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Analysis on Mathematical Model of Alcohol Consumption

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Abstract
A mathematical model, $PHTQ$, was developed in this project to study alcohol consumption patterns and strategies to combat addiction. The model focuses on four groups: non-drinkers $P(t)$, heavy drinkers $H(t)$, drinkers in treatment $T(t)$, and the alcohol quitters $Q(t)$. The impact of various factors on alcohol consumption was analysed through mathematical analysis and numerical simulations. The model's well-posedness was demonstrated by investigating positivity solutions and invariant regions. The dynamic behaviour and basic reproduction number (R_0) were calculated, indicating stability conditions. When $R_0 < 1$, the alcohol-free equilibrium point (E_0) is asymptotically stable, while $R_0 > 1$ leads to the existence of an endemic equilibrium (E^*) that is asymptotically stable according to the Routh-Hurwitz criterion. MATLAB simulations were conducted to validate the analytical results. In conclusion, this mathematical model provides valuable insights into the dynamics and factors influencing alcohol consumption. To address the alcoholism epidemic, it is recommended to reduce interactions between non-drinkers and heavy drinkers and increase the number of individuals entering treatment.
Keywords: Alcohol consumption; Existence and uniqueness; Basic reproduction number; Stability analysis; Routh-Hurwitz

Introduction

Alcoholism is a widespread problem causing significant health and social issues globally. Alcohol misuse leads to physical, psychological, and economic consequences, including accidents and chronic diseases. Alcohol-related deaths account for a significant percentage of global mortality. Prevention and treatment methods, such as rehabilitation, have been developed to address alcohol-related problems. Mathematical modelling has been used to study the dynamics of alcoholism and its transmission in communities.

Preventing alcoholism involves reducing stress, limiting alcohol consumption, imposing taxes on alcohol, raising awareness, and providing treatment options. Treatment approaches include cautious cessation, medication, and psychological support. Mathematical models have been used to explore the impact of alcohol on disease transmission and to find strategies for combating addiction.

Khajji et al. (2020) developed the $PMHT^rT^pQ$ model to analyse the influence of addiction treatment centres on different groups of drinkers. Their findings emphasized the importance of educational programs and treatment facilities in assisting vulnerable individuals. Chinnadurai (2020) used eigenvalue methods and the Routh Hurwitz criteria to analyse the stability of equilibrium states in relation to alcoholism. However, understanding their proof may be challenging due to differences in matrix elements compared to the mathematical model.

Overall, mathematical modelling provides valuable insights into alcoholism dynamics, aiding in prevention and treatment efforts. The remaining sections of the paper are structured as follows. Section 2 presents the formulation of the mathematical model and derives fundamental properties by analyzing the equilibrium point. Stability analysis of the model is covered in Section 3. Section 4 discusses the issue of parameter sensitivity. Numerical simulations of the model are presented in Section 5, where parameter estimation takes place. Finally, Section 6 concludes the paper.

Model Formulation and Description

In this paper, we formulate a mathematical model for the dynamics of alcoholism and divide the population in four compartments: Non-drinkers $P(t)$, Heavy drinkers $H(t)$, Drinkers in treatment $T(t)$ and Alcohol quitters $Q(t)$. The mathematical model diagram is as in Figure 1.

