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Free Convection Flow of Newtonian Fluid with Accelerated Channels

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Abstract
The main purpose of this research is to formulate the mathematical models and solutions for the free convection flow in Newtonian fluid in channels with accelerated effects. Using appropriate dimensionless variables, the dimensional governing equations of momentum and energy are reduced to dimensionless equations subjected to the associated initial and boundary conditions. The analytical solutions are obtained by using Laplace transform technique. Dimensionless parameters are obtained through dimensionless processes such as Grashof number Gr, Prandtl number $\Pr$, accelerated R and time t. The mathematical findings for velocity and temperature are graphically plotted to investigate the influence of dimensionless variables on profiles. It is observed that fluid velocity increases with increasing of Gr, t and R, whereas it decreases with increasing of $\Pr$. Besides that , it is found that temperature profiles decreases with a high value of Prandtl number $\Pr$, while increase with high value of time t. In order to validate the results, the obtained results in limiting cases are compared with the published results and it is found to be in a mutual agreement.
Keywords: Newtonian fluid; free convection flow; accelerated in channels; laplace transform

Introduction

Fluids liquefy continuously, which means they always flow. A fluid is a substance that has no distinct shape and easily yields to external pressure. When at rest, a fluid is any liquid, gas, or material that cannot withstand a shearing or tangential force. When the previously stated forces are applied to the fluid, it undergoes a continuous change in shape. In addition, fluids are also classified into the following properties which are viscosity, conductivity, compressible, incompressible and density. These characteristics are usually due to their inability to withstand a shear stress in static equilibrium. Only normal, compressive stress, known as pressure, may be applied to perfect fluids. Since real fluids possess viscosity, they can withstand minimal amounts of shear stress.

Meanwhile shear stress is the unit area quantity of force acting on a specific fluid parallel to a tiny piece of the surface. Fluids can also be classified as one of the following based on the connection between shear stress and strain rate which can be divided into two types, Newtonian fluids and non-Newtonian fluids. Newtonian fluids are typically liquids that contain low molecular weight compounds. To see more, Newtonian fluids are excellent for lubrication application. Although these fluids change viscosity as a function of temperature, they do not change viscosity as a function of shear. Motor oil is an excellent example for this application. It remains fluid at low temperatures while remaining thin at high engine running temperatures.

In this study, free convection flow in Newtonian fluids is considered a difference in density which is caused by difference in temperature and it is referred to as a free convection flow field. The density field is not uniform as a result of the difference in temperature. Because of the gravitational field and the difference in the density field, buoyancy will cause a flow current. A free convection heat transfer is often significantly smaller than a forced convection heat transfer. It is thus only essential when there is no external flow. As we know, free convection flow occurs when there is a difference in density which is caused by difference in temperature for example, sea breeze and also land breeze.

There are various ways and methods to solve the problems of Newtonian fluids, wall shear stress and channel flow. Most of researchers used mathematical modeling to solve the problem by using analytical methods or numerical methods. One of the analytical methods that can be used to solve linear problem equations is Laplace Transform method. Laplace function, is a function that has a finite number of breaks and does not blow up to infinity anywhere, it is said to be a piecewise continuous function. Laplace transform helps in the solution of differential equations by reducing the differential equation toan algebraic problem.

Most of the researchers are more interested in studying non-Newtonian fluid flow compared to Newtonian fluid. Nevertheless, there are still a few researchers who are interested in investigating the effect of Newtonian fluid in different types of boundary conditions. Majority of the researchers' investigations focused on heat transfer analysis that happens in Newtonian fluid . Nonetheless, there are still no researchers who investigated the topic of free convection flow of Newtonian type fluid in channels with accelerated by using Laplace transform. The goal of this research is to analyze the behavior of free convection flow of Newtonian fluid in channels using the Laplace transform with the accelerated effect.The purpose of this study is to use the Laplace transform method to obtain the analytical solutions of the velocity and temperature profiles for the problem of wall acceleration effect on free convection flow of Newtonian fluid in the channel. It also seeks to obtain the mathematical models and solutions for free convection flow of Newtonian fluid in channel with the effect of wall shear stress obtained by using Laplace transform method. To analyze the behavior of mathematical solutions profiles (velocity and temperature) with different physical flow parameters by using MATHCAD software.

