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# Analysis of Three Species Lotka-Volterra Predator-Prey Models with Omnivory

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**Abstract**
Predator-prey model is the study that depicts the interaction between predator and prey in real life. It is a prominent subject among researchers who are interested in mathematical ecology. This model implies that predator populations negatively impact prey populations. The predator-prey model system is modelled using ordinary differential equations to describe the dynamic behaviour of the systems. The purpose of the study is to examine the predator-prey model with omnivore as the top predator on the stability based on the concept of Lotka-Volterra in the predator-prey model. The objectives of this study are i) to formulate model of Lotka-Volterra predator-prey with omnivory, ii) to solve the predator-prey model using analytical solution, iii) to analyze the predator-prey model with omnivore as the top predator. In analysing the models, the stability of the equilibrium point is obtained and described by using the properties of the eigenvalues and the Routh-Hurwitz Criteria. Last but not least, numerical simulation and graph analysis are provided to demonstrate the stability of equilibrium points.
**Keywords** Predator-prey model; omnivory; stability; equilibrium points

## Introduction

The Predator-prey model is the oldest studies or in other words the classical mathematical model and it is also called as first model that illustrate the relationship between two species which are predators and prey. This model is thought to indicate the predator populations have adverse effects on the prey populations and this system using Lotka-Volterra model developed by Vito Volterra and Alfred Lotka in 1925 (Boyce and DiPrima, 2010).

Al-khedhairi et al., (2018) said that numerous intriguing dynamical characteristics, including the stability of equilibrium points, the occurrence of Hopf bifurcation, and the existence of limit cycles have been found. Therefore, this study concern on analyzing the stability of the predator-prey model with omnivory by using the method of differential equation.

The ecological behavior of omnivory, which is generally described as eating on more than one trophic level, should be commonplace. The omnivore should do better in a variety of habitats than certain more specialised feeders, at the most fundamental level, since it is a generalist. The apparent fact that omnivory is uncommon in nature and unstable in food webs, however, has somehow replaced the conundrum of which, why, and how creatures are omnivorous (Agrawal, 2003).

Using three species and an omnivore as the top predator, this paper will use two distinct Lotka-Volterra prey-predator equations. This study examines three hypothetical food webs consisting of omnivorous animals organised in a predator-prey hierarchy. This study will examine the Lotka-Volterra model of food webs for these three species, taking into account the presence of omnivores. In this paper, we will use the concept of equilibrium points to analyse the dynamics of interactions between three species in an environment where an omnivore serves as a top predator.

**Predator-prey model with omnivore as top predator**

There are two predators and one prey model is considered. Figure 1 demonstrates the schematic interactions between three predator-prey species and the omnivore system.

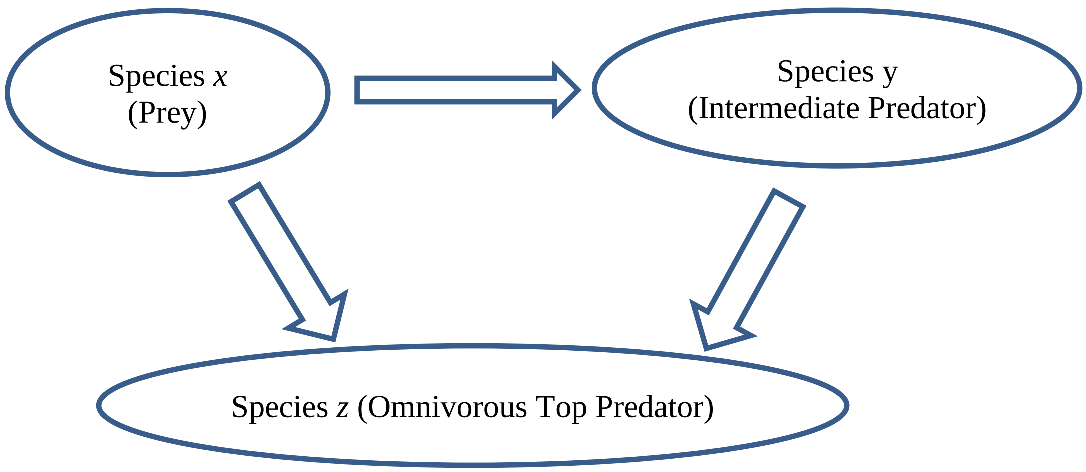


Figure 1 The feeding connection within a food chain involving omnivores. The direction of arrow is from prey to omnivore as top predator.

