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Mathematical Modelling of Semi-Active and Active Car Suspension System

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Abstract
A mechanical system of springs or shock absorbers linking the wheels and axles to the vehicle's chassis makes up the car suspension system. So, a suspension system's main goal is to enhance ride quality and road handling. Two suspensions, an active suspension and a semi-active suspension, are selected to support this project in order to compare the behaviour of both systems. State space form is used to present the derivations of the mathematical modelling for both suspension. By developing analysis that were simulated using the MATLAB programme, a comparison between semi-active and active suspension is presented.

Keywords: Semi-active suspension; Active suspension; Mathematical modelling; State space equation; MATLAB

1. Introduction

Suspension system is one of the most important and basic systems in a vehicle. The major purpose of any vehicle suspension system is to maximize the friction between the road surface and the tires to provide the stability steering and good handling of the vehicle. To achieve the stability and rides comfort, there were three important principles must be resolved which is road isolation, road handling and cornering. Numerous studies have been conducted in other to achieve stability and rides comfort. Vehicle suspension system consists of 3 elements which are wishbones, spring and the shock absorber. These 3 elements are to filter and transmit forces exerted between the vehicle body and the road. The spring is important as it carries the body mass and isolates the vehicle form uneven road surface. This contributes to drive comfort. Furthermore, damper system also contributes to safety as it absorbs the damping of the body and wheel oscillations.

2. Semi-active Suspension System

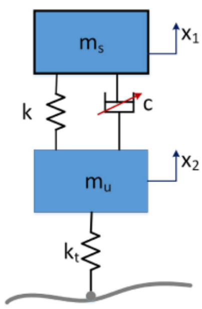


Fig. 1 Semi-active suspension model

This suspension is often used when in case of extreme driving conditions where an active shock absorber that is automatically controlled by regulator (c). It is similar to passive suspension with the only difference being having a variable damping coefficient but still fixed spring constant. Tesfaye and Hirpa (2018) analysed that compared with the fully active system the semi-active suspension requires less energy, it is cheaper, and it is simplest in design. The semi-active suspension model can be seen from Figure 1 Tesfaye and Hirpa (2018).

3. Active Suspension System

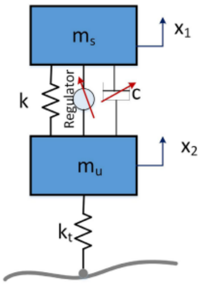
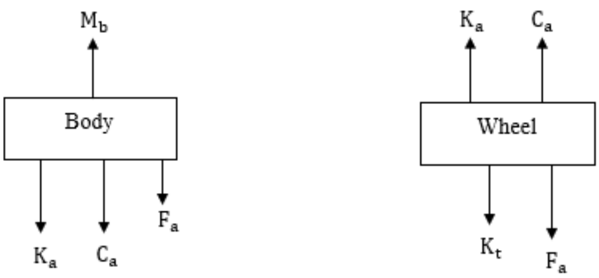


Fig. 2 Active suspension model

An active suspension includes an actuator that can supply active force regulated by a controlling algorithm which gathered from attached vehicle sensors. From Figure 2 Tesfaye and Hirpa (2018), active suspension comprises an actuator, sensors and central processing unit (CPU). Sensors measure the acceleration and the programmable CPU computes and guides the actuator that produces the additional forces when desired. Thus, this suspension system has better benefits over the other types because of its adaptability to the road profile while driving. However, Sharp and Hassan (1986) states that the passive system is more widely used because of the simplicity and comparable lower weight.

4. Mathematical Modelling of Active Suspension System

For this active suspension system, the modelling analysis start with the free body diagram of the sprung mass (body part) and unsprung mass (tire part).



(a) (b)

(a) Sprung mass and (b) Unsprung mass.

For M_w,

$$M_w x''_w = C_a(x'_b - x'_w) + K_a(x_b - x_w) - K_t(x_w - x_r) - F_a \quad (1)$$

where K_t *is the stiffness of the tire*, by rearranging the question we get

$$M_w x''_w = -(K_a + K_t)x_w + K_a x_b + C_a x'_b - C_a x'_w + K_t x_r - F_a \quad (2)$$

move M_w *to the left hand side, we obtain*

(3)

$$x''_w = \frac{-(K_a + K_t)x_w + K_a x_b + C_a x'_b - C_a x'_w + K_t x_r - F_a}{M_w}$$

Let the state variables selected as

$X_1 = x_b$ is the body displacement
$X_2 = x'_b$ is the car velocity
$X_3 = x_w$ is the wheel displacement
$X_4 = x'_w$ is the vertical wheel velocity

Thus, in state space the equation can be write as

$$X'_1 = x'_b \tag{4}$$

$$X'_2 = x''_b = \frac{-K_a x_b + K_a x_w - C_a x'_b + C_a x'_w + F_a}{M_b} \tag{5}$$

$$X'_3 = x'_w \tag{6}$$

$$X'_4 = x''_w = \frac{-(K_a + K_t)x_w + K_a x_b + C_a x'_b - C_a x'_w + K_t x_r - F_a}{M_w} \tag{7}$$