The study of Newtonian type fluid with heat energy may be solved theoretically using Laplace transform. Many publications and research journals have solved the problem utilizing various sorts of boundary problems such as in channel and accelerated. Most likely, the researcher utilized a variety of approaches to tackle the problem in different sorts of boundary problems, but the Laplace transform method was prioritized. The goal of this study is to demonstrate the mathematical solutions of free convection flow in Newtonian type fluid through channels and with accelerated effect using the Laplace transform method to solve dimensionless equations with appropriate dimensionless limits.

Newtonian Fluid by Using Laplace Transform

In recent years, Newtonian fluid has been extensively explored and debated in terms of its applications with the Laplace transform approach. In mathematics, the Laplace transform is a powerful integral transform that is used to translate a function from the time t-domain to the s-domain. The Laplace transform may also be used to solve linear differential equations with stated initial and boundary conditions. By using the Laplace transform approach, a study about the second Stokes problem for Newtonian fluids is established by Fetecau et al. (2008). They observed that when transient motion stops for high values of t, steady-state solutions that are periodic in time and independent of the original conditions are used to describe the fluid motion. It comes to the conclusion that as frequency and velocity rise, time decreases. In the same year, Siddique (2008) has found a Newtonian fluid flowing in a helical pattern inside of an infinite circular cylinder with rotational and longitudinal shear stress. The exact solution is determined by using the Laplace transform and finite Hankel transforms.

Free Convection Flow of Newtonian Fluid in Channel

Given its broad applicability, Newtonian fluid flow between plates in a channel is particularly popular among researchers. Lee et al. (2017) has done research regarding a quick capillary-pressure driven micro-channel to show zebrafish blood's Newtonian fluid behavior at high shear rates. To adjust the pressure-driven microfluidic channels, water was utilized. In order to reduce manufacturing error, each micro-channel was calibrated against water. In order to forecast changes in viscosity throughout embryonic development, a calibration curve for viscosity as a function of hematocrits was constructed.

Thus, the dynamic viscosity throughout development as well as the Newtonian fluid behavior of zebrafish blood at high shear rates were revealed by our capillary pressure-driven micro-channel. A study for free convection flow of Newtonian fluid in a channel with heated vertical channel was done by Hamza (2018). The effects of Navier slip and Newtonian heating on the transient/steady flow of an exothermic fluid in a vertical parallel plate are considered. In an asymmetrically heated open-ended channel, the interaction of radiation with turbulent natural convective flow of dry and humid air has been studied computationally by Tkachenko et al. (2019). A year after, Mahmood et al. (2020) analyze the

fluid flow in the channel with power law effects as well as shear thinning and thickening. It was determined that the vortex exhibits a consistent pattern for lower Reynold number magnitudes in contrast to higher magnitudes of. During the same year, the effect of thermal radiation, heat generation and induced magnetic field on free convection flow of couple stress fluid in an isoflux isothermal vertical channel has been investigated. The magnetic Prandtl number and heat production coefficient were found to increase the induced current density. By comparison the radiation parameter and Hartmann number work together to reduce the induced current density.

Free Convection Flow in Newtonian Fluid with Accelerated

A few papers have discussed Newtonian fluid with accelerated. Zimmels (1983) did a study regarding particulate motion in Newtonian fluids is examined in a finite system with a constant total volume. Using updated hindrance factors as a function of particle concentration, equations of motion are developed. It was determined that the fluid acceleration associated with the motion of the solid should be accounted for in the equation of motion, and that the average acceleration of the counter-lowing fluid, defined on a per unit solid volume basis, is equal in magnitude to the solid acceleration. After many years passes, another study was done by Ferreira & Chhabra (1998) regarding the transient motion of a sphere falling through a Newtonian fluid has been studied using an Abraham/Wadell drag. This investigation also measured the sphere's acceleration and final velocity. This study demonstrated that it was possible to analytically describe the transient motion of a falling sphere, and both the time and distance required by the sphere to accelerate from an initial velocity to a velocity were found to be described by three different analytical expressions depending on the value of the sphere-fluid combination parameter.