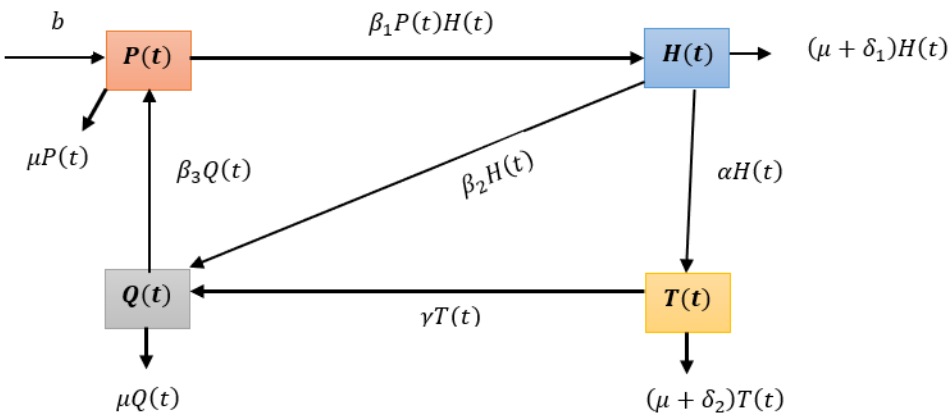

Figure 1 Compartmental diagram of drinkers' class

The governing nonlinear system of differential equations represent the model are given by

$$\frac{dP(t)}{dt} = b - \beta_1 P(t)H(t) - \mu P(t) + \beta_3 Q(t)$$
$$\frac{dH(t)}{dt} = \beta_1 P(t)H(t) - (\beta_2 + \mu + \delta_1 + \alpha)H(t)$$
$$\frac{dT(t)}{dt} = \alpha H(t) - (\mu + \delta_2 + \gamma)T(t)$$
$$\frac{dQ(t)}{dt} = \gamma T(t) + \beta_2 H(t) - (\beta_3 + \mu)Q(t)$$

where $P(0) \geq 0, H(0) \geq 0, T(0) \geq 0$ and $Q(0) \geq 0$ are the given initial states.
The model variables and their descriptions are tabulated in Table 1 and the model parameters, and their description are tabulated in Table 2.

Table 1: Model variables and their description at any time

Variables	Description of the variables
$\boldsymbol{P(t)}$	Non-Drinker population at time
$\boldsymbol{H(t)}$	Heavy Drinker population at time
$\boldsymbol{T(t)}$	Drinkers in Treatment population at time
$\boldsymbol{Q(t)}$	Alcohol quitter population at time

Table 2: Model parameters and their description

Parameters	Description of parameters
$\boldsymbol{b}$	Birth rate
$\boldsymbol{\mu}$	The natural death rate which refers to any death rate not caused by alcohol
$\boldsymbol{\alpha}$	Proportion of drinkers entering treatment compartment
$\boldsymbol{\beta_1}$	The rate of recruitment to alcoholism

	due to encounter between non-drinkers and heavy drinkers
$\boldsymbol{\beta_2}$	The rate of recruitment from heavy to drinkers in treatment
$\boldsymbol{\beta_3}$	The rate at which recovered drinkers' relapse to non-drinkers
$\boldsymbol{\delta_1}$	Drinking induced death rate of heavy drinkers
$\boldsymbol{\delta_2}$	Drinking induced death rate of drinkers in treatment
$\boldsymbol{\gamma}$	The rate of recovered drinkers from treatment

Positivity Solution of the Model

To show that the model system is to be epidemiologically meaningful and well posed, it is needed to prove that all the state variables are non-negative.

Assume that all the state variables are continuous. Then from the model system of the first equation we have:

$$\frac{dP(t)}{dt} = b - \beta_1 P(t)H(t) + \beta_3 Q(t) - \mu P(t),$$

$$\frac{dP(t)}{dt} \geq -\beta_1 P(t)H(t) - \mu P(t),$$

$$\frac{dP(t)}{dt} \geq -\mu P(t),$$

$$\int \frac{dP(t)}{P(t)} \geq \int -\mu\, dt,$$

$$P(t) \geq e^{-\mu t + c},$$

$$P(t) \geq Ae^{-\mu t},$$

where constant $A = e^C$.
Assuming initial value of $P(t)$ is $P(0) = P_0$,

$$P(t) \geq P_0 e^{-\mu t} \geq 0.$$

Similarly,

$$H(t) \geq H_0 e^{-(\beta_2+\mu+\delta_1+\alpha)t} \geq 0,$$
$$T(t) \geq T_0 e^{-(\mu+\delta_2+\gamma)t} \geq 0,$$
$$Q(t) \geq Q_0 e^{-(\beta_3+\mu)t} \geq 0.$$

It has been showed that the solution $P(t), H(t), T(t), Q(t)$ of the $PHTQ$ model are positive for all $t \geq 0$.

