Proceedings of Science and Mathematics
Faculty of Science, Universiti Teknologi Malaysia

Vol. 18, 2023, page 136 – 144

Solving Mixed Convection Boundary Layer Flow of Viscoelastic Hybrid Nanofluid Past Over a Sphere

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Abstract
Over the last decade, nanofluid has been increasing in prominence for technological fields such as solar energy, biomedical applications, heat exchanges, nuclear reactor cooling and electronics device cooling systems that can help increase thermal conductivity and viscosity. However, hybrid nanofluid was introduced by many researchers to improve the thermal conductivity and heat transfer of fluid. This research investigates the problem of mixed convection boundary layer flow of viscoelastic hybrid nanofluid past. Generally, this research investigates the presence of Copper (Cu) and Aluminium Oxide (Al_2O_3) numerically and graphically to compare nanofluid and hybrid nanofluid using mathematical models that have been developed. The mathematical results are solved by obtaining the dimensionless governing equations, and transformations of the governing equations into ODEs. The MATLAB BVP4C solver was used to investigate the flow of viscoelastic hybrid nanofluid past over a sphere by giving velocity and temperature profile graphs with considering the different value of viscoelastic parameter (K), Prandtl number (Pr), mixed convection parameter (λ), and nanoparticles volume fraction ($\emptyset$).

Keywords MATLAB BVP4C solver; viscoelastic; hybrid nanofluid; sphere; boundary layer flow

1. Introduction

The name of nanofluid was introduced by Choi (1995) defined as nanofluid as manufactured colloids composed of a base fluid and nanoparticles. Recently, nanofluids play an important role in many technological fields including solar energy, biomedical applications, heat exchanges, nuclear reactor cooling and electronics device cooling systems to have better heat transfer. It is due to nanofluid can make size of thermal become more smaller to make the system more compact saving in term material weight and price in industry and technological fields. Mahian et al. (2019) agreed by saying that nanofluid are more stable than microfluids and have a greater capacity to improve heat conduction. In addition, the rate of heat exchange in thermal system is increase when used nanofluid, then the nanofluid gives better thermal conductivity compared to base fluid. According to Gamachu & Ibrahim (2021) the viscoelastic that also known as non-Newtonian fluid is used in favour of hybrid nanofluids that apply magnetic field influences normal to fluid flow.

However, hybrid nanofluid has been a productive research subject among scientists due to their potential thermal properties and functions, which give an outstanding outcome when compared to nanofluids in increasing the rate of heat transfer (Gul et al., 2021). Hybrid nanofluid can be defined as two or more nanoparticles mixed with base fluid to obtain better thermophysical and rheological characteristics in viscosity. Muneeshwaran et al. (2021), also agreed the hybrid nanofluids were created to increase thermophysical properties of base fluid and heat transfer characteristics. Gul et al. (2021) investigate that by increasing nanoparticle volume fraction ($\emptyset$), the hybrid nanofluid enhance the thermal conductivity from 5.8% to 11.947% meanwhile in case of nanofluid only enhance the thermal conductivity from 2.576% to 5.197%. Muneeshwaran et al. (2021) said that the metal and metal oxide particle composition are mostly distributed in the base fluid. Hybrid nanofluids can achieve high thermal conductivity as well as improved chemical inertness and stability by combining metal and metal oxide nanoparticles such as Copper (Cu) and Aluminium Oxide (Al_2O_3).

The flow of hybrid nanofluid is used to improve the conventional nanofluid heat transfer efficiency. Armaghani et al. (2021) studied the efficiency of Aluminium Oxide (Al_2O_3) with Multi-walled carbon nanotubes (MWCNT) in the various temperature between 25℃ to 50℃ and volume fraction between 0.125% to 1.5%. It shows their thermophysical properties such as dynamic viscosity, thermal conductivity, pumping power, convection heat transfer and heat transfer efficiency were increasing compared to nanofluid.

Wahid et al. (2022) explained that mixed convection is the form of the free convection also known as natural force and forced convection which process of heat transfer that involves the flow of fluid from hotter to colder material. Generally, free convection is a combined motion caused by a difference in density, meanwhile forced convection is a combined motion caused by an external source. Waini et al. (2022) studied the mixed convection flow and heat transfer over an exponentially stretching or shrinking vertical surface in a hybrid nanofluid that shows the velocity profile is decelerated when the mixed convection is reduced in a porous medium of a vertical surface.

