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# Rate of gold growth and thermal distribution profile on Printed circuit board and full chip system

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### Abstract

Thermal effects become gradually more significant as devices get smaller on-chip. In manufacturing nanoscale electronic devices, it is hard to use measuring devices and sensors to monitor various performance parameters such as temperature of device and the process of growing thin gold wire on printed circuit board using CVD process. This is especially true if the chip and PCB are very powerful and advanced since the components are cramped in a small space. Hence, it is wise to develop a mathematical model to model temperature and the process of growing golden wire on various regions of PCB.

This project is mathematical modeling of rate of gold growth in CVD process during manufacturing of electronic devices such as printed circuit boards. The other model is the temperature profile of printed circuit board. The mathematical model is governed by parabolic and elliptic PDE respectively, and the solution of the models are finite difference method, with Explicit and Implicit method respectively. Computation will be conducted by using MATLAB software for both methods, and expected results are presented in the form of pioneer result which a simpler 2D case result for temperature profile before the construction of 3D model and the result is the 3D temperature profile obtained along with graph of rate of gold growth in PCB.

**Keywords:** Printed Circuit Board; Partial Differential Equations; Temperature profile; Rate of gold growth; Mathematical modelling.

### Introduction

Constant technological improvement has significantly increased the computing capacity of computing devices. Electronic devices and printed circuit boards (PCB) generate heat when it is operating, and growth of computing power is usually accompanied by an increase of heat flux density of the computing device (Ramakrishnanet al., 2021). Moreover, the heat generated is directly proportional to the central processing unit (CPU) utilization. (Wang et al., 2021). Excess heat generated by electronics can cause a device to become inefficient, unreliable and fail prematurely. Smaller devices such as smartphones and tablets usually does not have room to accommodate much cooling components.

The process of developing smaller devices with the same computation caliber is the driving factor of development of more advanced chips and processors. It is a process of utilizing as much space as possible in a certain volume and cramping components very close to their limit. A very important factor to be considered is the heat generation and dissipation of devices. As components are packed closely together, it is not feasible to put measuring devices on each power generating component to observe the temperature of different areas in chip and PCB.

On the other hand, the introduction of CVD technique revolutionized manufacturing of electronic devices. The technique enabled us to directly grow or print the wire on circuit board. Monitoring rate of gold growth in multilayered full chip system is also important as printed gold wires are important connectors between components on PCB and it requires precise positioning and size especially on smaller devices.

A solution to those problems is by developing appropriate mathematical models and using them to predict temperature behavior of the system. Rather than measuring every piece of printed gold wires on PCB in the CVD process, it is wiser to develop a mathematical model and use it to predict rate of gold growth on the PCB. By solving the mathematical models, we can study the temperature behavior to gain insight and make predictions, and thus making better decisions in engineering a thermal and gold wire printing solution for the specific system.

This research aims to find out how rate of gold growth in manufacturing of multilayer printed circuit board (PCB) and temperature of small components in multilayer printed circuit board. This research also aims to solve the mathematical model developed using numerical methods approach, and to analyze rate of gold growth in manufacturing of multilayer PCB and temperature profile of PCB.

## Materials and methods

In this research, we discuss briefly about formulation of mathematical model for modelling rate of gold growth and temperature profile model using parabolic differential equation and elliptic PDE respectively. Method and procedure that are necessary to solve differential equation in the mentioned model includes discretization by using appropriate finite difference methods and writing MATLAB code to solve and plot the result of the models.

In this research, a parabolic differential equation is used to model the rate of gold growth.

$$\frac{\partial U}{\partial t} = \phi \frac{\partial^2 U}{\partial x^2} + \lambda \, , 0 \leq x \leq L \; and \; 0 < t$$

With initial condition

$$U(x,0) = \sin(\pi x) \, , 0 \leq x \leq T$$

And boundary condition

$$U(0,t) = 0 \, , U(L,t) = 0 \, , 0 \leq x \leq L$$

Where $\lambda$ represent the incoherent ultraviolet radiation. Φ is the diameter of gold growth. $U$ indicates the rate of gold growth while $t$ is time and $x$ is the spatial coordinate in $x$ direction.

By using finite element method, with central difference, rate of gold growth at the point $i, j$, ($U_{i,j}$ ) can be expressed as:

$$U_{i,j} = U_{i,j+1} - \phi\Delta t \frac{U_{i+1,j} - 2U_{i,j} + U_{i-1,j}}{\Delta x} + \lambda$$

The computing molecule of the above equation is shown in the figure below.