Research Methodology

Let consider the unsteady free convection flow of Newtonian fluid past through two vertical channels which are separated by distance d with constant temperature. The x -axis is in upward direction and y -axis is in normal direction to the plates. At time $t'\leq 0$, the plates $y'=0$ starts moving with velocity $U(t')$ in the x' direction and its temperature is raised or lowered to T_w while the plate $y'=d$ is kept fixed and is maintained at T_d . The geometry of the problem is presented in Figure 1

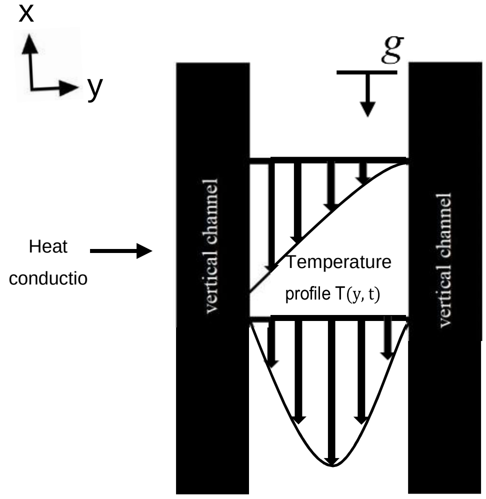

Figure 1

Applying the Boussinesq approximation, the free convection flow is governed by the equations.

$$\frac{\partial u'}{\partial t'} = \upsilon \frac{\partial^2 u'}{\partial t'^2} + g\beta(T' - T'_d) \tag{3.1}$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p} \frac{\partial^2 T'}{\partial y'^2} \tag{3.2}$$

In order to transform the governing equations (3.1) and (3.2) and the corresponding conditions of equations of initial and boundary conditions into dimensionless form. The dimensionless variable are substituted and simplified the equations. Lastly, reduced number of variables by grouping them into dimensionless parameter that can be obtained as

$$Gr = \frac{d^3}{v^2} g\beta(T'_w - T'_d),\ \Pr = \frac{\mu C_p}{k} \tag{3.3}$$

where, Gr is the Grashof number and Pr is the Prandtl number. Thus, dimensionless momentum and energy equations can be written as

$$\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial t} = -GrT, \tag{3.4}$$

$$\frac{\partial^2 u}{\partial y^2} = \Pr \frac{\partial T}{\partial t}, \tag{3.5}$$

with associated dimensionless initial and boundary conditions

$$u(y,0) = 0 \qquad ;0 \le y \le 1, \tag{3.6}$$

$$u(0,t) = F(t) \qquad ;t > 0,$$

$$u(1,t)=0 \qquad t>0.$$

and

$$T(y,0)=0 \qquad ;0\le y\le 1, \tag{3.7}$$

$$T(0,t)=1 \qquad ;t>0,$$

$$T(1,t)=0 \qquad t>0.$$

where $F(t)=\dfrac{U(t')d}{v}$, is dimensionless velocity.

Analytical solutions of velocity and temperature profiles can be obtained by applying Laplace Transform method into the dimensionless momentum and energy equations. Associated dimensionless initial and boundary conditions are used while performing the Laplace Transform technique.
The inverse Laplace transform of equation is obtained using the inverse Laplace transform formula which can be obtained as

$$T(y,t)=\sum_{n=0}^{\infty}(erfc(\frac{(y+2n)\sqrt{\Pr}}{2\sqrt{t}})-erfc(\frac{(2+2n-y)\sqrt{\Pr}}{s\sqrt{t}})). \tag{3.8}$$