2. Mathematical Model

The problem of two-dimensional (2D) mixed convection boundary layer flow of viscoelastic hybrid nanofluid past over a sphere is presented as well as taken into consideration the convective boundary condition by using two nanoparticles which are Cu and Al_2O_3 with the base fluid Carboxymethyl cellulose solution (CMC).

The flow of the fluid in this study is considered moving past over a sphere of a radius a, which is immersed in a viscoelastic hybrid nanofluid. The assumption that the velocity outside the boundary donate as $\bar{u}_e(\bar{x})$, also u and v are the velocity components of the fluid along x and y. By Salleh et al. (2021), T_w denote as surface temperature of the sphere while T_∞ is ambient temperature of hybrid nanofluid and the researchers proposed $T_w < T_\infty$ correspond to a cooling sphere (opposing flow) and $T_w > T_\infty$ corresponds to a heating sphere (assisting flow). Figure 1 showed that the physical model and coordinate system of the problem from Alzu'bi et al. (2022).

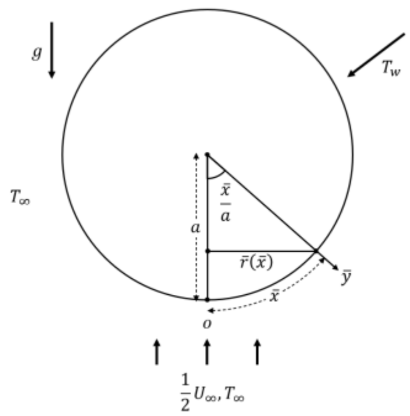

Figure 1 Coordinates system and flow model

The mathematical model for the problem consists of three equations as follow:

Continuity equation:

$$\frac{\partial}{\partial\bar{x}}(\bar{r}\,\bar{u})+\frac{\partial}{\partial\bar{y}}(\bar{r}\,\bar{v})=0. \quad (1)$$

Momentum equation:

$$\rho_{hnf}\left(\bar{u}\frac{\partial\bar{u}}{\partial\bar{x}}+\bar{v}\frac{\partial\bar{u}}{\partial\bar{y}}\right)$$
$$=\rho_{hnf}\left(\bar{u}_e\frac{\partial\bar{u}_e}{\partial\bar{x}}\right)+\mu_{hnf}\frac{\partial^2\bar{u}}{\partial\bar{y}^2}+k_0\left(\frac{\partial}{\partial\bar{x}}\left(\bar{u}\frac{\partial^2\bar{u}}{\partial\bar{y}^2}\right)+\bar{v}\frac{\partial^3\bar{u}}{\partial\bar{y}^3}\right.$$
$$\left.+\frac{\partial\bar{u}}{\partial\bar{y}}\frac{\partial^2\bar{v}}{\partial\bar{y}^2}+g(\rho\beta)_{hnf}(T-T_\infty)\sin\left(\frac{\bar{x}}{\alpha}\right)\right). \quad (2)$$

Energy equation:

$$\bar{u}\frac{\partial T}{\partial\bar{x}}+\bar{v}\frac{\partial T}{\partial\bar{y}}=\alpha_{hnf}\frac{\partial^2 T}{\partial\bar{y}^2}. \quad (3)$$

Subject to boundary conditions

$$\bar{u}=0,\quad \bar{v}=0,\quad T=T_w,\quad on\ \bar{y}=0,\quad \bar{x}\geq 0,\quad \bar{u}=\bar{u}_e(\bar{x}), \quad (4)$$
$$\frac{\partial\bar{u}}{\partial\bar{y}}=0,\quad T=T_\infty,\quad \text{as } \bar{y}\to\infty,\quad \bar{x}\geq 0.$$