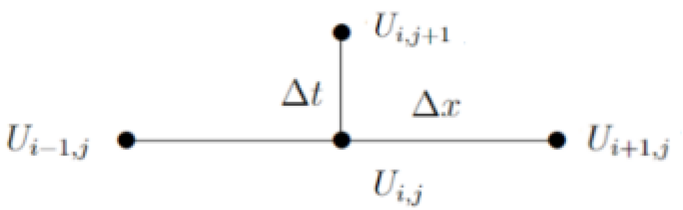


**Figure 1** Computing molecule of rate of gold growth.

On the other hand, to generate the temperature profile (3D) of the printed circuit board, we use the Poisson heat equation with the following assumptions.

1. There are no power sources inside the chip. Electronics manufacturing in a micro scale fabricates components and devices that dissipate power such that it is very close to the surface.
2. Derivative of T = 0 for steady state temperature distribution.
3. Power dissipation on the surface can be expressed and described in boundary condition.

Thus, the Poisson heat equation can be simplified and expressed in:

$$\Delta^2 T(x, y, z) = 0$$

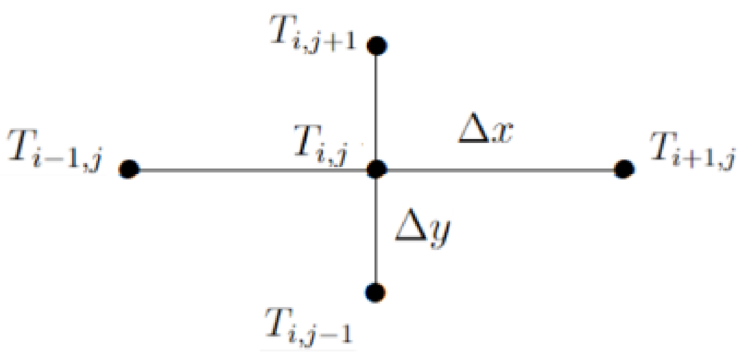


**Figure 2** Computing molecule of Temperature profile.

## Results and discussion

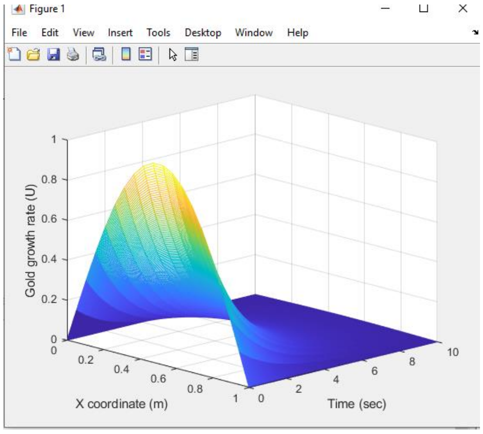


**Figure 3** Preliminary result (Rate of gold growth)

The result obtained from modeling rate of gold growth indicates that to build wire on PCB, rate of gold growth on center of cross section of wire is higher than the rest of the region of wire. On the edge of wire, the rate of gold growth gradually decreases and reaches zero. It is to prevent gold wire from overgrowing into other regions of PCB where wires should not be. As time increases the amount of gold should reach the desired level thus rate of gold growth will decrease to zero everywhere on the region of wire once wire printing process is completed.

| **Parameters and conditions** | **notation** | **value** |
|---|---|---|
| Diameter of gold growth | $\phi$ | 0.06 |
| Incoherent ultraviolet radiation | $\lambda$ | 0 |
| Initial condition | $U(x,0)$ | $3.651 \times 10^{\{-7\}}(\sin\left(\frac{\pi x}{1.27}\right))$ |
| Boundary condition 1 | $U(0,t)$ | 0 |
| Boundary condition 2 | $U(1.27,t)$ | 0 |