The following step is to get the momentum solution for equation. The analytical momentum solution for partial differential equation with initial and boundary conditions by using Laplace transform method.
By taking $F(t)=Rt$ (Seth et al, 2016) into equation, obtained as $u(0,t)=Rt$ where R is acceleration parameter. Hence, the associated transformed boundary conditions are

$$\bar{u}(0,s)=\frac{R}{s^2}, \tag{3.9}$$
$$\bar{u}(1,s)=0.$$

In order to calculate the Laplace transform, substitute geometrical series equation

$$\begin{aligned}\bar{u}(y,s)=&\frac{1}{s^2}\cdot e^{y\sqrt{s}}\cdot R-\left[\left[R\cdot\sum_{n=0}^{\infty}(\frac{1}{s^2}\cdot e^{-(2n-y)\sqrt{s}})+\frac{Gr}{b_1}\cdot\sum_{n=0}^{\infty}(\frac{1}{s^2}\cdot e^{-(2n-y)\sqrt{s}})\right]+\frac{Gr}{b_1}\frac{1}{s^2}\cdot e^{y\sqrt{s}}\right]\\&+R\left[\sum_{n=0}^{\infty}\frac{1}{s^2}\cdot e^{-(2n+y)\sqrt{s}}\right]+\frac{Gr}{b_1}\left[\sum_{n=0}^{\infty}\frac{1}{s^2}\cdot e^{-(2n+y)\sqrt{s}}\right]\\&+\frac{Gr}{b_1}\left[\sum_{n=0}^{\infty}\frac{1}{s^2}\cdot e^{-(2n\sqrt{\Pr}+y\sqrt{\Pr})\sqrt{s}}+\sum_{n=0}^{\infty}\frac{1}{s^2}\cdot e^{-(2n\sqrt{\Pr}+2\sqrt{\Pr}-y\sqrt{\Pr})\sqrt{s}}\right].\end{aligned} \tag{3.10}$$

By using Inverse Laplace transform, $u(y,t)$ in equation above and rewrite equation as

$$u(y,t) = Ru_1(y,t) - Ru_2(y,t) - \frac{Gr}{b_1}u_3(y,t) - \frac{Gr}{b_1}u_4(y,t) + Ru_5(y,t) + \frac{Gr}{b_1}u_6(y,t) + \frac{Gr}{b_1}u_7(y,t) + \frac{Gr}{b_1}u_8(y,t). \tag{3.11}$$

Results and discussion

This problems are solved using the Laplace transform method and the results are visually shown using MATHCAD software. The purpose is to make data analysis and comprehension easier by visually displaying the relationship between flow parameters.

To have a better understanding of the fundamental problem, parametric research is conducted and numerical results are obtained. The numerical results for velocity and temperature are computed and visually presented for different values of the important parameters time t, Grashof number Gr, Prandtl number Pr, and acceleration R. Figures 1-4 depict physical behaviour velocity, whereas Figures 5-6 provide temperature profiles. The effects of the parameters displayed in the graphs are examined. Furthermore, the findings and graphs show that all beginning and boundary criteria are met.

Figure 1 illustrates velocity profiles and depicts a range of thermal Grashof number Gr. According to Makinde et al. (2010), the thermal Grashof number Gr in the boundary layer is the ratio of buoyancy to viscous forces. Free convection is distinguished by an increase in as a result of the thermal buoyancy force. When velocity rises, so does the thermal Grashof number (Rajput and Kumar, 2012). As a result, fluid density decreases and the momentum equation suffers minor viscous effects, causing fluid velocity to rise. According to Saqib et al. (2018), when is negative, the buoyant force opposes fluid flow; when is positive, the buoyant force promotes fluid flow.

The impact of acceleration with a period of time R on velocity profiles is seen in Figure 2. The velocity grows as the increased with the passing of time. According to Seth et al. (2016), thermal buoyancy force and medium permeability tended to increase fluid flow in the velocity profile. The fluid flow directions are becoming more rapid as time passes.

