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Determination of Hamiltonian Polygonal Paths in Assembly Graph of $FTTM_7$ Using Ω-algebra

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Abstract
Fuzzy Topographic Topological Mapping $(FTTM)$ or $FTTM_1$ is a model designed to present a 3D view of unbounded single current source of epileptic foci which uses the magnetic field data as input. $FTTM_2$ is designed as a grayscale image by transforming magnetic field data of $FTTM_1$. Afterwards, $FTTM_3$ was introduced based on the electrical field and it is anticipated that there will be more of its extended versions in future. Therefore, the concept of sequence, namely $FTTM_n$ was introduced in which each version of $FTTM$ is connected by the respective homeomorphism. As a new sequence, it is necessary to ponder its features. This study focuses on $FTTM_7$. Some concepts of geometric, graph and number theories are utilized to analyze $FTTM_7$. Geometrical features of $FTTM_7$ is precisely obtained. Determination of Hamiltonian paths is performed by algorithms using Ω-algebra as base. Due to the high number of possible paths, visualization whereby every Hamiltonian path is possibly viewed is necessary. As result, the study verifies the number of paths as proposed by Ahmad et. al. in 2019 and concurred to Fibonacci number as stated by Burns et. al. in 2013.
Keywords: Fuzzy Topographic Topological Mapping; Omega algebra; Hamiltonian paths; sequence of $FTTM_7$; visualization of $FTTM_7$

Introduction

The human brain is an incredibly complex and vital organ [1]. It consists of the cerebral cortex, which is folded to fit inside the skull. The cerebral cortex contains billions of neurons that form a vast network responsible for processing information. During information processing, the brain generates a weak magnetic field due to the flow of electrical currents in the neural system. This magnetic field can be measured non-invasively using a technique called magnetoencephalography (MEG), which is safe and rapidly advancing in the study of brain function. MEG can localize and characterize the electrical activity of the brain by detecting the associated magnetic fields [2]. Epileptic foci, specific brain regions that trigger epileptic seizures, produce unique magnetic fields. Locating these foci accurately is crucial for successful epilepsy surgery. MEG is one of the invasive methods used to locate epileptic foci.

The neuromagnetic inverse problem refers to the challenge of determining the direction, location, and magnitude of a current source in the brain based on the magnetic field measurements obtained from a Superconducting Quantum Interference Device (SQUID) [3]. Fuzzy Topographic Topological Mapping (FTTM) is a model that addresses this problem by combining fuzzy topological space, magnetic field analysis, graph theory, and topological mapping. It consists of 4 different topological spaces, which are magnetic contour plane (MC), base magnetic plane (BM), fuzzy magnetic field (FM) and topographic magnetic field (TM). FTTM minimizes the difference between calculated and measured values to identify the best location of a current source, such as an epileptic focus. Unlike other methods, FTTM is efficient, requires no prior information, and saves time in determining the location of current sources [4].

A sequence of $\boldsymbol{FTTM}$ namely $\boldsymbol{FTTM_n}$ is designed symmetrically [5]. By the homeomorphism characteristics of each component in $\boldsymbol{FTTM}$, it is possible to generate new $\boldsymbol{FTTM}$. $\boldsymbol{FTTM_2}$ is designed as a grayscale image in which it transforms the magnetic field data of $\boldsymbol{FTTM_1}$ [5]. It can be used to solve inverse neuromagnetic problems of multiple current sources. $\boldsymbol{FTTM_3}$ is designed with the electrical field generated by epileptic foci. Afterwards, several research has defined, extended, created and implement the $\boldsymbol{FTTM}$ in solving inverse neuromagnetic problem for epileptic seizures.

In this paper, we will discuss about the 7th sequence of $\boldsymbol{FTTM}$, namely $\boldsymbol{FTTM_7}$. First, the geometrical features of $\boldsymbol{FTTM_7}$ is precisely obtained. Assembly graph is defined in the model and Hamiltonian polygonal paths is determined by algorithms construction with concept of $\boldsymbol{\Omega}$-algebra. Then, 49312 Hamiltonian polygonal paths is successfully shown and visualized in appropriate ways.