**Table 1** Parameters, initial and boundary conditions for Rate of gold growth in PCB.

| | **0** | **0.1659** | **0.3318** | **0.4978** | **0.6637** | **0.8296** | **1** |
|---|---|---|---|---|---|---|---|
| **0.00E+00** | 0 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 |
| **1.41E-01** | 1.22E-07 | 1.14E-07 | 1.08E-07 | 1.01E-07 | 9.50E-08 | 8.92E-08 | 8.39E-08 |
| **2.82E-01** | 2.29E-07 | 2.15E-07 | 2.02E-07 | 1.90E-07 | 1.78E-07 | 1.68E-07 | 1.58E-07 |
| **4.23E-01** | 3.08E-07 | 2.90E-07 | 2.72E-07 | 2.56E-07 | 2.40E-07 | 2.26E-07 | 2.12E-07 |
| **5.64E-01** | 3.51E-07 | 3.30E-07 | 3.10E-07 | 2.91E-07 | 2.73E-07 | 2.57E-07 | 2.41E-07 |
| **7.06E-01** | 3.51E-07 | 3.30E-07 | 3.10E-07 | 2.91E-07 | 2.73E-07 | 2.57E-07 | 2.41E-07 |
| **8.47E-01** | 3.08E-07 | 2.90E-07 | 2.72E-07 | 2.56E-07 | 2.40E-07 | 2.26E-07 | 2.12E-07 |
| **9.88E-01** | 2.29E-07 | 2.15E-07 | 2.02E-07 | 1.90E-07 | 1.78E-07 | 1.68E-07 | 1.58E-07 |
| **1.13E+00** | 1.22E-07 | 1.14E-07 | 1.08E-07 | 1.01E-07 | 9.50E-08 | 8.92E-08 | 8.39E-08 |
| **1.27E+00** | 2.02E-22 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 |

**Table 2** Table of numerical solution of rate of gold growth model.

The aim of this research is to develop a mathematical model to model a 3D temperature profile for PCB. Before modeling temperature behavior of PCB, a mathematical model to model a 2D case is developed to give insight into adding another dimension and making it 3D. The following figure is a 2D case temperature profile of a PCB, with different temperature represented as colors.

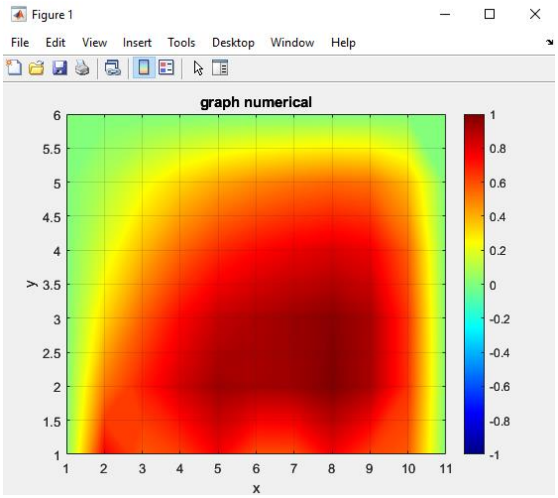


**Figure 4** Preliminary result (2D case temperature profile)

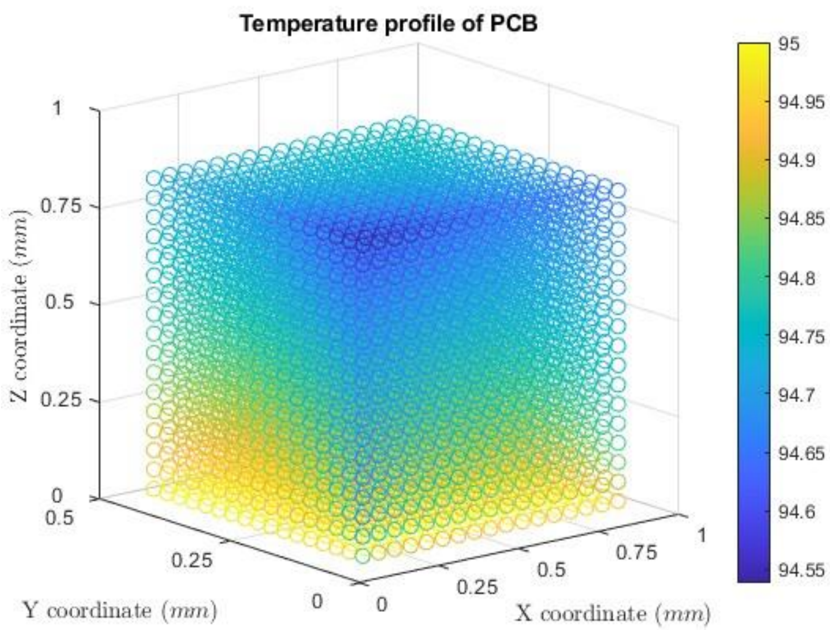


**Figure 5** Modelling result (3D case temperature profile)

After extending the MATLAB code for 2D temperature profile, the code can now model temperature profile of 3D PCB and visualize the result in 3D scatter plot. A scatter plot is chosen since it allows us to observe temperature in all nodes defined in the PCB and not only on the surface. The following figure is a 3D case temperature profile of a PCB, with different temperature represented as colors.