The model in this study is analysing three species Lotka-Volterra food web with omnivory. The population change through time according to each species. The predator-prey models with omnivory can be governed by the following system of differential equations.

$$\begin{aligned}
\frac{dx}{dt} &= x(1 - x - y - \mu z) = f(x, y, z), \\
\frac{dy}{dt} &= y(-d_1 + \alpha x - \beta z) = g(x, y, z), \\
\frac{dz}{dt} &= z(-d_2 + \gamma x + \delta y) = h(x, y, z),
\end{aligned} \tag{1}$$

where *x*, *y* and *z* respectively represent the resource, intermediate predator and top predator. The parameters $d_1$ and $d_2$ represent the death rates of intermediate and top predator respectively. Symbol $\mu$ represents the contribution of the resource to the top predator. Whereas α and β represent the contribution rates of the resource and top predator. While parameter $\gamma$ and $\delta$ indicate the birth rate of top predator contribute by resource and intermediate predator respectively.

By examining the model, there are three equilibrium points (EPs) which are non-trivial, semi-trivial, or coexistence could be found. The equilibrium points will be calculated. To find the equilibrium points, the system in (1.1), (1.2), and (1.3) is set to zero such as:

$$\begin{aligned} \frac{dx}{dt} &= x(1 - x - y - \mu z) = 0, \\ \frac{dy}{dt} &= y(-d_1 + \alpha x - \beta z) = 0, \\ \frac{dz}{dt} &= z(-d_2 + \gamma x + \delta y) = 0. \end{aligned} \tag{2}$$

Therefore, the equilibrium points for the predator-prey model with omnivore as top predator are:

1) $\bar{E}_0 = (x, y, z) = (0,0,0)$,
2) $\bar{E}_1 = (x, y, z) = (1,0,0)$,
3) $\bar{E}_2 = (x, y, z) = \left(\frac{d_2}{\gamma}, 0, \frac{\gamma - d_2}{\mu\gamma}\right)$, $\gamma > d_2, \mu > 0$.
4) $\bar{E}_3 = (x, y, z) = \left(\frac{d_1}{\alpha}, 1 - \frac{d_1}{\alpha}, 0\right)$. $\alpha > d_1$.

To determine the stability of these equilibrium points, the Jacobian Matrix will be used and can be obtained as

$$J = \begin{pmatrix} \frac{df}{dx} & \frac{df}{dy} & \frac{df}{dz} \\ \frac{dg}{dx} & \frac{dg}{dy} & \frac{dg}{dz} \\ \frac{dh}{dx} & \frac{dh}{dy} & \frac{dh}{dz} \end{pmatrix} = \begin{pmatrix} 1 - 2x - y - \mu z & -x & -\mu x \\ \alpha y & -d_1 + \alpha x - \beta z & -\beta y \\ \gamma z & \delta z & -d_2 + \gamma x + \delta y \end{pmatrix}. \tag{3}$$

All the equilibrium points that have been identified before may be used to verify the systems' stability. If the eigenvalues satisfy the characteristic of eigenvalues and Routh-Hurtwiz conditions, then the equilibrium point is stable. Then, by using all of the stable equilibrium points, we can see how each species is biologically interpreted.