Invariant Region

The model under consideration monitors populations as such, we assume that all the variables and parameters of the model are positive for all $t \geq 0$. To show that the solution of the $PHTQ$ model is bounded, it is needed to prove that the total population size $N(t)$ is bounded.

$$N(t) = P(t) + H(t) + T(t) + Q(t)\,.$$

Differentiating both sides with respect to time t gives

$$\frac{dN}{dt} = \frac{dP(t)}{dt} + \frac{dM(t)}{dt} + \frac{dH(t)}{dt} + \frac{dT(t)}{dt} + \frac{dQ(t)}{dt}\,,$$

$$\frac{dN}{dt} = b - \mu N - \delta H.$$

Ignore death due to alcohol, which is the last term in the above equation,

$$\frac{dN}{dt} \leq b - \mu N.$$

Using separation of variables and taking integration on both sides,

$$b - \mu N \geq Ae^{-\mu t}.$$

By applying the initial condition $N(0) = N_0$

$$N \leq \frac{b}{\mu} - \left(\frac{b - \mu N_0}{\mu}\right)e^{-\mu t}.$$

At $t \to \infty$

$$N \leq \frac{b}{\mu}$$

which implies that $0 \leq N \leq \frac{b}{\mu}$. Thus, the feasible solution set of the model system enter and remain in the region for all time t,

$$\Omega = \left\{(P, H, T, Q) \in \mathbb{R}_t^4; P + H + T + Q \leq \frac{b}{\mu}\right\}.$$

Therefore, the $PHTQ$ model is well posed epidemiologically so it is sufficient to study the dynamics of the model system in the domain Ω.

Alcohol-Free Equilibrium

Let $E_0 = (P^*, H^*, T^*, Q^*)$ represent alcoholic free equilibrium from model $PHTQ$. Drinker population achieved only in heavy drinkers and drinkers in treatment compartment, hence $H^* = T^* = 0$. To obtain equilibrium point, setting 0 in the model system, then alcohol free equilibrium point will be:

$$b - \beta_3 Q^* - \mu P^* = 0$$
$$-(\beta_3 + \mu)Q^* = 0.$$

Hence, we have

$$Q^* = 0, \qquad b - \mu P^* = 0$$
$$P^* = \frac{b}{\mu}.$$

Therefore, an alcoholic free equilibrium point, E_0 is

$$E_0 = (P^*, H^*, T^*, Q^*) = \left(\frac{b}{\mu}, 0, 0, 0\right).$$

Basic Reproduction Number

The basic reproduction number, R_0 measures the average number of heavy drinkers generated when a single heavy drinker in a susceptible population. The value of R_0 will indicate whether the epidemic could occur or not. We obtain the basic reproduction number, R_0 of the system by Next-Generation Matrix Method. The next generation matrix compromises two matrices F and V, whose elements in matrix constitute the new infections that will arise and the transfer of infections from one compartment to another respectively.

1. f_i be the rate of new infections in compartment i.
2. v_i be the rate transfer of individuals in and out of compartment i.

