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Effect of Microgravity on Breast Cancer Growth by using One-Dimensional Hyperbolic Equation

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Abstract
Breast cancer is a globally prevalent and severe disease. Understanding cancer cell behavior is crucial for effective treatments. Microgravity, like in space, affects cellular functions. Using a one-dimensional hyperbolic equation, we assess microgravity's influence on breast cancer growth. Applying numerical methods, we analyze cell growth patterns. The equation represents cell diffusion and proliferation in a microgravity environment. By discretizing it into linear equations and employing the Jacobi and Gauss-Seidel methods, we iteratively update cell densities until convergence. Results reveal distinct behavior under microgravity, with slower cancer cell mobility and altered growth patterns. The Gauss-Seidel method demonstrates faster convergence, making it suitable for modeling breast cancer growth in microgravity. These findings highlight the need to consider environmental impacts on cancer cells. Numerical approaches like Jacobi and Gauss-Seidel provide valuable insights for specific interventions.
Keywords: Microgravity; Breast cancer; One-Dimensional Hyperbolic Equation; JB method, GS method

Introduction

Cancer growth in microgravity

Cancer, a complex and deadly disease, remains a global health challenge. Understanding how cancer cells behave in various situations is critical for creating successful treatments. Microgravity, which is experienced during spaceflight or mimicked using ground-based platforms, is one unique setting that has piqued the curiosity of cancer researchers. Microgravity's changing gravitational forces have been demonstrated to influence numerous aspects of cancer progression, such as cell proliferation, migration, and responsiveness to therapy. Investigating the dynamics of cancer growth in microgravity provides important insights into the underlying mechanisms of carcinogenesis and opens new avenues for cancer treatment techniques.

Several research have used both in vitro and in vivo models to study the effect of microgravity on cancer growth. Researcher investigated the behaviour of breast cancer cells in simulated microgravity circumstances.They discovered that microgravity affected the expression of genes involved in cell adhesion, migration, and angiogenesis, implying that tumour aggressiveness and metastatic potential may vary.

Cancer growth research in microgravity provides a unique chance to investigate the fundamental mechanisms that cause tumour progression. It enables researchers to look into changes in cellular behaviour, signalling pathways, and interactions in the tumour microenvironment. Furthermore, microgravity research discoveries have the potential to improve cancer research and therapy development on Earth, as they provide fresh insights into the fundamental processes driving cancer growth and response to treatment.

Breast Cancer

PDE models have recently been used in research to explore the dynamics of breast cancer. For example, created a mathematical model based on PDEs to simulate the growth and migration of breast tumour cells. The model included cell proliferation, cell adhesion, and extracellular matrix remodelling characteristics, offering light on the mechanisms behind tumour invasion and metastasis.

In addition, parabolic equations have been used to investigate the diffusion of biological compounds inside the microenvironment of a breast tumour. Researcher described the distribution of oxygen throughout breast tumours using a parabolic model. The model took into account aspects like oxygen consumption, blood flow, and tumour vasculature heterogeneity, providing insights into the establishment of hypoxic areas and their consequences for treatment response.

Furthermore, PDE and parabolic models have been used to study the impact of therapy on breast

cancer. For example, created a PDE-based model to predict the response of breast tumours following radiotherapy. The model included radiation transfer, tumour cell proliferation, and cell death, which helped optimise treatment strategies and forecast results.

These mathematical models provide unique insights into breast cancer behaviour and have the potential to assist treatment strategies. These models allow researchers to explore diverse scenarios and assess the efficiency of various therapeutic approaches by mimicking tumour growth, invasion, and treatment response.

The problem addressed in this research is to develop a mathematical model for breast cancer growth considering the influence microgravity conditions. The study aims to investigate the impact of gravity on breast cancer growth dynamics and compare it with the growth patterns observed in microgravity environments by using 1D hyperbolic equations. Additionally, the research seeks to explore the application of the Finite Difference Method to obtain a numerical solution for the mathematical model. Furthermore, the study aims to implement a sequential algorithm to efficiently analyze the performance of the model, specifically focusing on the growth of cells at the microgravity level. By addressing these research questions, this study aims to enhance our understanding of breast cancer growth mechanisms under varying gravitational conditions and provide insights into the computational methods and sequential applicable to such investigations.