Definition 1 [4]
Let $\boldsymbol{FTTM} = \{(\boldsymbol{M}, \boldsymbol{B}, \boldsymbol{F}, \boldsymbol{T}) : \boldsymbol{M} \cong \boldsymbol{B} \cong \boldsymbol{F} \cong \boldsymbol{T}\}$.

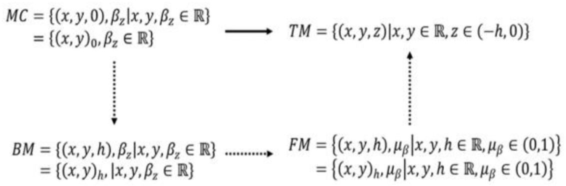

Figure 1 $FTTM$ model [9].

Definition 2 [3]
Let $\boldsymbol{FTTM_i} = (\boldsymbol{MC_i}, \boldsymbol{BM_I}, \boldsymbol{FM_i}, \boldsymbol{TM_i})$ such that $\boldsymbol{MC_i}, \boldsymbol{BM_I}, \boldsymbol{FM_i}, \boldsymbol{TM_i}$ are topological space with $\boldsymbol{MC_i} \cong \boldsymbol{BM_I} \cong \boldsymbol{FM_i} \cong \boldsymbol{TM_i}$. Then the sequence of $\boldsymbol{FTTM}$, that is $\boldsymbol{FTTM_1}$, $\boldsymbol{FTTM_2}$, ..., $\boldsymbol{FTTM_n}$ is homeomorphism sequentially such that $\boldsymbol{MC_i} \cong \boldsymbol{MC_{i+1}}$, $\boldsymbol{BM_i} \cong \boldsymbol{BM_{i+1}}$, $\boldsymbol{FM_i} \cong \boldsymbol{FM_{i+1}}$ and $\boldsymbol{TM_i} \cong \boldsymbol{TM_{i+1}}$.

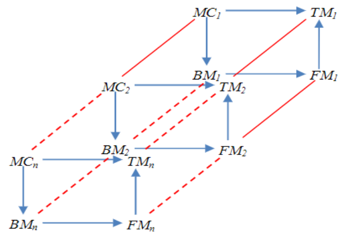

Figure 2 Sequence of $FTTM$, $FTTM_n$ model.

Definition 3 [6]
The vertices of FTTM are the 4 components in FTTM, in which the sequence of vertices of $\boldsymbol{FTTM_n}$ is defined as $\boldsymbol{vFTTM_1}$, $\boldsymbol{vFTTM_2}$, $\boldsymbol{vFTTM_3}$, $\cdots$ and given recursively by equation

$$\boldsymbol{vFTTM_n} = \boldsymbol{4n} \text{ for } \boldsymbol{n} \geq \boldsymbol{1}.$$

Definition 4 [6]
The edges of FTTM are the paths which connect the components of FTTM, in which the sequence of edges of $\boldsymbol{FTTM_n}$ can be defined as $\boldsymbol{eFTTM_1}$, $\boldsymbol{eFTTM_2}$, $\boldsymbol{eFTTM_3}$, $\cdots$ and given recursively by equation

$$\boldsymbol{eFTTM_n} = \boldsymbol{4} + (\boldsymbol{n} - \boldsymbol{1})\boldsymbol{8} \text{ for } \boldsymbol{n} \geq \boldsymbol{1}.$$

Definition 5 [6]
The faces of FTTM are the square in shapes built by 4 components of FTTM, in which the sequence of faces of $\boldsymbol{FTTM_n}$ can be defined as $\boldsymbol{fFTTM_1}$, $\boldsymbol{fFTTM_2}$, $\boldsymbol{fFTTM_3}$, $\cdots$ and given recursively by equation

$$\boldsymbol{fFTTM_n} = \boldsymbol{1} + (\boldsymbol{n} - \boldsymbol{1})\boldsymbol{5} \text{ for } \boldsymbol{n} \geq \boldsymbol{1}.$$