From the model system, the infected compartments to be

$$\frac{dH(t)}{dt} = \beta_1 P(t)H(t) - (\beta_2 + \mu + \delta_1 + \alpha)H(t)$$

$$\frac{dT(t)}{dt} = \alpha H(t) - (\mu + \delta_2 + \gamma)T(t)\,.$$

Define f_i and v_i as

$$f_i = \begin{pmatrix} \beta_1 P(t)H(t) \\ 0 \end{pmatrix},$$

$$v_i = \begin{pmatrix} (\beta_2 + \mu + \delta_1 + \alpha)H(t) \\ -\alpha H(t) + (\mu + \delta_2 + \gamma)T(t) \end{pmatrix}.$$

Evaluate F and V

$$F = \frac{\partial f_i}{\partial x_j}(E_0) = \begin{pmatrix} \beta_1 P^* & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} \beta_1 \frac{b}{\mu} & 0 \\ 0 & 0 \end{pmatrix},$$

$$V = \frac{\partial V_i}{\partial x_j}(E_0) = \begin{pmatrix} \beta_2 + \mu + \delta_1 + \alpha & 0 \\ -\alpha & \mu + \delta_2 + \gamma \end{pmatrix}.$$

Next generation matrix FV^{-1}

$$V^{-1} = \frac{1}{(\beta_2 + \mu + \sigma_1 + \gamma)(\delta_2 + \mu + \gamma)} \begin{pmatrix} \mu + \delta_2 + \gamma & 0 \\ \alpha & \beta_2 + \mu + \delta_1 + \alpha \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{\beta_1 + \mu + \delta_1 + \gamma} & 0 \\ \frac{\alpha}{(\beta_2 + \mu + \delta_1 + \gamma)(\delta_2 + \mu + \gamma)} & \frac{1}{\delta_2 + \mu + \gamma} \end{pmatrix}$$

$$FV^{-1} = \begin{pmatrix} \beta_1 \frac{b}{\mu} & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\beta_1 + \mu + \delta_1 + \gamma} & 0 \\ \frac{\alpha}{(\beta_2 + \mu + \delta_1 + \gamma)(\delta_2 + \mu + \gamma)} & \frac{1}{\delta_2 + \mu + \gamma} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{b\beta_1}{\mu(\beta_1 + \mu + \delta_1 + \gamma)} & 0 \\ 0 & 0 \end{pmatrix}.$$

The eigenvalue of the next generation matrix is

$$\lambda_1 = \frac{b\beta_1}{\mu(\beta_2 + \mu + \delta_1 + \alpha)}, \quad \lambda_2 = 0.$$

Then the basic reproduction number, R_0 which is the maximum eigenvalue of the matrix is

$$R_0 = \frac{b\beta_1}{\mu(\beta_2 + \mu + \delta_1 + \alpha)}.$$

Endemic Equilibrium Point

Endemic equilibrium achieved when alcoholism persists in the population in which we already showed the positivity solution of the model $(P(t) \geq 0, H(t) \geq 0, T(t) \geq 0, Q(t) \geq 0)$. It can be obtained by equating each of the model systems to zero. That is

$$\frac{dP}{dt} = \frac{dH}{dt} = \frac{dT}{dt} = \frac{dQ}{dt} = 0.$$

By simple calculations we obtained

$$P^* = \frac{\beta_2 + \mu + \delta_1 + \alpha}{\beta_1} = \frac{b}{\mu R_0}$$

$$H^* = \frac{\mu R_0 - \mu}{\beta_1} = \frac{B_1 b - \mu(\beta_2 + \mu + \delta_1 + \alpha)}{\beta_1(\beta_2 + \mu + \delta_1 + \alpha)}$$

$$T^* = \frac{\alpha H^*}{(\delta_2 + \mu + \gamma)}$$

$$\boldsymbol{Q^* = \frac{\gamma T^* + \beta_2 H^*}{\beta_3 + \mu}}.$$

Stability of Alcohol-Free Equilibrium

To determine the stability of the model at an alcohol-free point, we will consider the Jacobian matrix of the $\boldsymbol{PHTQ}$ model at alcohol free equilibrium, $\boldsymbol{E_0}$.