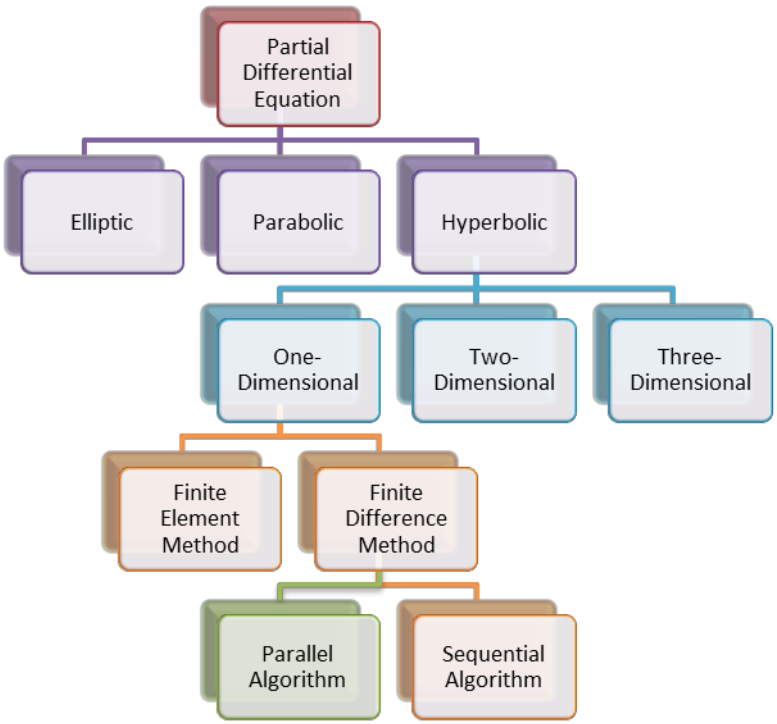

Figure 1 Research Scope

Gravitational Field on Earth

$$g_{EARTH} = \frac{GM_{EARTH}}{(r_{EARTH})^2} \quad (1)$$

$$g_{EARTH} = \frac{(6.67 \text{ x } 10^{-11} \text{ Nm}^2/kg^2)(5.97 \text{ x } 10^{24} \text{ kg})}{(6.37 \text{ x } 10^{6} \text{ m})^2}$$

$$g_{EARTH} = 9.81 \, N/kg$$

Gravitational Field in Microgravity State

$$g_{ISS} = \frac{GM_{EARTH}}{(r_{EARTH+ISS})^2} \quad (2)$$

$$g_{ISS} = \frac{(6.67 \text{ x } 10^{-11} \text{ Nm}^2/kg^2)(5.97 \text{ x } 10^{24} \text{ kg})}{(4.58 \times 10^{13} \text{ m})^2}$$

$$g_{ISS} = 8.69\ N/kg = \mu p(x,t)$$

Research Methodology

1D Problem Formulating

$$\frac{1}{c^2}\frac{\partial^2 p(x,t)}{\partial t^2} + \gamma \frac{\partial p(x,t)}{\partial t} = \nabla.\left(c\nabla p(x,t)\right) + \mu p(x,t) \quad (3)$$

c - the sound speed of the tissue being traversed and

x - space

t - time variables

γ - damping or attenuation parameter

$\mu p(x,t)$ – controllability parameters

Hyperbolic Partial Differential Equations

The wave equation is given by the differential equation:

$$\frac{\partial^2 u}{\partial t^2}(x,t) - \alpha^2 \frac{\partial^2 u}{\partial x^2}(x,t) = 0, 0 < x < L, t > 0 \quad (4)$$