Definition 6 [6]
The maximal assembly graph of $\boldsymbol{FTTM_n}$ is defined as

$$\boldsymbol{\Gamma_{FTTM_n}} = \boldsymbol{FTTM_n} - [\boldsymbol{E}(\boldsymbol{FTTM_1}) \cup \boldsymbol{E}(\boldsymbol{FTTM_n})] \text{ for } \boldsymbol{n} \geq \boldsymbol{3},$$

such that $|\boldsymbol{\Gamma_{FTTM_n}}|$ denotes the number of 4-valent vertices. Noted that the only assembly graph considered in this study is the maximal assembly graph of $\boldsymbol{FTTM_n}$ if there is not specific mentioned. Then, an assemble graph is exists in any sequence of $\boldsymbol{FTTM_n}$ for $\boldsymbol{n} \geq \boldsymbol{3}$.

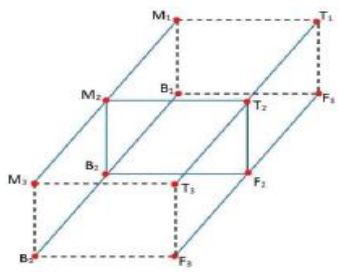

Figure 3 Maximum assembly graph of $FTTM_3$ model.

Methodology

To determine all of the possible Hamiltonian polygonal paths in assembly graph of $\boldsymbol{FTTM_7}$, $\boldsymbol{\Omega}$-algebra is used to design the algorithms for implementation. Definitions below shows the concept of $\boldsymbol{\Omega}$-algebra and vigesimal operation.

Definition 7 [5]

An omega algebra is a set of catalytic relations and can be modelled as an algebraic operation on a set $\boldsymbol{M}$ such that $\boldsymbol{\omega_n} : \boldsymbol{M_1} \times \boldsymbol{M_2} \times \ldots \times \boldsymbol{M_n} \rightarrow \boldsymbol{M}$, for $\boldsymbol{M_i} = \boldsymbol{M_j} = \boldsymbol{M}$. The $\boldsymbol{n}$-ary or arity of a function or operation is the number if argument or operands of the function. For instance,

$$\boldsymbol{\omega_2} : \boldsymbol{M} \times \boldsymbol{M} \rightarrow \boldsymbol{M}$$
$$\boldsymbol{\omega_3} : \boldsymbol{M} \times \boldsymbol{M} \times \boldsymbol{M} \rightarrow \boldsymbol{M}$$
$$\boldsymbol{\omega_4} : \boldsymbol{M} \times \boldsymbol{M} \times \boldsymbol{M} \times \boldsymbol{M} \rightarrow \boldsymbol{M}$$
$$\vdots$$
$$\boldsymbol{\omega_n} : \boldsymbol{M} \times \boldsymbol{M} \times \ldots \times \boldsymbol{M} \rightarrow \boldsymbol{M}$$

In other words, any two vertices in a given system are connected and one of the $\boldsymbol{n}$ elements will produce a final product from the connection.

Definition 8

A vigesimal operation based on $\boldsymbol{\Omega}$ – algebra is defined as below.

$$\boldsymbol{\omega_{20}} : *_{20} : \boldsymbol{V} \times \boldsymbol{V} \times \ldots \times \boldsymbol{V} \longrightarrow \boldsymbol{V}$$

such that there exists some $\boldsymbol{v_i}, \boldsymbol{v_j} \in \boldsymbol{V}$ and $\boldsymbol{v_i} *_{20} \boldsymbol{v_j} = \boldsymbol{v_j} \in \boldsymbol{V}$ through all other $\boldsymbol{v_k} \in \boldsymbol{V}$ (20 vertices with length 19).

$$v_a \Rightarrow v_b \Rightarrow v_c \Rightarrow \ldots \Rightarrow v_t$$

Figure 4 Example of a path with length 19 from v_a to v_t (passing $v_b, v_c, \ldots, v_s$).