where

$$\alpha_{hnf}=\frac{k_{hnf}}{(\rho C_p)_{hnf}},\quad a_{bf}=\frac{k_{bf}}{(\rho C_p)_{bf}}, \quad (5)$$
$$\mu_{hnf}=\frac{\mu_{bf}}{(1-\emptyset_{np1})^{2.5}(1-\emptyset_{np2})^{2.5}},$$
$$\rho_{hnf}=(1-\emptyset_{np2})\left[(1-\emptyset_{np1})\rho_{bf}+\emptyset_{np1}\rho_{np1}\right]+\emptyset_{np2}\rho_{np2},$$
$$(\rho C_p)_{hnf}=(1-\emptyset_{np2})\left[(1-\emptyset_{np1})(\rho C_p)_{bf}+\emptyset_{np1}(\rho C_p)_{np1}\right]+\emptyset_{np2}(\rho C_p)_{np2},$$
$$(\rho\beta)_{hnf}=(1-\emptyset_{np2})(1-\emptyset_{np1})(\rho\beta)_{bf}+\emptyset_{np1}(\rho\beta)_{np1}+\emptyset_{np2}(\rho\beta)_{np2},$$
$$\frac{k_{hnf}}{k_{bf}}=\frac{k_{np2}+(n-1)k_{bf}-(n-1)\emptyset_{np2}(k_{bf}-k_{np2})}{k_{np2}+(n-1)k_{bf}+\emptyset_{np2}(k_{bf}-k_{np2})},$$
$$\frac{k_{nf}}{k_{bf}}=\frac{(n-1)k_{bf}+k_{np1}-(n-1)\emptyset_{np1}(k_{bf}-k_{np1})}{(n-1)k_{bf}+k_{np1}+\emptyset_{np1}(k_{bf}-k_{np1})}.$$

For $\bar{x}$ and $\bar{y}$ are the Cartesian coordinates along the surface of the sphere. $\bar{y}$ is the coordinate measured normally to the surface of the sphere. $\bar{u}$ and $\bar{v}$ denote as velocity of components, g is the gravity acceleration, n is direction of flow, T is temperature of selected fluid, $k_0 > 0$ is the constant of the viscoelastic material (Walter's Liquid-B model), v is the kinematic viscosity, $\emptyset$ is the fraction of nanoparticles volume, np_1 and np_2 are represent the nanoparticles of Cu and Al_2O_3. bf, nf, hnf are represent base fluid, nanofluid and hybrid nanofluid respectively. β is coefficient of thermal expansions, C_p is heat capacitance, k is thermal conductivities, ρ is density, μ is dynamic viscosities and α is the thermal diffusivities. $\bar{u}_e(\bar{x})$ is the local free stream velocity outside the boundary layer, and $\bar{r}(\bar{x})$ is the radial distance from the symmetrical axis to the surface of the sphere which are given by

$$\bar{u}_e(\bar{x})=\frac{3}{2}U_\infty\sin\left(\frac{\bar{x}}{a}\right)\text{ and }\bar{r}(\bar{x})=\alpha\sin\left(\frac{\bar{x}}{a}\right). \quad (6)$$

Table 1 shows the thermophysical properties of the base fluid (CMC-water) and nanoparticles of Cu and Al_2O_3.

Table 1 Thermophysical properties of base fluid with nanoparticles of Cu and Al_2O_3.

Physical properties	Based fluid (water)	Copper (Cu)	Aluminium Oxide (Al_2O_3)
$\rho(kg/m^3)$	997.1	8933	3970
$C_p(J/kgK)$	4179	385	765
$k(W/mK)$	0.613	400	40
$\beta(1/K)$	21 × 10^{-5}	1.67 × 10^{-5}	0.85 × 10^{-5}

By Nazar et al. (2011) and Patil et al. (2021) the dimensionless variables are represented as

$$x = \frac{\bar{x}}{a}, \quad y = Re^{\frac{1}{2}}\left(\frac{\bar{y}}{a}\right), \quad u = \frac{\bar{u}}{U_\infty}, \quad v = Re^{\frac{1}{2}}\left(\frac{\bar{v}}{U_\infty}\right), \tag{7}$$

$$r(x) = \frac{\bar{r}\,\bar{x}}{a}, \qquad u_e(x) = \frac{\bar{u}_e\,(\bar{x})}{U_\infty}, \qquad \theta = \frac{(T - T_\infty)}{(T_w - T_\infty)}.$$

and $Re = \frac{U_\infty a}{v}$ is the Reynolds number. By using the Equation (7) substitute into Equation (1) to (3) to perform dimensionless equations.