$$\boldsymbol{J(E) = \begin{pmatrix} -\beta_1 H - \mu & -\beta_1 P & 0 & \beta_3 \\ 0 & \beta_1 P - (\beta_2 + \mu + \delta_1 + \alpha) & 0 & 0 \\ 0 & \alpha & -(\delta_2 + \mu + \gamma) & 0 \\ 0 & \beta_2 & \gamma & -(\beta_3 + \mu) \end{pmatrix}}$$

$$\boldsymbol{J(E_0) = \begin{pmatrix} -\mu & -\beta_1 \frac{b}{\mu} & 0 & \beta_3 \\ 0 & \beta_1 \frac{b}{\mu} - (\beta_2 + \mu + \delta_1 + \alpha) & 0 & 0 \\ 0 & \alpha & -(\delta_2 + \mu + \gamma) & 0 \\ 0 & \beta_2 & \gamma & -(\beta_3 + \mu) \end{pmatrix}}$$

where $\boldsymbol{P^*} = \frac{\boldsymbol{b}}{\boldsymbol{\mu}}$.

By using eigenvalue method,

Let $\boldsymbol{g = \beta_2 + \mu + \delta_1 + \alpha}$, $\boldsymbol{h = \delta_2 + \mu + \gamma}$

$$\boldsymbol{|J(E_0)| = \begin{vmatrix} -\mu - \lambda & -\beta_1 \frac{b}{\mu} & 0 & \beta_3 \\ 0 & \beta_1 \frac{b}{\mu} - g - \lambda & 0 & 0 \\ 0 & \alpha & -h - \lambda & 0 \\ 0 & \beta_2 & \gamma & -(\beta_3 + \mu) - \lambda \end{vmatrix} = 0}$$

$$(-\mu - \lambda)\left(\beta_1 \frac{b}{\mu} - g - \lambda\right)(-h - \lambda)(-(\mu + \beta_3) - \lambda) = 0$$

$$\begin{matrix} -\mu - \lambda = 0 \\ \lambda_1 = -u \end{matrix}, \quad \begin{matrix} \beta_1 \frac{b}{\mu} - a - \lambda = 0 \\ \lambda_2 = \beta_1 \frac{b}{\mu} - a \end{matrix}, \quad \begin{matrix} -b - \lambda = 0 \\ \lambda_3 = -b \end{matrix}, \quad \begin{matrix} -(\mu + \beta_3) - \lambda = 0 \\ \lambda_4 = -(\mu + \beta_3) \end{matrix}.$$

Here, $\lambda_1, \lambda_3, \lambda_4$ are clearly real and negative. Alcohol free equilibrium, E_0 is stable if $\lambda_2 < 0$. So,

$$\frac{\beta_1 b}{\mu} - g < 0$$

$$\frac{b\beta_1}{\mu(\beta_2 + \mu + \delta_2 + \alpha)} < 1$$

$$R_0 < 1.$$

Hence, the model system is asymptotically stable at alcohol free equilibrium, $E_0 = \left(\frac{b}{\mu}, 0,0,0\right)$ if $R_0 < 1$.

Stability of Endemic Equilibrium Point

To investigate the stability of endemic equilibrium, consider the Jacobian matrix of the model system at endemic equilibrium, $\boldsymbol{E^*}$.

$$\boldsymbol{J(E^*) = \begin{pmatrix} -\beta_1 H^* - \mu & -\beta_1 P^* & 0 & \beta_3 \\ 0 & \beta_1 P^* - (\beta_2 + \mu + \delta_1 + \alpha) & 0 & 0 \\ 0 & \alpha & -(\delta_2 + \mu + \gamma) & 0 \\ 0 & \beta_2 & \gamma & -(\beta_3 + \mu) \end{pmatrix}}$$

where

$$\boldsymbol{P^* = \frac{\beta_2 + \mu + \delta_1 + \alpha}{\beta_1}}$$

$$\boldsymbol{H^* = \frac{B_1 b - \mu(\beta_2 + \mu + \delta_1 + \alpha)}{\beta_1(\beta_2 + \mu + \delta_1 + \alpha)}}.$$