The Hamiltonian path in assembly graph of $\boldsymbol{FTTM_7}$ is an $\boldsymbol{\Omega}$-algebra, which undergo a vigesimal operation. The path contains 20 vertices with length 19. The Hamiltonian paths start with the vertex in the set of 4-valent vertices, then undergo vigesimal operation with other vertices in the set, and then end up with a vertex which is also in the set of 4-valent vertices. In order to find all possible Hamiltonian polygonal paths in assembly graph of $\boldsymbol{FTTM_7}$, a recursive algorithm is created by using backtracking concept to avoid heavy computation. Then, the recursive algorithm is included into the main algorithm as shown in figure 5.

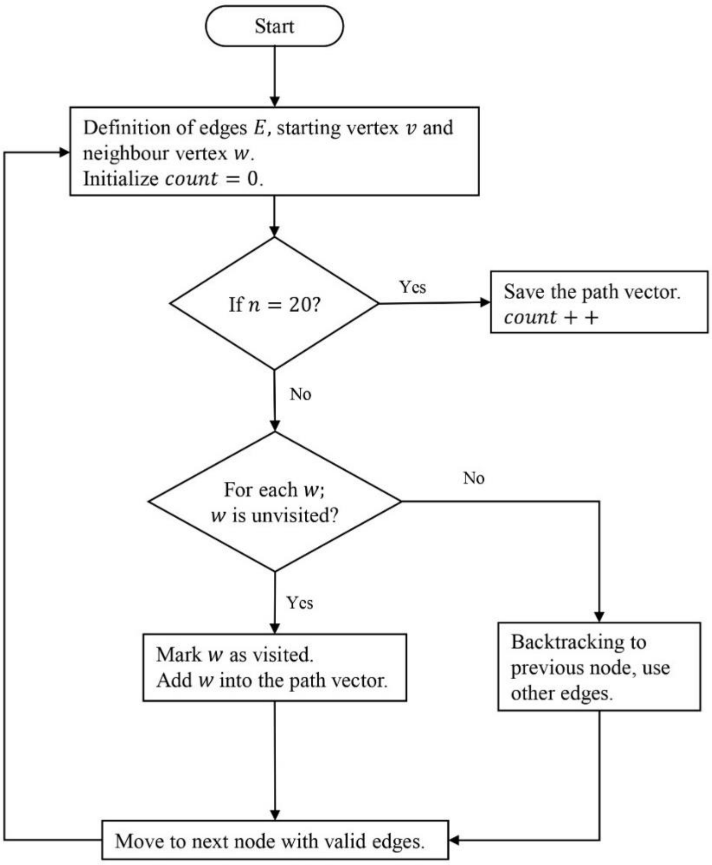

Figure 5 Recursive function algorithm to find all Hamiltonian paths.

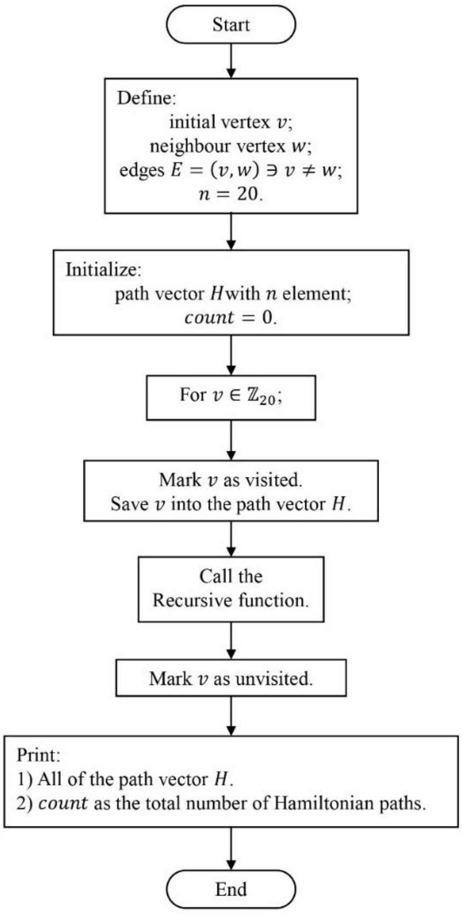

Figure 6 Main algorithm to find all Hamiltonian paths with 20 vertices.