Continuity equation:

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial y}(rv) = 0. \tag{8}$$

Momentum equation:

$$\begin{aligned}
&\left[(1-\emptyset_{np2})\left[(1-\emptyset_{np1}) + \emptyset_{np1}\frac{\rho_{np1}}{\rho_{bf}}\right] + \frac{\emptyset_{np2}\rho_{np2}}{\rho_{bf}}\right]\left(\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) \\
&= \left[(1-\emptyset_{np2})\left[(1-\emptyset_{np1}) + \emptyset_{np1}\frac{\rho_{np1}}{\rho_{bf}}\right] + \frac{\emptyset_{np2}\rho_{np2}}{\rho_{bf}}\right]\left(u_e\frac{\partial u_e}{\partial x}\right) \\
&+ \frac{1}{(1-\emptyset_{np1})^{2.5}(1-\emptyset_{np2})^{2.5}}\left(\frac{\partial^2 u}{\partial y^2}\right) \\
&+ K\left(\frac{\partial}{\partial x}u\frac{\partial^2 u}{\partial y^2} + v\frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2}\right) \\
&+ \lambda\theta\sin(x)\left[(1-\emptyset_{np2})(1-\emptyset_{np1}) + \emptyset_{np1}\frac{(\rho\beta)_{np1}}{(\rho\beta)_{bf}}\right. \\
&\left. + \emptyset_{np2}\frac{(\rho\beta)_{np2}}{(\rho\beta)_{bf}}\right].
\end{aligned} \tag{9}$$

Energy equation:

$$\begin{aligned}
&\left((1-\emptyset_{np2})\left[(1-\emptyset_{np1}) + \emptyset_{np1}\frac{(\rho C_p)_{np1}}{(\rho C_p)_{bf}}\right] + \emptyset_{np2}\frac{(\rho C_p)_{np2}}{(\rho C_p)_{bf}}\right)\left(u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y}\right) \\
&= \frac{1}{Pr}\left[\frac{k_{np2} + (n-1)k_{bf} - (n-1)\emptyset_{np2}(k_{bf} - k_{np2})}{k_{np2} + (n-1)k_{bf} + \emptyset_{np2}(k_{bf} - k_{np2})}\right]\frac{\partial^2\theta}{\partial y^2},
\end{aligned} \tag{10}$$

with boundary conditions

$$u = 0, \quad v = 0, \quad \theta' = -1, \quad on\; y = 0, \quad x \geq 0, \tag{11}$$

$$u = u_e\,(x) = \frac{3}{2}\sin x, \quad \frac{\partial u}{\partial y} = 0, \quad \theta = 0, \quad as\; y \to \infty, \quad x \geq 0.$$

Where $K = \frac{k_0 U_\infty}{a v \rho_{bf}}$ is the dimensionless viscoelastic parameter, $Pr = \frac{v}{a_{bf}}$ is the Prandtl number, $\lambda = \frac{Ge}{Re^2} = \frac{g\beta_{bf}(T_w - T_\infty)a}{{U_\infty}^2}$ is the constant mixed convection parameter. For $\lambda < 0$ correspond to a cooling sphere (opposing flow), $\lambda > 0$ corresponds to a heating sphere (assisting flow) and the forced convection flow when $\lambda = 0$. $K = 0$ is for the case of viscous (Newtonian) fluid.

To solve Equation (8) to Equation (10), with the boundary conditions Equation (11), the following variables are being assumed as

$$\psi = x r(x) f(x, y), \quad \theta = \theta(x, y). \tag{12}$$

where ψ denote as stream function which are,

$$u = \frac{1}{r}\frac{\partial \psi}{\partial y}, \quad v = -\frac{1}{r}\frac{\partial \psi}{\partial x}. \tag{13}$$

Substitute Equation (8) to Equation (11) with Equation (13), and consider

$$u_e\,(x) = \frac{\bar{u}_e\,(\bar{x})}{U_\infty} = \frac{3}{2}\sin x \text{ and } r(x) = \sin x.$$

Continuity equation:

$$\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial x \partial y} = 0. \tag{14}$$

Momentum equation:

$$\begin{aligned}
C_1 &\left[\frac{-\cos x}{\sin x}\left(\frac{\partial f}{\partial y}\right)^2 + x\left(\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x \partial y} - \frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}\right) + \left(\frac{x\cos x}{\sin x} + 1\right)\left(\left(\frac{\partial f}{\partial y}\right)^2 - \frac{\partial^2 f}{\partial y^2} f\right)\right] \\
&= C_1\left(\frac{9}{4}\frac{\sin x\,\cos x}{x}\right) + C_2\left(\frac{\partial^3 f}{\partial y^3}\right) \\
&+ K\left[x\frac{\partial^2 f}{\partial x \partial y}\frac{\partial^3 f}{\partial y^3} + 2\left(\frac{x\cos x}{\sin x} + 1\right)\left(\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}\right) + x\frac{\partial^4 f}{\partial x \partial y^3}\frac{\partial f}{\partial y}\right. \\
&- 2\left(\frac{x\cos x}{\sin x}\right)\left(\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}\right) - x\frac{\partial f}{\partial x}\frac{\partial^4 f}{\partial y^4} - \left(\frac{x\cos x}{\sin x} + 1\right)\frac{\partial^4 f}{\partial y^4} f \\
&\left. - x\frac{\partial^3 f}{\partial x \partial y^2}\frac{\partial^2 f}{\partial y^2} - \left(\frac{x\cos x}{\sin x} + 1\right)\left(\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial y^2}\right)\right] + C_3\lambda\theta\frac{\sin(x)}{x}.
\end{aligned} \tag{15}$$

Energy equation:

$$C_4\left[\left(x\frac{\partial f}{\partial y}\frac{\partial \theta}{\partial x}\right) - \left(\left(x\frac{\partial f}{\partial x}\right) + \left(1 + x\frac{\cos x}{\sin x}\right) f\right)\frac{\partial \theta}{\partial y}\right] = C_5\frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2}. \tag{16}$$

The boundary conditions are represented as

$$\begin{aligned}
&f = 0, \quad \frac{\partial f}{\partial y} = 0, \quad \theta' = -1 \quad on\; y = 0, \quad x \geq 0, \\
&\frac{\partial f}{\partial y} \to \frac{3}{2}\frac{\sin x}{x}, \quad \frac{\partial^2 f}{\partial y^2} = 0, \quad \theta \to 0, \quad as\; y \to \infty, \quad x \geq 0
\end{aligned} \tag{17}$$

where at the lower stagnation point of the sphere $x \approx 0$, the Equation (15) and Equation (16) become ordinary differential equations as shown below:

Momentum equation:

$$C_1\left(2ff''-f'^2+\frac{9}{4}\right)+C_2f'''+2K\left(f'f'''-ff''''-f''^2\right)+C_3\lambda\theta=0. \tag{18}$$

Energy equation:

$$2C_4f\theta'+C_5\frac{1}{Pr}\theta''=0. \tag{19}$$

Then, the boundary conditions turn to

$$f=0,\quad \frac{\partial f}{\partial y}=0,\quad \theta'=-1\quad on\ y=0,\quad x\geq 0, \tag{20}$$
$$\frac{\partial f}{\partial y}\to\frac{3}{2},\quad \frac{\partial^2 f}{\partial y^2}=0,\quad \theta\to 0,\quad as\ y\to\infty,\quad x\geq 0.$$

3. Research Methodology

The BVP4C solver can be used if the problem equations have converted to first order equation. Then the command syntax be used in the BVP4C solver is sol=bvp4c(odefun,bcfun,solinit) that integrates a system of differential equations, which odefun of the form $y' = f(x,y)$, bcfun is the boundary conditions described by and the initial solution guess solinit. The function of bvpinit is use to create the initial guess solinit, which also defines the points at which the boundary conditions in bcfun are enforced. The following equation is written as first order equation.

$$y_1=f,\quad y_2=f',\quad y_3=f'',\quad y_4=f''',\quad {y_4}'=f'''', \tag{21}$$
$$y_5=\theta,\quad y_6=\theta',\quad y_6'=\theta''.$$

Momentum equation:

$$y_4'=\frac{1}{y_1}\left[y_2y_4-{y_3}^2+\frac{1}{2K}\left(C_1\left(2y_1y_3-{y_2}^2+\frac{9}{4}\right)+C_2y_4++C_3\lambda y_5\right)\right]=0. \tag{22}$$

Energy equation:

$$2C_4y_1y_6+C_5\frac{1}{Pr}{y_6}'=0 \tag{23}$$

where

$$C_1=\left(1-\emptyset_{np2}\right)\left[\left(1-\emptyset_{np1}\right)+\emptyset_{np1}\frac{\rho_{np1}}{\rho_{bf}}\right]+\frac{\emptyset_{np2}\rho_{np2}}{\rho_{bf}},$$

$$C_2=\frac{1}{\left(1-\emptyset_{np1}\right)^{2.5}\left(1-\emptyset_{np2}\right)^{2.5}},$$