Subject to the boundary conditions

$$u(0,t) = u(L,t) = 0, t > 0 \quad (5)$$

And the initial conditions

$$u(x,0) = f(x), 0 \le x \le L \quad (6)$$

$$\frac{\partial u}{\partial t}(x,0) = g(x), 0 \le x \le L \quad (7)$$

To set up the finite-difference method, assume $u = f(x)$ is a function of the independent variables x and t. Subdivide the x-plane into sets of equal rectangles if sides $\delta x = h$ and $\delta t = k$. Writing this set of equations in matrix form gives,

$$\begin{bmatrix} w_{1,j+1} \\ w_{2,j+1} \\ \cdot \\ \cdot \\ \cdot \\ w_{m-1,j+1} \end{bmatrix} = \begin{bmatrix} 2(1-\lambda^2) & \lambda^2 & 0 & \cdot & \cdot & \cdot & 0 \\ \lambda^2 & 2(1-\lambda^2) & \lambda^2 & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \lambda^2 \\ 0 & \cdot & \cdot & \cdot & 0 & \lambda^2 & 2(1-\lambda^2) \end{bmatrix} \begin{bmatrix} w_{1,j} \\ w_{2,j} \\ \cdot \\ \cdot \\ \cdot \\ w_{m-1,j} \end{bmatrix} - \begin{bmatrix} w_{1,j-1} \\ w_{2,j-1} \\ \cdot \\ \cdot \\ \cdot \\ w_{m-1,j-1} \end{bmatrix}$$

Figure 2 Matrix Form

Discretization

We introduce a time grid t_n=nΔt for n=0,1,2,… and Δt is the time step size. We set p^n (x)=p(x,t_n)as the nth iterate of the pressure at the global point x. The time derivatives in (1) are discretized by centered difference formula, which gives the semi-discrete scheme as the following,

$$\frac{p^{n+1}-2p^n+p^{n-1}}{\Delta t^2} + \gamma c^2 \frac{p^{n+1}-p^{n-1}}{2\Delta t} = c^2\Delta^2 p^n + \mu p^n \text{n} \quad (8)$$

To derive the finite-difference discretization for the above partial differential equation (PDE), we will first use the centered difference formula to discretize the spatial derivatives, followed by the time derivatives.

Let's go over each method's steps which is JB and GS method.

Finite Difference Discretization using JB

Discretize the spatial derivatives using centered difference formula:

$$\nabla.\left(c\nabla p(x,t)\right) \approx \left(\frac{c^2}{h^2}\right)(p_{i+1}^n - 2p_i^n + p_{i-1}^n) \quad (9)$$

Now, discretize the time derivatives using centered difference formula:

$$\left(\frac{\partial^2 p(x,t)}{\partial t^2}\right) \approx \left(\frac{p_i^{n+1}+2p_i^n+p_i^{n-1}}{\Delta t^2}\right) \quad (10)$$

$$\left(\frac{\partial p(x,t)}{\partial t}\right) \approx \left(\frac{p_i^{n+1}-p_i^{n-1}}{2\Delta t}\right) \quad (11)$$

Substitute the discretization into the original PDE:

$$\left(\frac{p_i^{n+1}-2p_i^n+p_i^{n-1}}{\Delta t^2}\right) + \gamma\left(\frac{p_i^{n+1}-p_i^{n-1}}{2\Delta t}\right) = \left(\frac{c^2}{h^2}\right)(p_{i+1}^n - 2p_i^n + p_{i-1}^n) + \mu p_i^n \quad (12)$$

Simplify the equation to get the semi-discrete scheme using Jacobi method:

$$p_i^{n+1} = \left(\frac{k^2 2\alpha^2}{h^2}\right)p_{i+1}^n + p_{i-1}^n + 2\left(\frac{1-(k^2 2\alpha^2}{h^2}\right)p_i^n - p_i^{n-1} \quad (13)$$

Finite Difference Discretization using GS

Discretize the spatial derivatives using centered difference formula same with equation (9). Then, discretize the time derivatives using centered difference formula that is duplicate with equation (10) and (11). We substitute the discretization into the original PDE that is equivalent to equation (12). Next, we simplify the equation to get the semi-discrete scheme using Gauss-Seidel method:

$$p_i^{n+1} = \lambda^2(p_{i+1}^n) + 2(1-\lambda^2)(p_i^n) + \lambda^2(p_{i-1}^{n+1}) - b_i \quad (14)$$