**Results and Discussion**

This section will look at how the presence of omnivores at the top of the food chain affects the stability of the predator-prey model. Omnivores do not restrict their diet to solely species *x*, but instead devour them alongside intermediate predators. The following figures illustrate the outcome of a numerical simulation of the system's stable equilibrium points:

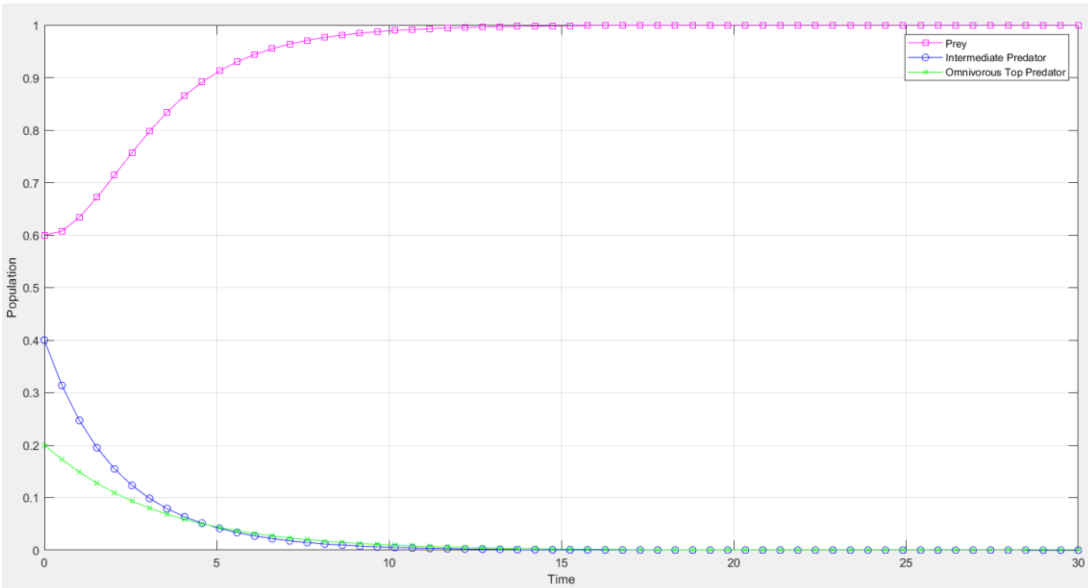


Figure 2 The graph of the population with omnivorous top predator versus time when (1,0,0) is a stable equilibrium point where $\alpha = 0.1$, $\beta = 0.2$, $\gamma = 0.1$, $\delta = 0.15$, $\mu = 0.1$, $d_1 = 0.5$, $d_2 = 0.4$.

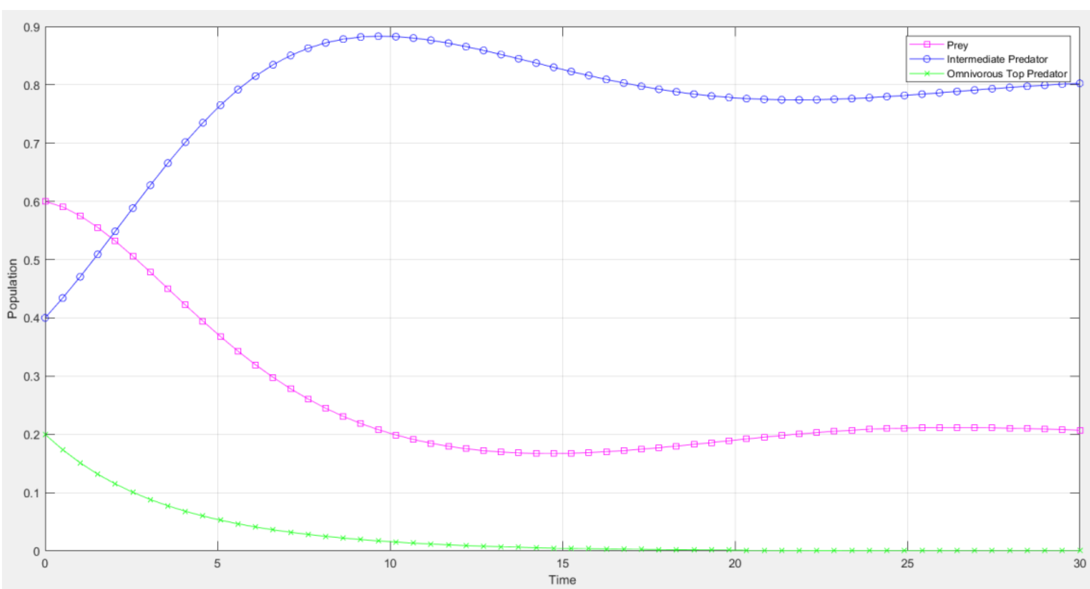

Figure 3 The graph of the population with omnivorous top predator versus time when (0.2,0.8,0) is a stable equilibrium point where $\alpha = 0.5$, $\beta = 0.2$, $\gamma = 0.1$, $\delta = 0.15$, $\mu = 0.1$, $d_1 = 0.1$, $d_2 = 0.4$.