Figures 3 and 5 show velocity and temperature trends with varying Prandtl numbers, Pr. The Prandtl number, Pr, is the kinematic viscosity-to-thermal diffusivity ratio (Das et al., 2015). Increasing the Prandtl number in the fluid flow can reduce heat conductivity while increasing viscosity. Because of the slow rate of thermal diffusion, the thickness of the velocity boundary layer grows. As a result, the fluid's viscosity rises, resulting in a decrease in fluid velocity. When velocity rises, so does the Prandlt number (Rajput and Kumar, 2011). Furthermore, when the Prandtl number increases, the thickness of the thermal boundary layer decreases, resulting in a reduction in temperature profiles. In heat transfer problems, the Prandtl number influences the thickness of the momentum and thermal boundary layers.

Figures 4 and 6 demonstrate how velocity and temperature profiles change over time t. When the value of time is raised, so do velocity and temperature. Because of the input of external energy, the particle velocity of the fluid increases as it ages. As a result of this process, fluid velocity and temperature are increasing.

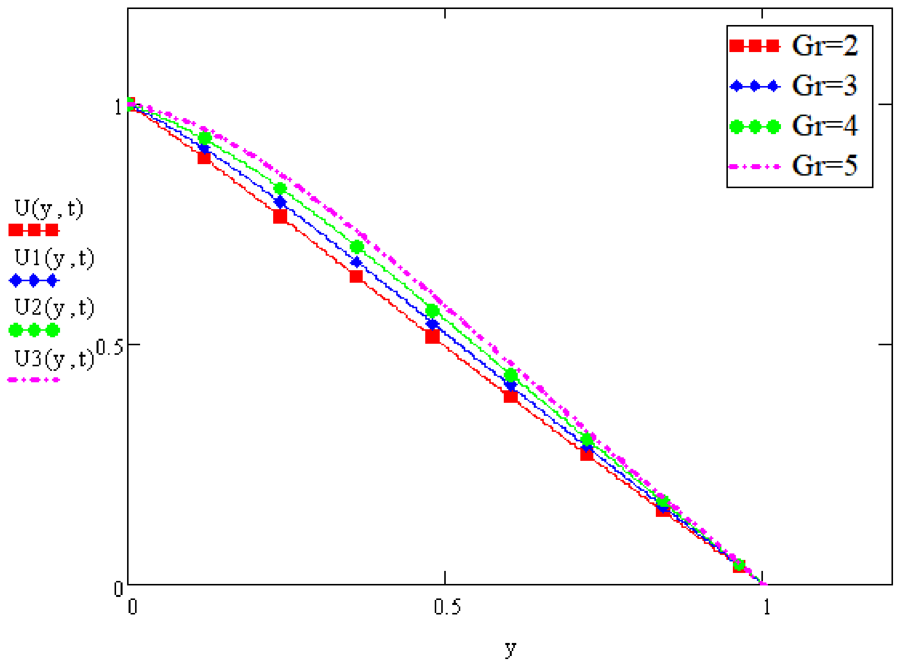

Figure 1 Velocity profiles for different values of Grashof numbers.

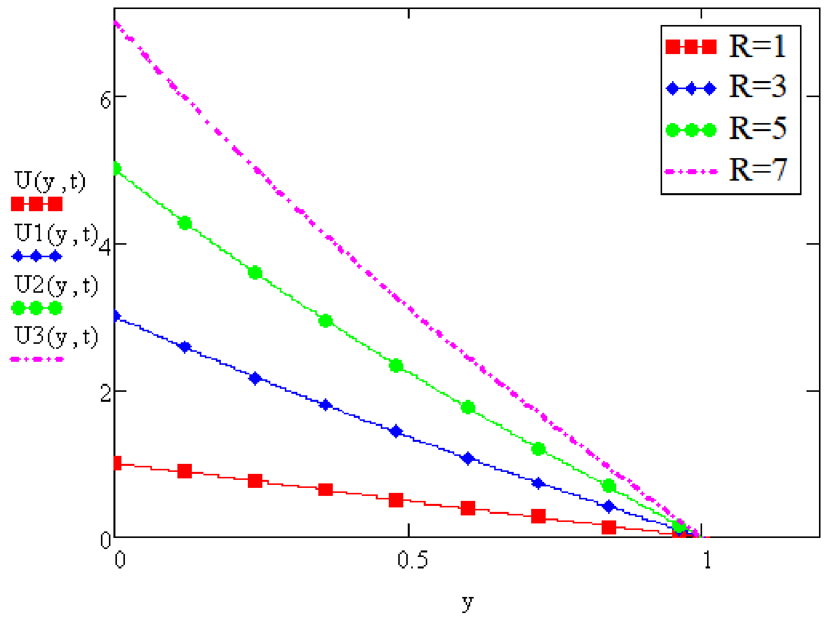

Figure 2 Velociy profiles for different values of accelerated with time.