Algorithm

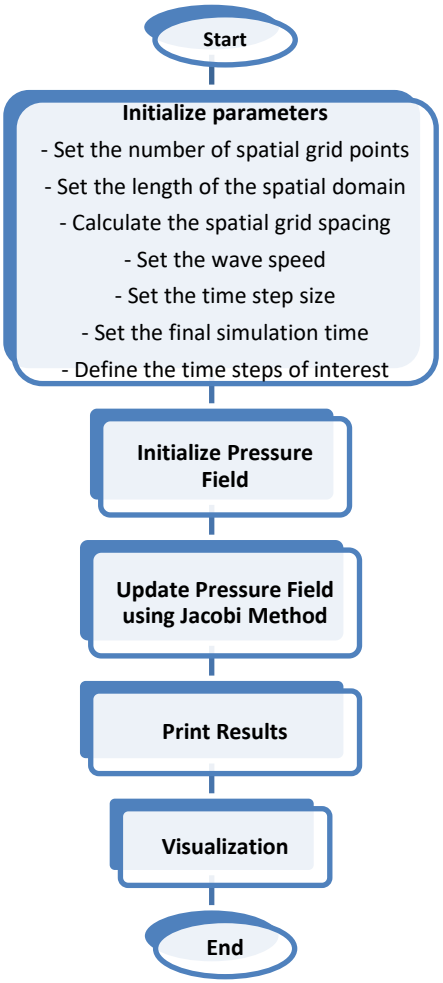

Figure 3 Simulation of Pressure Field Evolution using Jacobi Method and Gauss-Seidel Method

Results and Discussion

Pressure of Breast Cancer in 30 Days

Grid	Days					
	5	10	15	20	25	30
1	0	0	0	0	0	0
2	11933835.740	3.59651744e+08	9.11793814e+09	2.27513913e+11	5.66857414e+12	1.41214003e+14
3	24539416.280	6.82266162e+08	1.71506356e+10	4.27621226e+11	1.06535178e+13	2.65395712e+14
4	37451047.272	9.28384854e+08	2.31294890e+10	5.76185921e+11	1.43535733e+13	3.57566979e+14
5	45231143.510	1.06281733e+09	2.63184472e+10	6.55254155e+11	1.63223760e+13	4.06610392e+14
6	45231143.510	1.06281733e+09	2.63184472e+10	6.55254155e+11	1.63223760e+13	4.06610392e+14
7	37451047.272	9.28384854e+08	2.31294890e+10	5.76185921e+11	1.43535733e+13	3.57566979e+14
8	24539416.280	6.82266162e+08	1.71506356e+10	4.27621226e+11	1.06535178e+13	2.65395712e+14
9	11933835.740	3.59651744e+08	9.11793814e+09	2.27513913e+11	5.66857414e+12	1.41214003e+14
10	0	0	0	0	0	0

Table 1 Pressure of Breast Cancer in 30 Days by using Jacobi Method

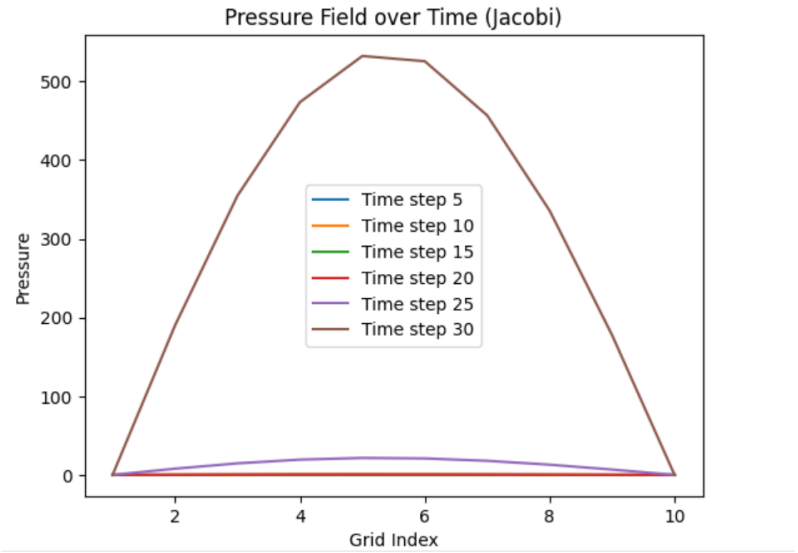

Figure 4 Graph of Pressure Field over Time (Jacobi)

Table 1 shows the pressure of breast cancer in 30 days by using Jacobi method in microgravity state computed using Python of mathematical model in this research. Figure 4 shows the pressure inside the breast for every 5 days within 1 month based on Table 1 by using Jacobi method in microgravity state. Therefore, the growth of breast cancer can be visualized from its pressure.