The visualization of Hamiltonian polygonal paths in assembly graph of $\boldsymbol{FTTM_7}$ can be reviewed as a two-dimensional graph. By a front view to the planes of $\boldsymbol{FTTM_7}$, the graph can be redrawn as shown in figure 7.

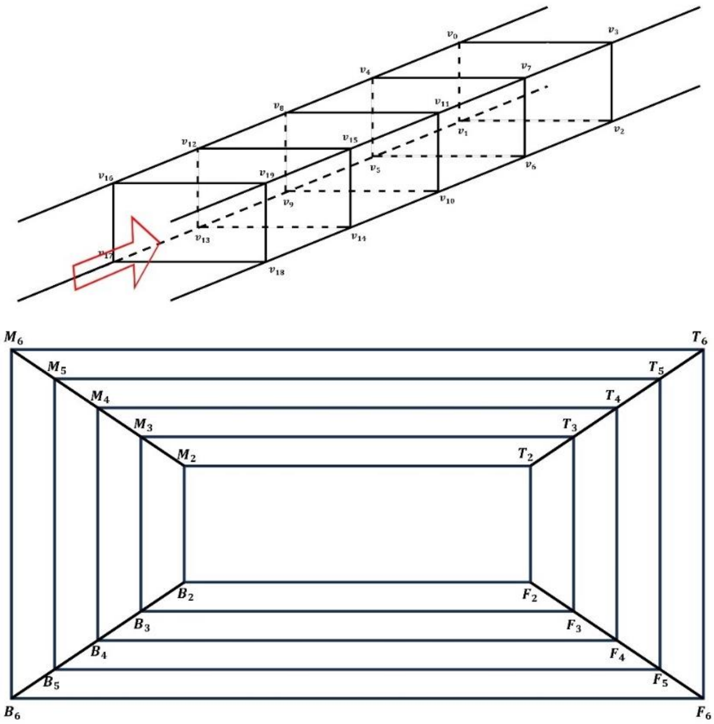

Figure 7 Front view of maximum assembly graph of $FTTM_7$ and its new representation.

By that, an algorithm is constructed to visualize the Hamiltonian paths with several colours by animation.

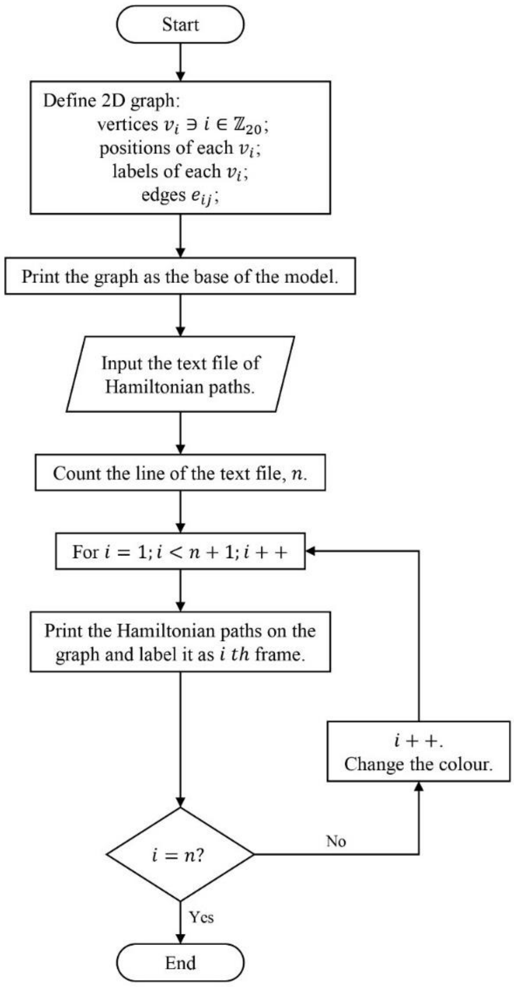

Figure 8 Algorithm of animation visualizing Hamiltonian paths.