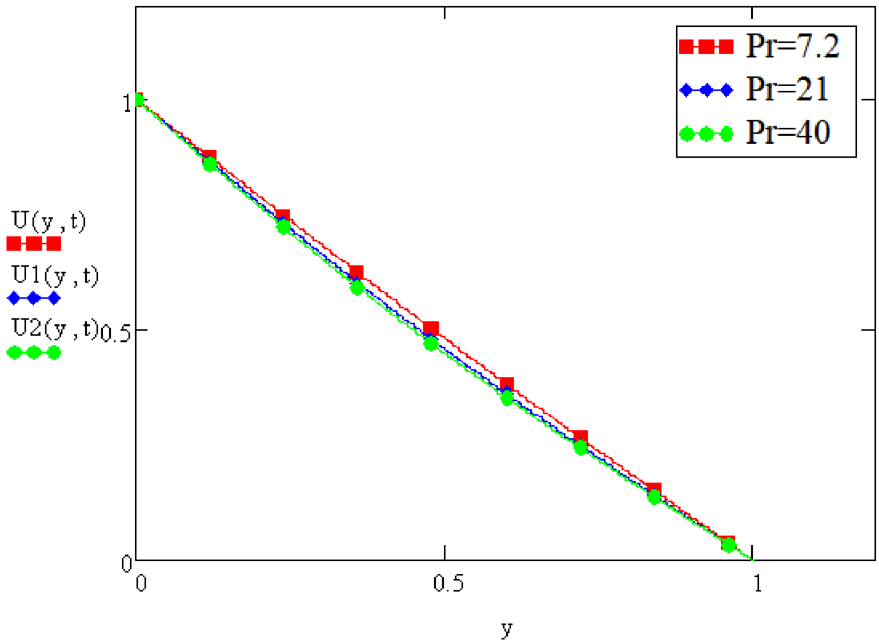

Figure 3 Velocity profiles for different values of Prandtl number Pr.

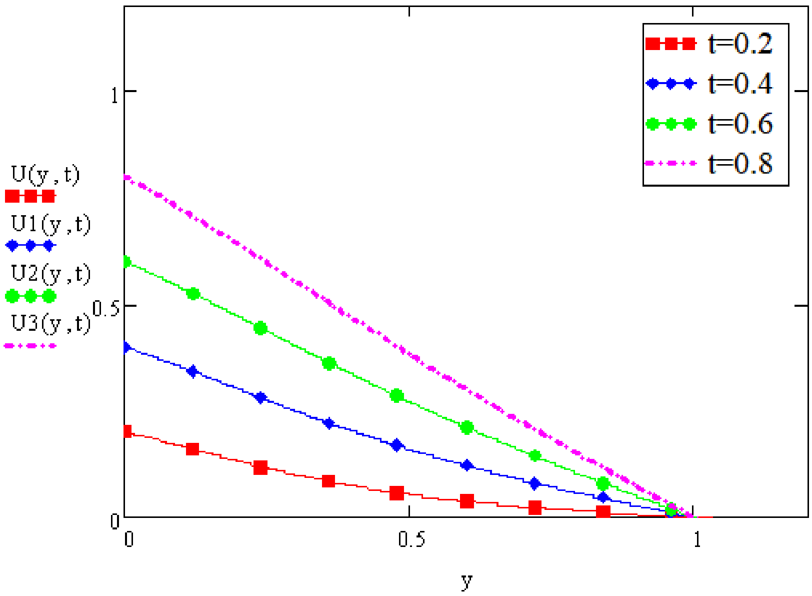

Figure 4 Velocity profiles for different values of time.