Grid	Days					
	5	10	15	20	25	30
1	6.94397193e-05	6.94397193e-05	6.94397193e-05	6.94397193e-05	6.94397193e-05	6.94397193e-05
2	9.03134678e-04	1.06381509e-01	2.25440468e+01	5.50657350e+03	1.39191059e+06	3.54563859e+08
3	2.11859737e-03	3.24643278e-01	7.25233602e+01	1.79455044e+04	4.54953987e+06	1.15966813e+09
4	3.65687325e-03	7.07047932e-01	1.67135294e+02	4.19206188e+04	1.06600951e+07	2.71905629e+09
5	5.43260585e-03	1.30545753e+00	3.25615803e+02	8.26854450e+04	2.10839049e+07	5.38107293e+09
6	7.34427960e-03	2.14478177e+00	5.59619974e+02	1.43524185e+05	3.66766770e+07	9.36513478e+09
7	9.23386386e-03	3.15111245e+00	8.49145985e+02	2.19272237e+05	5.61167162e+07	1.43336495e+10
8	1.06018178e-02	3.98092445e+00	1.09193496e+03	2.82999633e+05	7.24830703e+07	1.85172150e+10
9	9.54634955e-03	3.66833740e+00	1.01025204e+03	2.62044504e+05	6.71277838e+07	1.71497609e+10
10	0	0	0	0	0	0

Table 2 Pressure of Breast Cancer in 30 Days by using Gauss-Seidel Method

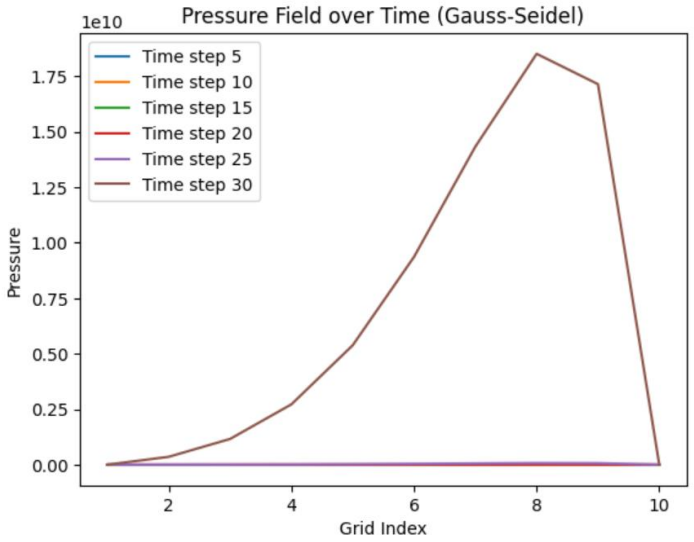

Figure 5 Graph of Pressure Field over Time (Gauss-Seidel)

Table 2 shows the pressure of breast cancer in 30 days by using Gauss-Seidel method in microgravity state computed using Python of mathematical model in this research. Figure 5 shows the pressure inside the breast for every 5 days within 1 month based on Table 2 by using Jacobi method in microgravity state. Therefore, the growth of breast cancer can be visualized from its pressure.