Results

The geometrical features of $FTTM_7$ is listed in table below.

Table 1: Geometrical features of $FTTM_7$.

$\boldsymbol{FTTM_n}$	$FTTM_7$
Vertices	28
Edges	52
Faces	31

The Hamiltonian polygonal paths in assembly graph of $FTTM_7$ is shown in figure below. The result shows that there are 49312 possible Hamiltonian paths.

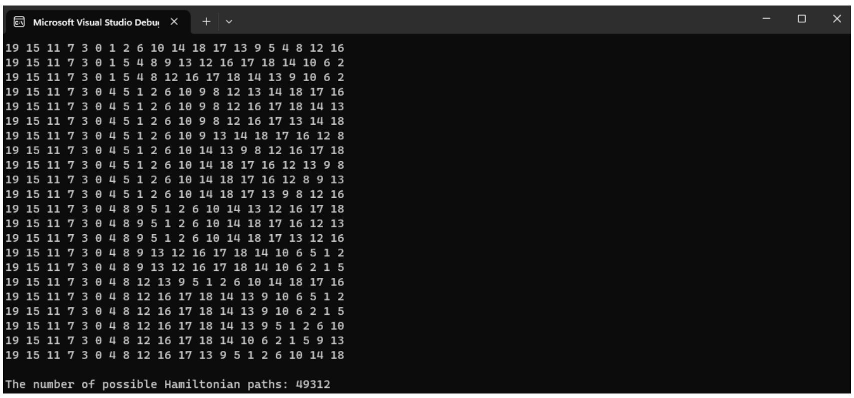

Figure 9 Output of possible Hamiltonian paths.

Example visualizations of Hamiltonian paths with their new representation is shown in figure below.

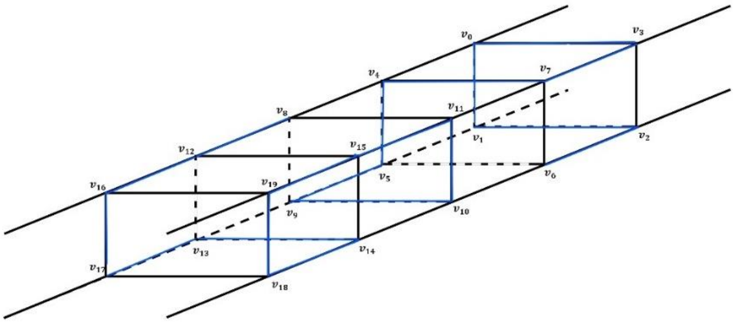

Figure 10 Example Hamiltonian paths 1.

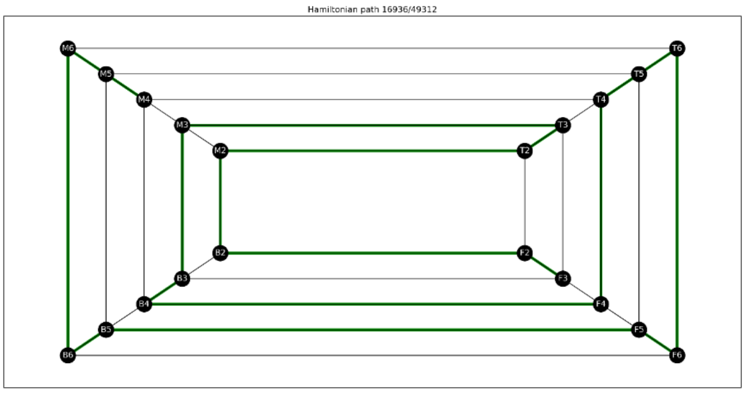

Figure 11 New visualization of example Hamiltonian paths 1 from the animation.