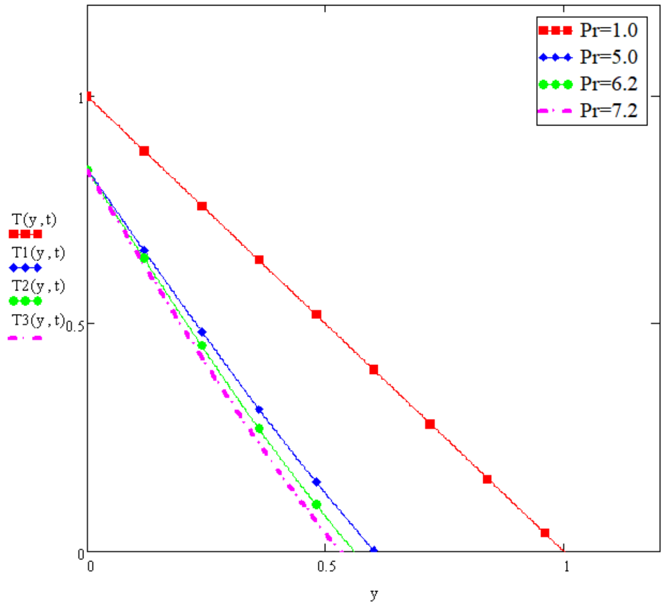

Figure 5 Temperature profiles for different values of Prandtl numbers.

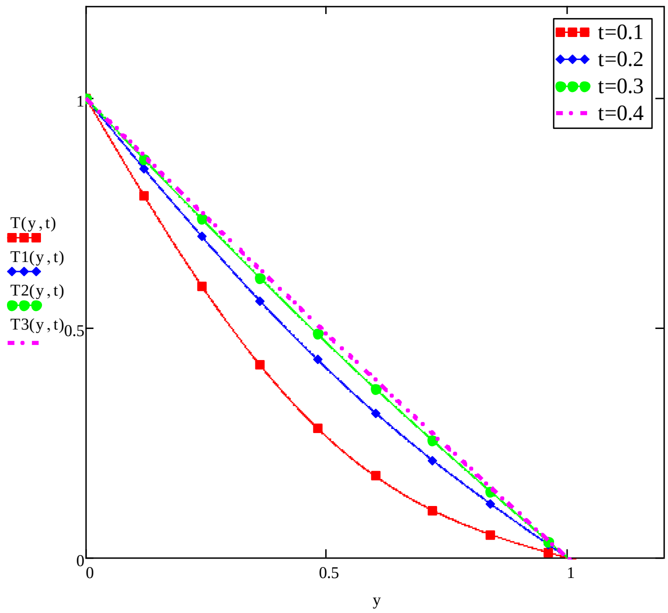

Figure 6 Temperature profiles for different values of times.

Conclusion

This thesis examined analytical solutions for unstable free convection flow of Newtonian fluid in accelerated channels. The Laplace and inverse of Laplace transform methods were used to solve this problem analytically. Finally, solutions with linked initial and boundary conditions are sufficient. For various values of the parameters involved in this issue, the resultant solutions are visually shown.

Previous research produced dimensional momentum and energy controlling equations with suitable initial and boundary conditions. Making use of dimensionless formats. Analytical solutions are then derived using the inverse Laplace transform. To summarised and showed the findings by integrating parameters into visuals. Figures 1 to 6, on the other hand, show the effect of linked components on velocity $u(y,t)$ and temperature profiles $T(y,t)$. Grashof number, Gr time, t and velocity grow with time, but Prandtl number Pr decreases with time t. As the Prandtl number, Pr, climbs and time t, increases, the temperature profiles get flatter.

To summarise, the purpose of this research is to find solutions for Newtonian fluid convection flow with accelerated channels. The study also investigates the impact of several aspects on the solutions, namely the velocity and temperature profiles equations, in order to gain a better understanding of how these parameters influence the results.

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