Number of Iterations

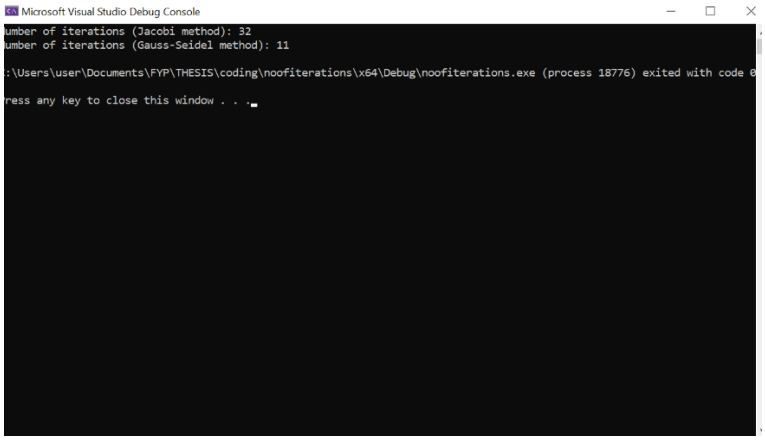

Figure 6 Results for number of iterations between JB and GS

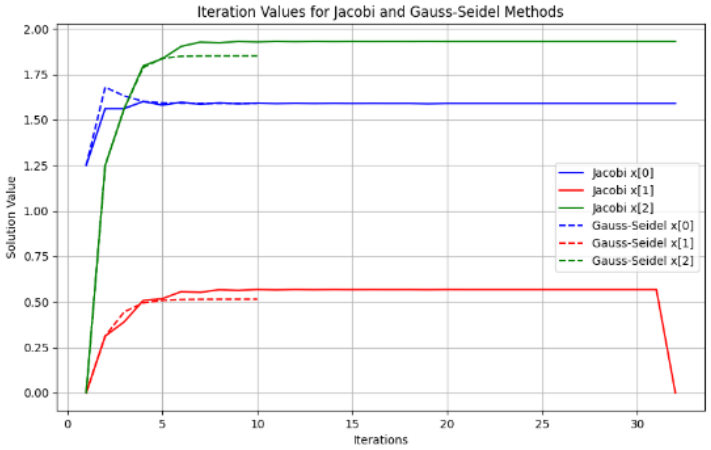

Figure 7 Graph for number of iterations between JB and GS

The Jacobi method required 32 iterations to achieve convergence, whereas the Gauss-Seidel method displayed faster convergence, achieving the target tolerance level in only 11 iterations. This suggests that the Gauss-Seidel method was more efficient than the Jacobi method, as it reached convergence in much less iterations. As a result, the Gauss-Seidel approach has the potential to reduce computational time and resources.