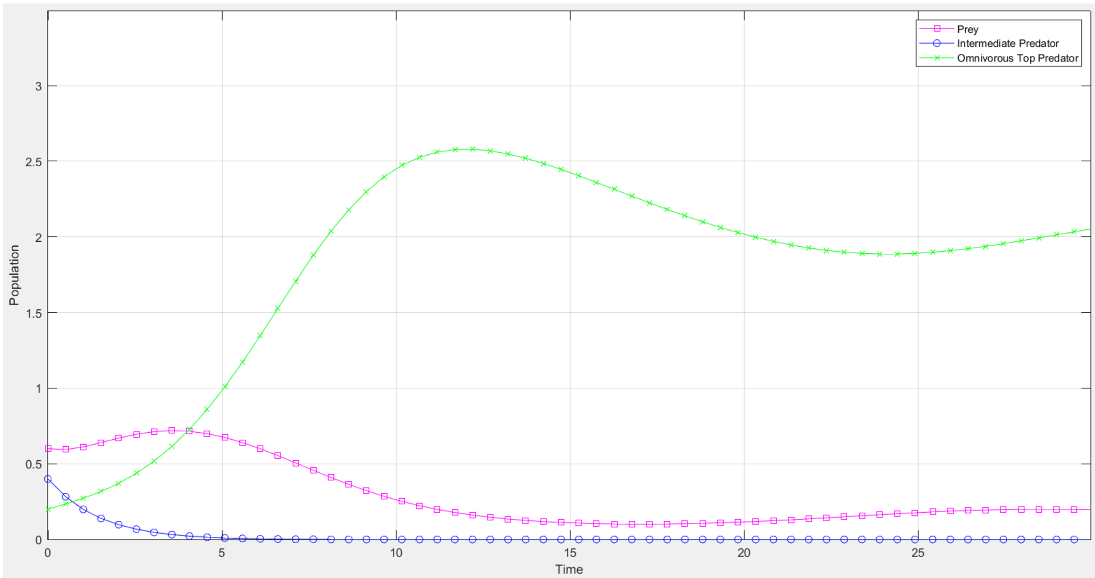


Figure 4 The graph of the population with omnivorous top predator versus time when (0.1667,0,2.0833) is a stable equilibrium point where $\alpha = 0.1$, $\beta = 0.2$, $\gamma = 0.6$, $\delta = 0.15$, $\mu = 0.4$, $d_1 = 0.7$, $d_2 = 0.1$.

The initial condition used same values for all these numerical simulations which are $x(0) = 0.6$, $y(0) = 0.4$, $z(0) = 0.2$. Figure 2 shown that two populations which are intermediate predator and omnivorous top predator move towards extinction due to death rate for both populations are high. So, only prey will stay alive in the long term. The population of omnivorous top predator tends to extinction due to higher rate of death of omnivorous top predators as can be seen in figure 3. Hence, no predation to the intermediate predators and they will be increased due to the food source which is prey just for them. In figure 4, omnivorous top predators got the highest density of species in the ecosystem even intermediate predators which are the food source of omnivorous top predators reduce and tend to extinction. Since omnivore can consume and subsist on both plant and animal stuff, prey population is enough as food resource for omnivorous top predator in the ecosystem.

## Conclusion

From this study, we have found the omnivorous top predator has advantage to survive in this ecosystem and the high rate of death of the species causing the population to decrease and becomes extinct meanwhile the lowest death rate will result the highest density of species in an ecosystem. Other than that, the equilibrium points are always semi-trivial because no all three populations extinct and survive. As a conclusion, we can see how omnivores act as top predator in the ecosystem. They have a choice of food sources to survive whether prey or intermediate predator becomes extinct.

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