$$C_3=\left(1-\emptyset_{np2}\right)\left(1-\emptyset_{np1}\right)+\emptyset_{np1}\frac{(\rho\beta)_{np1}}{(\rho\beta)_{bf}}+\emptyset_{np2}\frac{(\rho\beta)_{np2}}{(\rho\beta)_{bf}},$$

$$C_4=\left(1-\emptyset_{np2}\right)\left[\left(1-\emptyset_{np1}\right)+\emptyset_{np1}\frac{(\rho C_p)_{np1}}{(\rho C_p)_{bf}}\right]+\emptyset_{np2}\frac{(\rho C_p)_{np2}}{(\rho C_p)_{bf}},$$

$$C_5=\frac{k_{np2}+(n-1)k_{bf}-(n-1)\emptyset_{np2}\left(k_{bf}-k_{np2}\right)}{k_{np2}+(n-1)k_{bf}+\emptyset_{np2}\left(k_{bf}-k_{np2}\right)}.$$

Boundary conditions

$$y_1(0) = 0, \quad y_2(0) = 0, \quad y_6(0) = -1, \tag{24}$$
$$y_2(\infty) = \frac{3}{2}, \quad y_3(\infty) = 0, \quad y_5(\infty) = 0.$$

4. Results and Discussion

Figure 2 is obtained for velocity and temperature profiles of hybrid nanofluid at lower stagnation point $x \approx 0$ by considering different values of *K* when Pr=1, $\emptyset = 0.1$ and $\lambda = 1$. The result shows when the value of *K* increases then the velocity profiles decrease, but the temperature profiles increase. Based on the figure when x increases then the velocity profiles have higher value and the temperature profiles have lower value compared to nanofluid.

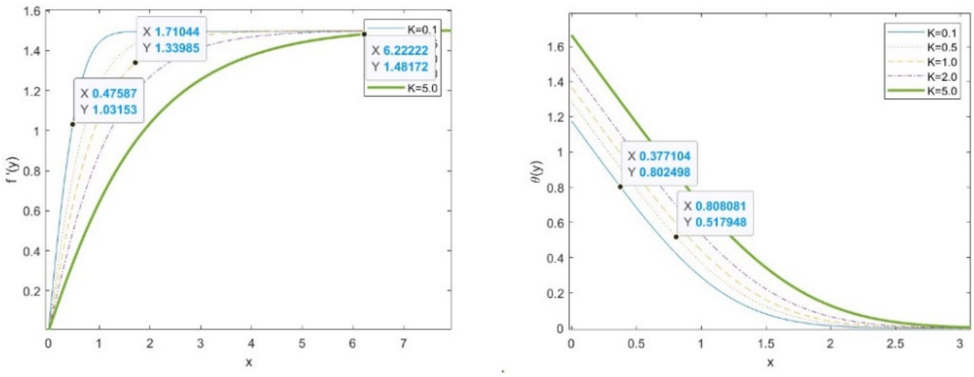

Figure 2 Velocity and temperature profile for different value of *K.*

Figure 3 is obtained for velocity and temperature profile of hybrid nanofluid at lower stagnation point $x \approx 0$ by considering different values of Pr when *K*=1, $\emptyset = 0.1$ and $\lambda = 1$. Based on the figures, it is noticed when the value of Pr increases, then the velocity and temperature profiles decrease due to if values of Pr are higher, then the viscous forces become stronger, and the thermal forces become weaker. According to the figure, it shows that when x increases then the value of velocity higher, and the value of temperature lower compared to nanofluid.

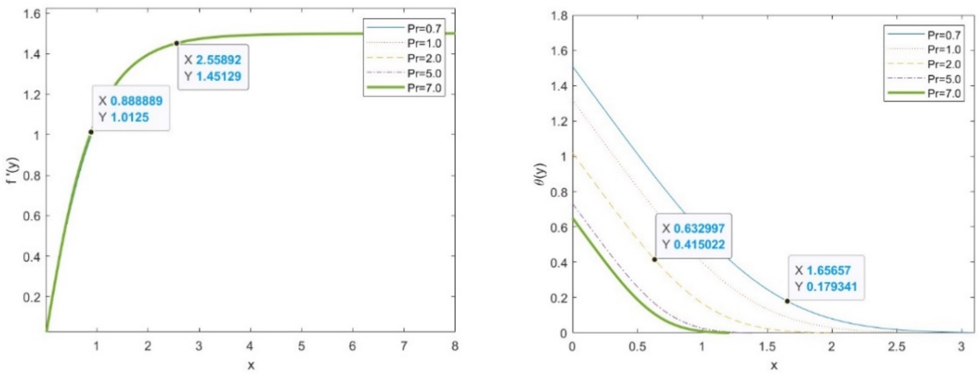

Figure 3 Velocity and Temperature profile for different value of Pr.