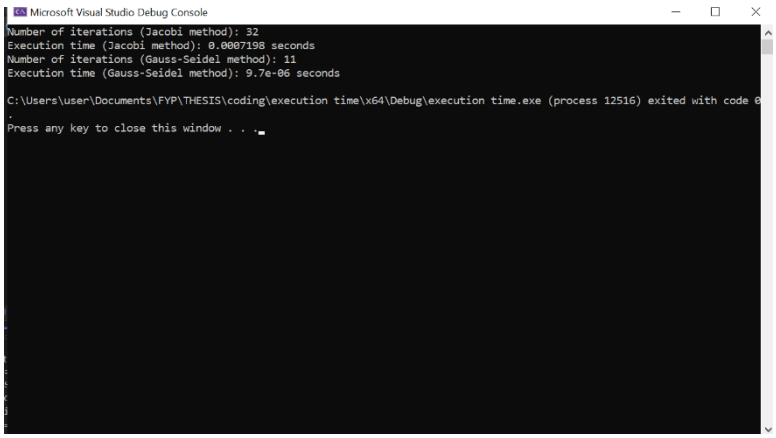

Figure 8 Results for execution time between JB and GS

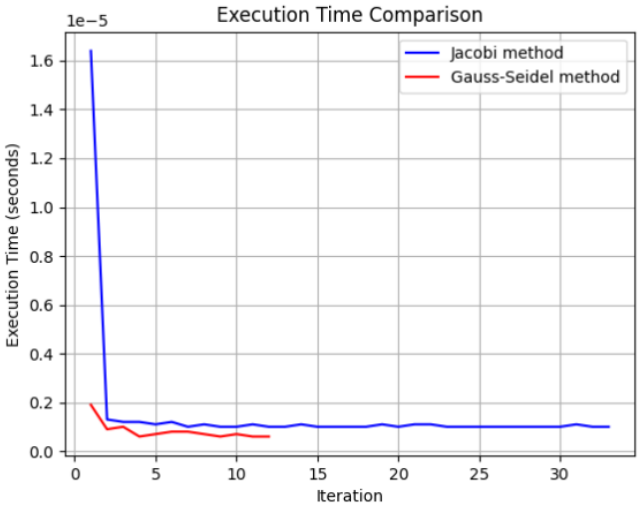

Figure 9 Graph for execution time between JB and GS

The comparison of execution times for the Jacobi and Gauss-Seidel algorithms yields some interesting findings. The Jacobi technique displayed an efficient computing performance with a reasonably short execution time of 0.0007198 seconds. On the other hand, the Gauss-Seidel method had an execution time of 9.7e-0.6 seconds, which was much faster than the Jacobi method. This indicates that the Gauss-Seidel approach is highly efficient in terms of calculation.

Computational Cost

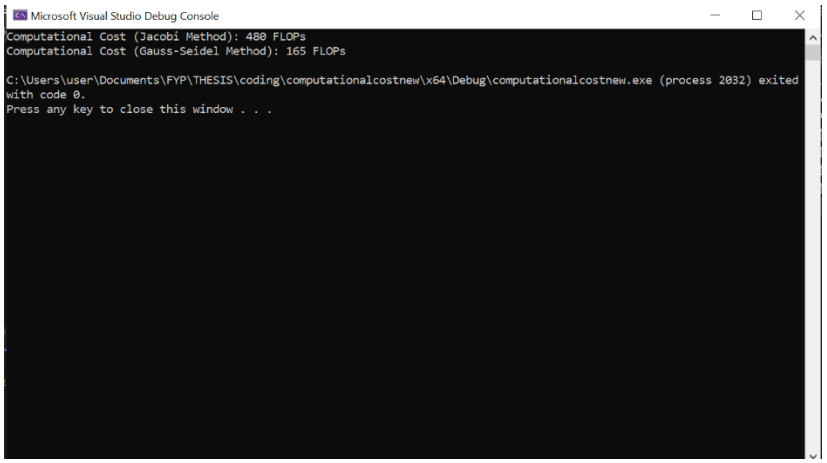

Figure 10 Results for computational cost between JB and GS

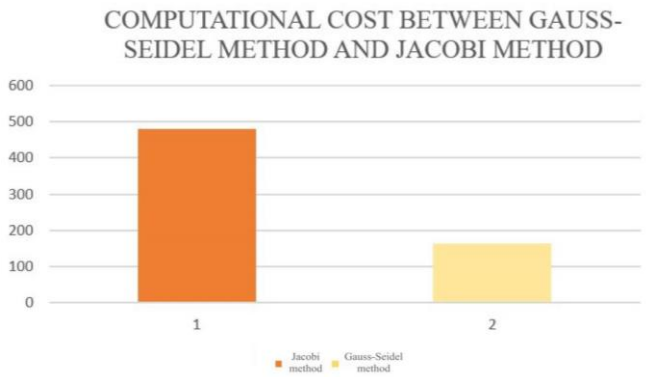

Figure 11 Chart for computational cost between JB and GS

The Jacobi approach has a computational cost of 380 FLOPs (Floating Point Operations), whereas the Gauss-Seidel method had a substantially reduced cost of 165 FLOPs. When compared to the Jacobi technique, the Gauss-Seidel method is more computationally efficient and takes less operations to achieve convergence. The lower computational cost of the Gauss-Seidel method suggests that it can save computational resources and reduce the overall processing time.

Root Mean Square Error (RMSE)

