0.2803 \\ 1 \end{bmatrix}$$

$$-12.285 - 0.5652\mathrm{f}_0 e^{-0.8301t}\begin{bmatrix} -1.3833 \\ 1 \\ 0.8783 \end{bmatrix}$$

$$+$$

$$\begin{bmatrix} 7.404 - 0.6841\sin\left(\frac{\pi}{12}t\right) + 2.0108\cos\left(\frac{\pi}{12}t\right) \\ 5.1852 + 2.1781\sin\left(\frac{\pi}{12}t\right) - 3.7462\cos\left(\frac{\pi}{12}t\right) \\ 0.3704 + -6.8527\sin\left(\frac{\pi}{12}t\right) - 10.3676\cos\left(\frac{\pi}{12}t\right) \end{bmatrix}. + \begin{bmatrix} 0.3989\,f_0 \\ 0.7407f_0 \\ 1.4815f_0 \end{bmatrix}.$$

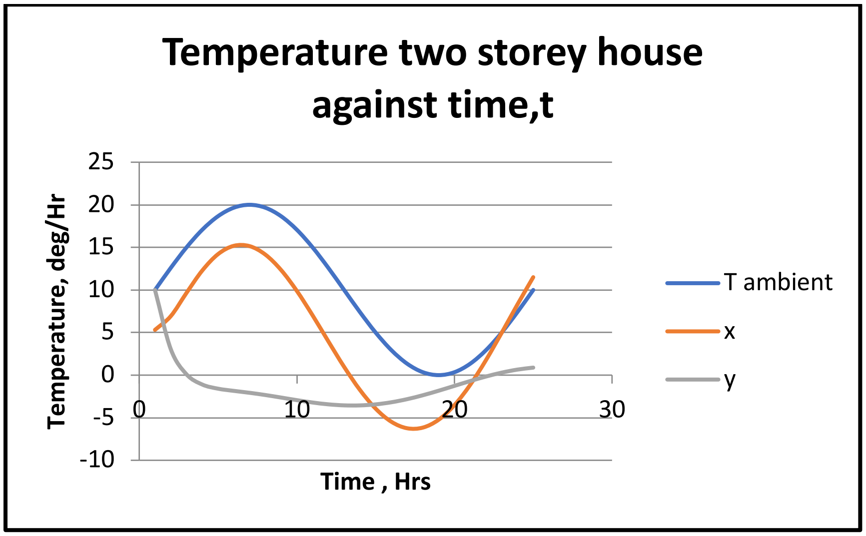

Figure 1

Examine the temperature change graphs. When the heater is off, Figure displays the temperature curves on the lower and higher floors of the home: It is clear that when the outside temperature rises, the temperature within the home does so gradually. The temperature on the ground level fluctuates in the following range due to the inertia of heat transmission.

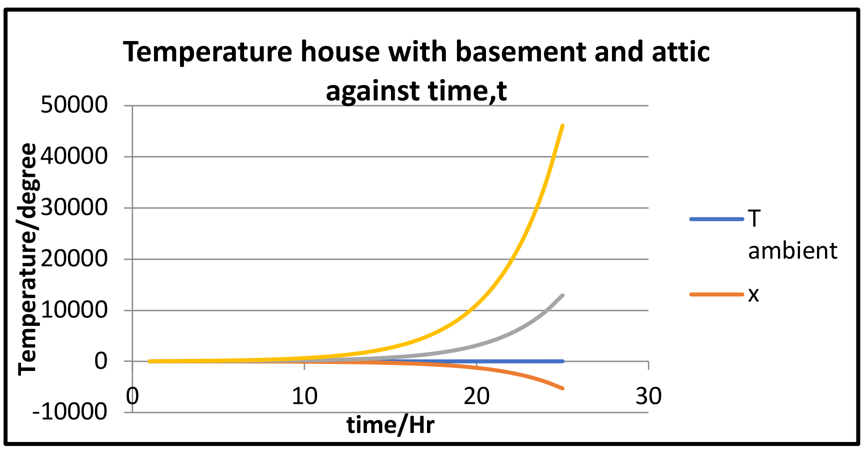

Figure 2

Based on the result obtained, the modelling for three systems of equation involving two heat sources is not being modelled successfully because the value of the temperature inside the house on each floor is inconsistent and show no relationship between ambient temperature and the temperature inside the house.