| **Number of segments** | **Number of nodes** | **Maximum absolute error** | **Runtime (s)** |
|---|---|---|---|
| $4 \times 4 \times 4$ | $5 \times 5 \times 5$ | 0.0206 | 1.015646 |
| $8 \times 8 \times 8$ | $9 \times 9 \times 9$ | 0.0105 | 1.006979 |
| $16 \times 16 \times 16$ | $17 \times 17 \times 17$ | 0.0075 | 1.006090 |

**Table 3** Result of convergence test and runtime.

| **z=1mm** | **y=0 mm** | **y=0.25mm** | **y=0.5mm** | **y=0.75mm** | **y=1mm** |
|---|---|---|---|---|---|
| **x=1mm** | 94.5905584 | 94.59971622 | 94.6734528 | 94.71935332 | 94.71811394 |
| **x=0.75mm** | 94.59536135 | 94.60249347 | 94.67513729 | 94.72037501 | 94.71940979 |
| **x=0.5mm** | 94.6356767 | 94.64123119 | 94.70039548 | 94.73762693 | 94.73687521 |
| **x=0.25mm** | 94.66547436 | 94.66980021 | 94.71903609 | 94.75049234 | 94.7499069 |
| **x=0mm** | 94.66540954 | 94.66877851 | 94.71841641 | 94.75011648 | 94.74966054 |

| **z=0.75mm** | **y=0mm** | **y=0.25mm** | **y=0.5mm** | **y=0.75mm** | **y=1mm** |
|---|---|---|---|---|---|
| **x=1mm** | 94.65656754 | 94.68146108 | 94.72303356 | 94.74942557 | 94.7460566 |
| **x=0.75mm** | 94.66962332 | 94.68901042 | 94.72761247 | 94.75220282 | 94.74957906 |
| **x=0.5mm** | 94.69351197 | 94.70861066 | 94.74126319 | 94.76241445 | 94.76037107 |
| **x=0.25mm** | 94.71051652 | 94.72227539 | 94.7508639 | 94.76979689 | 94.7682055 |
| **x=0mm** | 94.71034032 | 94.71949814 | 94.74917941 | 94.76877519 | 94.76753582 |

| **z=0.5mm** | **y=0mm** | **y=0.25mm** | **y=0.5mm** | **y=0.75mm** | **y=1mm** |
|---|---|---|---|---|---|
| **x=1mm** | 94.65607264 | 94.72374028 | 94.77258078 | 94.79626101 | 94.78710319 |
| **x=0.75mm** | 94.69156192 | 94.74426153 | 94.78502755 | 94.80381035 | 94.79667824 |
| **x=0.5mm** | 94.72332932 | 94.76437182 | 94.79768217 | 94.81322583 | 94.80767133 |
| **x=0.25mm** | 94.74239898 | 94.77436291 | 94.80363243 | 94.81802993 | 94.81370408 |
| **x=0mm** | 94.74192003 | 94.76681357 | 94.79905352 | 94.81525268 | 94.8118837 |

| **z=0.25mm** | **y=0mm** | **y=0.25mm** | **y=0.5mm** | **y=0.75mm** | **y=1mm** |
|---|---|---|---|---|---|
| **x=1mm** | 94.57213495 | 94.75607467 | 94.83038415 | 94.85894543 | 94.83405189 |
| **x=0.75mm** | 94.66860482 | 94.81185721 | 94.86421797 | 94.87946668 | 94.86007957 |
| **x=0.5mm** | 94.72641921 | 94.83798429 | 94.87857222 | 94.88974674 | 94.87464805 |
| **x=0.25mm** | 94.75979832 | 94.84668529 | 94.88180217 | 94.89256765 | 94.88080878 |
| **x=0mm** | 94.7584964 | 94.82616404 | 94.8693554 | 94.88501831 | 94.87586049 |

**Table 4** Tabulated result for temperature profile of PCB for different values of z.

## Conclusion

This research will provide new insights into simulating thermal distribution and heat transfer in printed circuit boards and multi-layer full chip systems. By using mathematical both models, we can predict changes in temperature in electronic components, and monitoring rate of gold growth in process of printing golden wire in PCB. We can also engineer a most efficient thermal solution for the PCB and full chip system according to the results. Hence, the temperature of the system can be lowered and preventing loss of efficiency due to overheating. Through this research, the community can benefit from accessibility of devices with higher performance.

Next, researchers may extend this study to simulate heat transfer in different heat generating components thus gaining an idea of which region in a system is more likely to be vulnerable to overheating. This is important because through accurate prediction we can provide an insight to help build a most efficient thermal solution according to the system, especially when devices and components are getting smaller.

Moreover, the analysis and technique presented in this study will convey information in applying numerical methods to solve real life problems and it will be beneficial to researchers that are interested and willing to do further research about numerical methods.

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