From the Jacobian matrix, we obtained the polynomial equation of the point which is

$$\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0$$

where

$$a_1 = (\delta_2 + u + \gamma + \beta_3 + \mu) + (\beta_1 H^* + \mu - \beta_1 b + \beta_2 + \mu + \delta_1 + \alpha)$$

$$a_2 = (\delta_2 + \mu + \gamma)(\beta_3 + \mu) + (\beta_1 H^* + \mu - \beta_1 b + \beta_2 + \delta_1 + \mu + \alpha)(\delta_2 + 2\mu + \beta_3 + \gamma) - (\beta_1 H + \mu)(\beta_1 P^* - \beta_2 - \mu - \delta_1 - \alpha)$$

$$a_3 = (\beta_1 H^* + \mu - \beta_1 P^* + \beta_2 + \mu + \delta_1 + \alpha)(\delta_2 + \mu + \gamma)(\beta_3 + \mu) - (\beta_1 H^* + \mu)(\beta_1 P^* - \beta_2 - \delta_1 - \alpha - \mu)(\delta_2 + 2\mu + \beta_3 + \gamma)$$

$$a_4 = -(\beta_1 H^* + \mu)(\beta_1 P^* - \beta_2 - \mu - \delta_1 - \alpha)(\delta_2 + \mu + \gamma)(\beta_3 + \mu).$$

Routh-Hurwitz criterion is applied to analyse the stability of the system at endemic equilibrium. The Hurwitz matrix evaluated at the characteristic equation is as follows

$$H = \begin{bmatrix} a_1 & a_3 & 0 & 0 \\ 1 & a_2 & a_4 & 0 \\ 0 & a_1 & a_3 & 0 \\ 0 & 1 & a_2 & a_4 \end{bmatrix}.$$

According to Routh-Hurwitz criteria, since it is a fourth order characteristic equations, there are four conditions for the system to be stable.

1. $\det(H_1) = a_1 > 0$
2. $\det(H_2) = a_1 a_2 - a_3 > 0$
3. $\det(H_3) = a_3(\det(H_2)) - a_1^2 a_4 > 0$
4. $det(H_4) = a_4\big(det(H_3)\big) > 0.$

Thus, we can show that the endemic equilibrium, E^* of the model system is asymptotically stable if and only if these four conditions are satisfied.

Sensitivity Analysis of Basic Reproduction Number, R_0

Sensitivity analysis is a technique for determining the effect of changes in input parameters or assumptions on the output or outcomes of a mathematical or computer model. Let

$$\Gamma_m^{R_0} = \frac{m}{R_0}\frac{\partial R_0}{\partial m},$$

denote the sensitivity index of $R_0 = \frac{bB_1}{\mu(\beta_2+\mu+\delta_1+\alpha)}$ with respect to the parameter m. We obtain

$$\Gamma_{\beta_1}^{R_0} = 1 > 0,$$

$$\Gamma_{\mu}^{R_0} = \frac{-(\beta_2 + 2\mu + \delta_1 + \alpha)}{bB_1(\beta_2 + \delta_1 + \alpha + \mu)} \le 0,$$

$$\Gamma_{\delta_1}^{R_0} = \frac{-\alpha}{bB_1(\beta_2 + \delta_1 + \alpha + \mu)} \le 0,$$

$$\Gamma_{\alpha}^{R_0} = \frac{-\beta_2}{bB_1(\beta_2 + \delta_1 + \alpha + \mu)} \le 0\,.$$

The basic reproduction number (R_0) is influenced by various factors as follows:

1. β_1: R_0 increases proportionally with an increase in β_1 and decreases proportionally with a decrease in β_1.

2 μ, δ_1, β_2 and α: R_0decreases with an increase in these parameters, as they exhibit an inverse relationship with R_0. However, increasing μ, δ_1 and β_2 is not feasible.

Considering the sensitivity of R_0, efforts should focus on reducing the rate of recruitment to alcoholism through encounters between non-drinkers and heavy drinkers (β_1) and increasing the proportion of drinkers entering treatment (α). Reducing β_1 is particularly important since R_0 is highly responsive to changes in this parameter.