Conclusion

To conclude the research finding, the objective of this research is being achieved whereby the equation for home heating system of two storey house with one heating sources on the ground floor. Unfortunately, the modelling for a two storey house with attic and two heat sources located at the ground floor and the upper floor is not successfully modelled. The model is being tested and the graph outcome from the model is interpreted well. The relationship between ambient temperature and the temperature inside the house also being studied and the two model is being compared.

References

[1] Patil, D. P. , Shirsath, D. S., Gangurde, V. S. (2022. 6 June). Application of Soham Transform in Newtons Law of Cooling, ISSN (Online): 2320-9364, ISSN (Print): 2320-9356. Retrieved : 31 December 2022

[2] G.W. Richmann, Physics Proceedings Moscow, (1956), pp. 69–571.

[3] Hassan, A., Zakari, Y., (2018). Application of first order differential equation in temperature problems. Department of Mathematics, Usmanu Danfodiyo University, Sokoto. Nigeria

[4] Hulbert LE, Nottage HB, Franks CV. Heat flow analysis in panel heating or cooling sections: case I-uniformly spaced pipes buried within a solid slab. ASHVE Trans 1950;57:221e32.

[5] IEA, Energy-efficient buildings: heating and cooling, Technology Roadmap, IEA Sustainable Energy Policy and Technology directorate, 2011.

[6] Kuijer, L., Watson, M., (2017). That's when we started using the living room': Lessons from a local history of domestic heating in the United Kingdom. University of Sheffield, Department of Geography, Western Bank, Sheffield, S10 2TN, United Kingdom, http://dx.doi.org/10.1016/j.erss.2017.04.010

[7] Lakatos, Á., (2018). J Therm Anal Calorim 133, 329–335. https://doi.org/10.1007/s10973-017-6686-5

[8] Maruyama , S. Moriya, S., (2020) . Newton's Law of Cooling: Follow up and exploration , retrieved from : http://www.elsevier.com/locate/hmt

[9] Neelesh Soni a , Debojit Sharma a , Mustafa M. Rahman b , Prashanth R. Hanmaiahgari c , V. Mahendra Reddy d,. (2021). Mathematical Modeling of Solar Energy based Thermal Energy Storage for House Heating in Winter.

[10] Nellis, G., Klein. S., (2009). Heat Transfer. Cambridge University Press,includes bibliographical references and index. ISBN 978-0-521-88107-4

[11] Scala graduum Caloris, Calorum Descriptions & signa, Philos. Trans. 22 (270) (17 01) 824–829

[12] Sumithra, B., (2017, 7 July). Engineering Applications of Differential equations. Department of Mathematics, Anna University, BIT Campus, Tiruchirappalli - 620 024, Tamilnadu, India.

[13] UK becomes first major economy to pass net zero emissions law, Department for Business, Energy & Industrial Strategy and The Rt Hon Chris Skidmore MP (2019, 27 June)

[14] Balaji, Nallaval & Mani, Monto & Reddy, B.. (2013). Thermal Performance of the Building Walls.

[15] C.HenryEdwards, E.David Penney., (2008) *Elementary Differential Equations,6thEdition.*

[16] Víctor,P.A., Carolina, A.F., Ana, M.I., José, L.V. (2018, 15 December). Impact of climate change on heating and cooling energy demand in a residential building in a Mediterranean climate,Energy; https://doi.org/10.1016/j.energy.2018.09.015

[17] Ng, W.T., Zakia, S.A., Hagishimac, A., Rijald, H.B., Zakariae, M.A., Yakub, M., (2019). Effectiveness of free running passive cooling strategies for indoor thermal environments: Example from a two-storey corner terrace house in Malaysia.

[18] Raji, A.W., Mohamad, M.N., (2015, February). Differential Equation for engineering students, pp 196.