Figure 4 is obtained for velocity and temperature profile of hybrid nanofluid at lower stagnation point $x \approx 0$ by considering different values of λ when *K*=1, $\emptyset = 0.1$ and Pr=1. Based on figures below, it is observed the increment of the value λ, the velocity profiles decrease, but the temperature profiles increase. The figures below show that when x increase then the velocity profiles give higher value, and the temperature profiles give lower value compared to nanofluid.

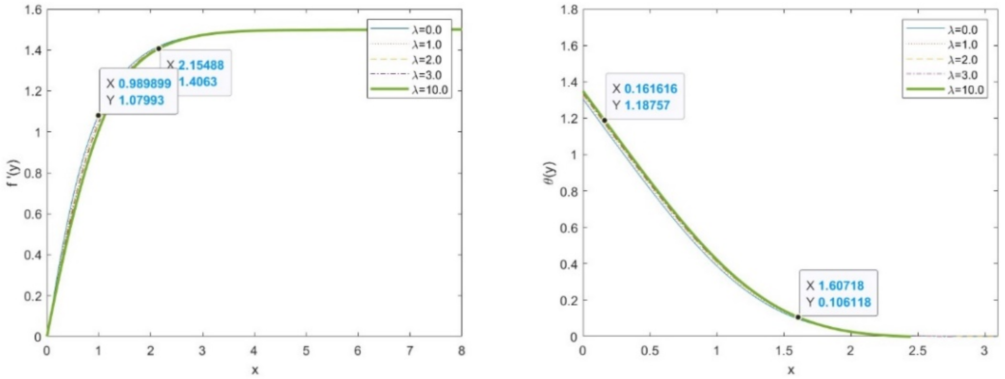

Figure 4 Velocity and temperature profile for different value of λ.

Figure 5 are obtained from velocity and temperature profile of hybrid nanofluid at lower stagnation point $x \approx 0$ by considering different values of $\emptyset$ when K=1, $\lambda = 1$ and Pr=1. By increment of the value $\emptyset$, the velocity profiles will decrease, and the temperature profiles will increase. According to the figure, it shows that, when x increase then the velocity profiles give higher value, and the temperature profiles give lower value compared to nanofluid. It is due to lower specific heat of nanoparticles and its higher thermal conductivity than that of the base fluid.

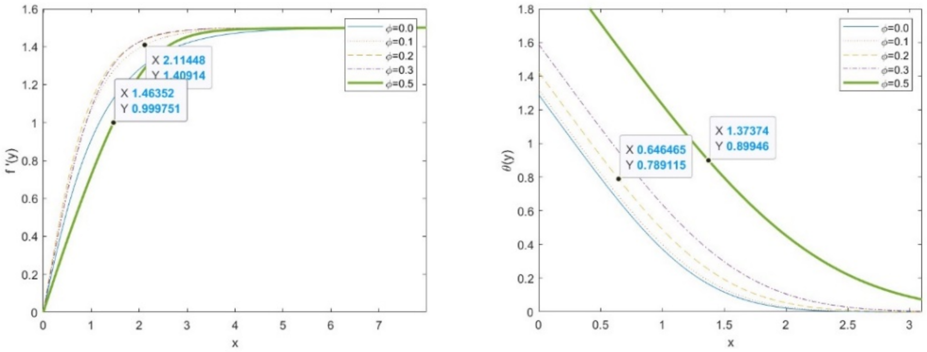

Figure 5 Velocity and temperature profile for different value of $\emptyset$.

Conclusion

From the results obtained, when the value of K, λ and $\emptyset$ increases, the velocity profile will decrease, but the temperature profile will be increasing. However, the velocity and temperature profiles of hybrid nanofluids are inversely proportional to the Pr.

In this research, it is worth mentioning that using more than one nanoparticle in base fluid can improve the heat transfer characteristics of the fluid. This is because hybrid nanofluids have better thermal conductivity compared to nanofluid. The problem that related to flow hybrid nanofluid can be used widely because hybrid nanofluid play an important role in many technological fields including solar energy, biomedical applications, heat exchanges, nuclear reactor cooling and electronics device cooling systems.