Numerical simulation of $PHTQ$ model

All parameter values are utilized, and the outcomes are visually depicted in a time-series diagram to facilitate the observation of the behavior of each equilibrium point derived from the $\boldsymbol{PHTQ}$ model. The resulting graph is further discussed to elucidate the process of obtaining each equilibrium point based on the provided parameter values.

The numerical simulation of the model is performed by adopting the value of parameters used by Satana and Kassaye (2022) in Table 3.

Table 3: Value of parameters at alcohol-free equilibrium point

Parameters	Value
$\boldsymbol{b}$	0.4
$\boldsymbol{\mu}$	0.25
$\boldsymbol{\alpha}$	0.7
$\boldsymbol{\beta_1}$	0.7
$\boldsymbol{\beta_2}$	0.2
$\boldsymbol{\beta_3}$	0.1
$\boldsymbol{\delta_1}$	0.35
$\boldsymbol{\delta_2}$	0.03
$\boldsymbol{\gamma}$	0.09

It is important to emphasize that the equilibrium point can be achieved by considering the basic reproduction number $\boldsymbol{R_0}$ derived with the parameter values substituted as shown in Table 3. $\boldsymbol{R_0} = \boldsymbol{0.74667}$ indicating that the epidemic is in alcohol-free phase.

As per the analysis, when $R_0 < 1$, the model system is asymptotically stable, indicating the asymptotic stability of the alcohol-free equilibrium E_0. This numerical verification is depicted in Figure 2 and initial condition of each compartment are $P(0) = 50, H(0) = 30, T(0) = 20, Q(0) = 15$.

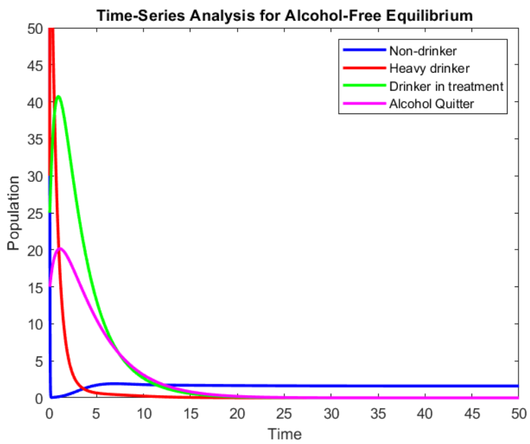

Figure 2 Time-series analysis on alcohol-free equilibrium point.

Effect of contact rate $\boldsymbol{\beta_1}$ on Heavy Drinker

In the context of this study, the graph presented regarding the impact of contact rate β_1 on heavy drinkers, (H). The simulation involved various values of β_1 while keeping the other parameters constant. According to Figure 3, a higher β_1 leads to an elevated likelihood of individuals being heavy drinkers, increasing exposure between people. As a result, the community must decrease the contact rate in order to minimize the number of individuals who consume alcohol within the population.

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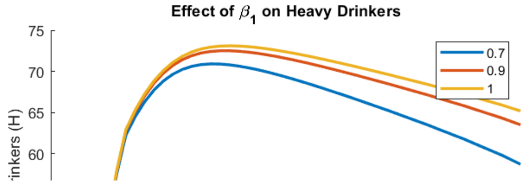

Figure 3 The effect of β_1 on heavy drinkers

Effect of recovered rate γ on drinker in treatment

In this study, we investigate the outcomes of simulations about the impact of the recovery rate, γ on heavy drinkers currently participating in treatment, $T(t)$. It is clear from Figure 4 that the number of people who are in the treatment category decreases as the recovery rate of treatment, γ increases. This indicates that when a person falls into the treatment category, society must increase treatment approaches to support their recovery. Therefore, we can successfully eliminate the scourge of alcoholism from the community if we increase the recovery rate, γ for those undergoing treatment and facilitate their transfer into the group of alcohol quitters.