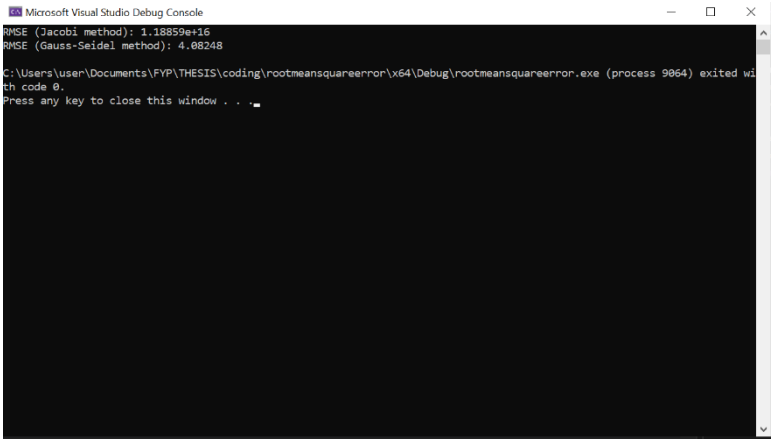

Figure 12 Results for RMSE between JB and GS

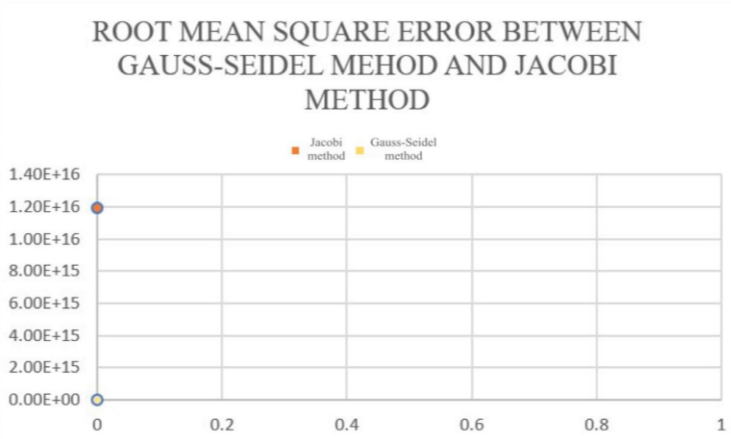

Figure 13 Chart for RMSE between JB and GS

The Jacobi technique produced an RMSE of 1.18859e+16, but the Gauss-Seidel method produced a much lower RMSE of 4.08248. This significant difference in RMSE shows that the Gauss-Seidel approach is more accurate than the Jacobi method for the given linear system of equations. Based on these results, the Gauss-Seidel method outperforms the Jacobi method in terms of solution accuracy. The Gauss-Seidel method's lower RMSE indicates its ability to better approximate the true solution, resulting in more exact results for solving linear systems of equations.

Conclusion and Recommendations

Research Outcomes

	GS Method	JB Method
Number of Iterations	11	32
Execution Time	9.7e-0.6 s	0.0007198 s
Computational Cost	165 FLOPs	380 FLOPs
RMSE	4.08248	1.18859e+16

Table 3 Research Outcomes of JB and GS

Conclusions

The Jacobi and Gauss-Seidel methods exhibit distinct behavior in resolving linear systems of equations. The Gauss-Seidel approach outperformed the Jacobi method in several aspects, making it a better choice for real-world applications.

Compared to Jacobi, Gauss-Seidel converged faster with only 11 iterations, while Jacobi required 32 iterations. The execution time for Gauss-Seidel was 9.7e-06 seconds, much shorter than Jacobi's 0.0007198 seconds, indicating higher computing efficiency. Gauss-Seidel also had a lower computational cost of 165 FLOPs, while Jacobi needed 380 FLOPs, making it more resource-efficient.

Furthermore, Gauss-Seidel achieved a lower RMSE of 4.08248, indicating higher accuracy compared to Jacobi's RMSE of 1.18859e+16. Overall, Gauss-Seidel excelled in convergence speed, execution time, computing cost, and solution accuracy, making it the preferred choice for solving linear systems of equations. These findings offer valuable insights for selecting the right approach in applications requiring prompt and accurate linear system solutions.

Recommendations

The research findings comparing Jacobi and Gauss-Seidel methods can guide future investigations into iterative techniques in microgravity studies, emphasizing the computational efficiency and accuracy of Gauss-Seidel. Prospective research can explore using iterative methods, particularly Gauss-Seidel, for computational modeling of breast cancer in microgravity settings. This entails analyzing breast cancer cell behavior, tumor growth patterns, drug responses, and other factors influenced by microgravity.

Further studies can assess the performance of various numerical algorithms, including Jacobi and Gauss-Seidel, for modeling microgravity's effects on breast cancer. Evaluating convergence rates, computational costs, accuracy, and efficiency can lead to efficient and precise numerical simulations, offering insights into breast cancer mechanisms and guiding potential strategies for diagnosis, treatment, and prevention in microgravity conditions.

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