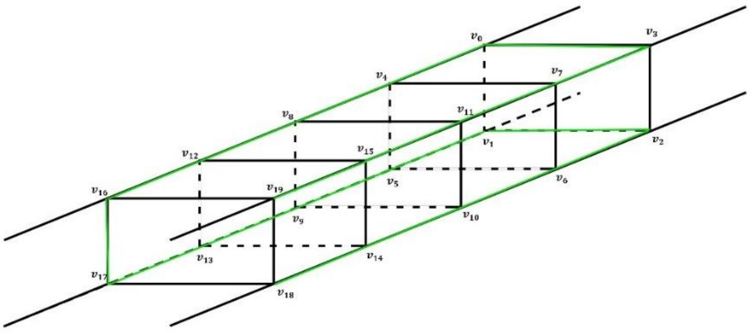

Figure 12 Example Hamiltonian paths 2.

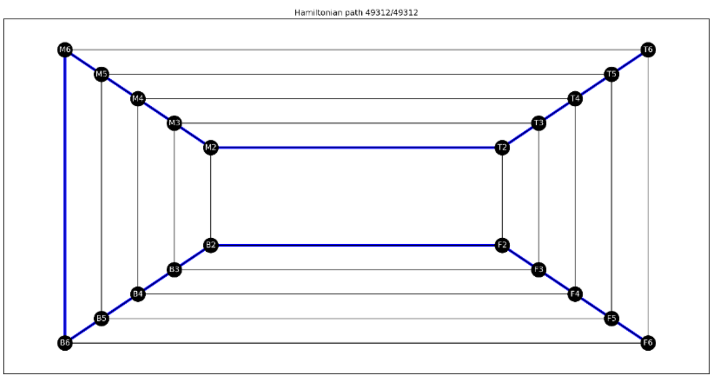

Figure 13 New visualization of example Hamiltonian paths 2 from the animation.

Discussion

As a result, 49312 Hamiltonian polygonal paths in assembly graph of $FTTM_7$ are successfully visualized. Therefore, the number of Hamiltonian polygonal paths in assembly graph of $FTTM_7$ is obtained visually and concurred to the number given by Ahmad et. al. in 2019 [7].

By previous definition, $|\Gamma_{FTTM_7}|$ represents the number of 4-valent vertices in $FTTM_7$. Therefore, $|\Gamma_{FTTM_7}| = 20$. According to [8], if Γ is a simple assembly graph with $|\Gamma| = k$, and $\mathcal{C}$ is the collection of all sets of Hamiltonian polygonal paths of Γ, then

$$|\mathcal{C}| \leq F_{2k+1}$$

where F_k is the k-th Fibonacci number. Therefore, for $FTTM_7$,

$$|\mathcal{C}_{FTTM_7}| = 49312 \leq 165580141 = F_{2(20)+1}$$

means that the number of Hamiltonian polygonal paths in assembly graph of $FTTM_7$ is concurred to 41-th Fibonacci number.

Conclusion

In this study, the maximum assembly graph of $FTTM_7$ is defined based on its 4-valent vertices. Then, Ω-algebra is found to be suitable for construction of algorithms in determining Hamiltonian polygonal paths in assembly graph of $FTTM_7$. Animation is constructed to visualize all of the Hamiltonian polygonal paths. Lastly, the number of Hamiltonian polygonal paths in assembly graph of $FTTM_7$ is verified as 49312 and concurred to 41-th Fibonacci number. To conclude, the research study has successfully determined and visualized the Hamiltonian polygonal paths in assembly graph of $FTTM_7$ with the concept of Ω-algebra. The outcomes of study verified the number of paths and concurred to k-th Fibonacci sequence as stated by previous research. The study contributes to the understanding of $FTTM_7$, which advancing knowledge of $FTTM$ model and provides potential to improve in solving inverse neuromagnetic problem. Development of algorithms such as recursive function and animation by using programming languages gives a physical approach in path determination and visualization. Application of algebraic set such as Ω-algebra in terms of algorithms construction for path determination and visualization provides a good insight into algebra analysis and coding theory.

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