References

[1] Alzu'bi, O. A. S., Alwawi, F. A., Swalmeh, M. Z., Sulaiman, I. M., Hamarsheh, A. S., and Ibrahim, M. A. H. (2022). Energy Transfer through a Magnetized Williamson Hybrid Nanofluid Flowing

around a Spherical Surface: Numerical Simulation. *Mathematics*, *10*(20). https://doi.org/10.3390/math10203823

[2] Armaghani, T., Sadeghi, M. S., Rashad, A. M., Mansour, M. A., Chamkha, A. J., Dogonchi, A. S., and Nabwey, H. A. (2021). MHD mixed convection of localized heat source/sink in an Al2O3-Cu/water hybrid nanofluid in L-shaped cavity. *Alexandria Engineering Journal*, *60*(3), 2947–2962. https://doi.org/10.1016/j.aej.2021.01.031

[3] Choi, S. U., & Eastman, J. A. (1995). *Enhancing thermal conductivity of fluids with nanoparticles* (No. ANL/MSD/CP-84938; CONF-951135-29). Argonne National Lab.(ANL), Argonne, IL (United States).

[4] Gamachu, D., & Ibrahim, W. (2021). Mixed convection flow of viscoelastic Ag-Al2O3/water hybrid nanofluid past a rotating disk. *Physica Scripta*, *96*(12). https://doi.org/10.1088/1402-4896/ac1a89

[5] Gul, T., Ali, B., Alghamdi, W., Nasir, S., Saeed, A., Kumam, P., … Jawad, M. (2021). Mixed convection stagnation point flow of the blood based hybrid nanofluid around a rotating sphere. *Scientific Reports*, *11*(1), 1–15. https://doi.org/10.1038/s41598-021-86868-x

[6] Mahian, O., Kolsi, L., Amani, M., Estellé, P., Ahmadi, G., Kleinstreuer, C., … Pop, I. (2019). Recent advances in modeling and simulation of nanofluid flows-Part I: Fundamentals and theory. *Physics Reports*, *790*, 1–48. https://doi.org/10.1016/j.physrep.2018.11.004

[7] Muneeshwaran, M., Srinivasan, G., Muthukumar, P., and Wang, C. C. (2021). Role of hybrid-nanofluid in heat transfer enhancement – A review. *International Communications in Heat and Mass Transfer*, *125*(May), 105341. https://doi.org/10.1016/j.icheatmasstransfer.2021.105341

[8] Nazar, R., Tham, L., Pop, I., & Ingham, D. B. (2011). Mixed Convection Boundary Layer Flow from a Horizontal Circular Cylinder Embedded in a Porous Medium Filled with a Nanofluid. *Transport in Porous Media*, *86*(2), 517–536. https://doi.org/10.1007/s11242-010-9637-1

[9] Salleh, S. N. A., Bachok, N., and Pop, I. (2021). Mixed convection stagnation point flow of a hybrid nanofluid past a permeable flat plate with radiation effect. *Mathematics*, *9*(21), 3323–3333. https://doi.org/10.3390/math9212681

[10] Patil, P. M., Shankar, H. F., Hiremath, P. S., & Momoniat, E. (2021). Nonlinear mixed convective nanofluid flow about a rough sphere with the diffusion of liquid hydrogen. *Alexandria Engineering Journal*, *60*(1), 1043–1053. https://doi.org/10.1016/j.aej.2020.10.029

[11] Tahar Tayebi & Ali J. Chamkha (2016). Free convection enhancement in an annulus between horizontal confocal elliptical cylinders using hybrid nanofluids, Numerical Heat Transfer, Part A: Applications, 70:10, 1141-1156

[12] Wahid, N. S., Md Arifin, N., Khashi'ie, N. S., Pop, I., Bachok, N., & Hafidzuddin, M. E. H. (2022). MHD mixed convection flow of a hybrid nanofluid past a permeable vertical flat plate with thermal radiation effect. *Alexandria Engineering Journal*, *61*(4), 3323–3333. https://doi.org/10.1016/j.aej.2021.08.059

[13] Waini, I., Ishak, A., & Pop, I. (2020). Mixed convection flow over an exponentially stretching/shrinking vertical surface in a hybrid nanofluid. *Alexandria Engineering Journal*, *59*(3), 1881–1891. https://doi.org/10.1016/j.aej.2020.05.030