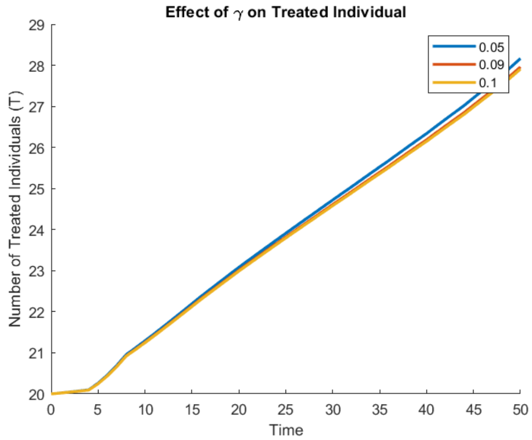

Figure 4 Effect of γ on drinkers in treatment

Endemic Equilibrium Point

When considering the scenario in which $R_0 > 1$, it is discovered that the reproduction number of the endemic equilibrium, as derived from the information presented in Table 4 by Satana and Kassaye (2022) is $R_0 = 1.74545 > 1$.

This finding provides compelling evidence that within the population, there coexist individuals who abstain from drinking, heavy drinkers, individuals undergoing treatment for drinking, and those who have successfully recovered. This coexistence serves as a clear indication of the presence of an alcohol abuse problem within the community. Moreover, individuals struggling with drinking issues consistently demonstrate a propensity to convert more non-drinkers into heavy drinkers, thus destabilizing the equilibrium of a non-drinking state. Figure 5 furnishes numerical evidence that substantiates the existence of this phenomenon.

Table 4: Value of parameters at endemic equilibrium point

Parameters	Value
b	0.8
μ	0.25
α	0.5
β_1	0.8
β_2	0.6
β_3	0.01
δ_1	0.3
δ_2	0.03
γ	0.5

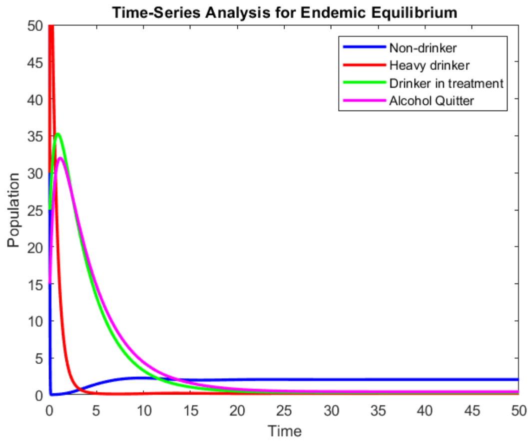

Figure 5 Time-series analysis at endemic equilibrium point

Conclusion

This study examines the transmission of infectious diseases through alcohol consumption using the $PHTQ$ Model. The model consists of four compartments representing different groups of individuals. The aim is to understand the dynamics and reduce the number of alcohol consumers. Equilibrium points and the basic reproduction number are determined using mathematical methods. The stability analysis indicates that the alcohol-free equilibrium and endemic equilibrium are stable when the reproduction number $R_0 < 1$. The sensitivity analysis identifies β_1 as a significant factor, suggesting that reducing β_1 is important in combating the alcoholism epidemic. Numerical simulations using MATLAB validate the mathematical findings. The results highlight the importance of minimizing contact between non-drinkers and heavy drinkers and maximizing the number of individuals seeking treatment to control the alcoholism epidemic. Mathematical models emphasize treatment's importance in reducing alcohol consumption. Continuous evaluation and monitoring are crucial for intervention success. Mathematical models provide valuable insights into outcomes. Regular monitoring of alcohol consumption, treatment results, and intervention impact enables evidence-based decisions. Future research should collect real-world data, use context-specific values, and explore additional contributing factors to control and prevent alcoholism.

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