Then, formulate the state space representation of the active suspension system with the general form

$$X' = Ax + Bu$$
$$y = Cx + Du$$

in state space matric form,

$$\begin{bmatrix} x'_b \\ x''_b \\ x'_w \\ x''_w \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{K_a}{M_b} & -\frac{C_a}{M_b} & \frac{K_a}{M_b} & \frac{C_a}{M_b} \\ 0 & 0 & 0 & 1 \\ \frac{K_a}{M_w} & -\frac{C_a}{M_w} & \frac{-(K_a - K_t)}{M_w} & \frac{C_a}{M_w} \end{bmatrix} \begin{bmatrix} x_b \\ x'_b \\ x_w \\ x'_w \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M_b} \\ 0 \\ -\frac{1}{M_w} \end{bmatrix} F_a + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{K_t}{M_w} \end{bmatrix} x_r \tag{8}$$

$$\begin{bmatrix} x'_b \\ x''_b \\ x'_w \\ x''_w \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{K_a}{M_b} & -\frac{C_a}{M_b} & \frac{K_a}{M_b} & \frac{C_a}{M_b} \\ 0 & 0 & 0 & 1 \\ \frac{K_a}{M_w} & -\frac{C_a}{M_w} & \frac{-(K_a - K_t)}{M_w} & \frac{C_a}{M_w} \end{bmatrix} \begin{bmatrix} x_b \\ x'_b \\ x_w \\ x'_w \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{1}{M_b} & 0 \\ 0 & 0 \\ -\frac{1}{M_w} & \frac{K_t}{M_w} \end{bmatrix} \begin{bmatrix} F_a \\ x_r \end{bmatrix} \tag{9}$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_b \\ x'_b \\ x_w \\ x'_w \end{bmatrix} \tag{10}$$

5. Linear Quadratic Requlator approach

The LQR approach will be used for the controller design for the active suspension system, as it is one of more classic control options for linear MIMO time-invariant systems and is simple enough to design. The primary function of an LQR controller is to minimise the cost function, J, the performance index is as below:

(11)

$$J = \frac{1}{2}\int_0^t (x^t Qx + u^t Ru)dt$$

where x^t is the state vector and contains the system state variables and the systems control input.

After trial and error of changing the nonzero elements in the Q matrix and the input weighting of the R matrix, the final weighting matrices Q and R are shown below:

$$Q = \begin{bmatrix} 1760 \times 10^6 & 0 & 0 & 0 \\ 0 & 11.6 \times 10^6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, R = 0.01$$

The performance of a suspension system purely determined by the springs and dampers that are all fixed. In comparison to passive suspension, for both active and semi-active an actuator is added to the suspensions transforming the suspension from passive to active and semi-active.

Table 1 Parameters for active suspension system

Parameters	Value
M_b	5333 kg
M_w	906.5 kg
K_a	430 kN/m
K_t	2440 kN/m
C_a	1190 Ns/m

6. Results and discussion

Analysis of the results of the passive suspension system of the quarter car model and then analyse the response of the active suspension system with the state space controller. The impact of the LQR controller will also be assessed to see the significance of its impact on the system.

When comparing the two systems, the criterions that will be compared are the following

The Settling Time: It is a measured period the system oscillates until it reaches a specific limit where it begins to match the desired value.

The Rise Time: How fast system will respond and the time for the system response to reach a specific percentage within the specified value.

The Peak Time: Peak time is the time that requires the system to reach the maximum overshoots point.

Overshoot: A measure of how the system starts to move away from the desired response at the beginning.

Steady-State Error: The measure of the final error from the desired response.

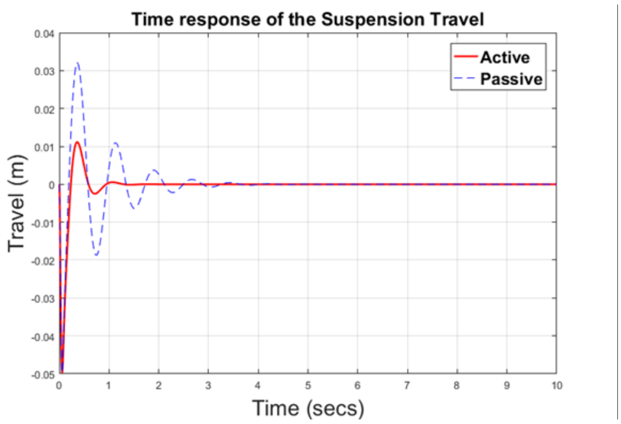


Fig. 3 Time response of the suspension travel for the active suspension system

Figure 3 shows the effect of adding a feedback LQR controller has on the system in regards to suspension travel when compared to the passive suspension system, a similar overshoot of 0.05 m is present in contrast to the passive suspensions. Compared to the passive system, however, the active suspensions see a significant reduction in the settling time at 1.86 seconds which is a 50% decrease of the response rate for the passive suspensions, therefore giving a faster response and improving the ride comfort and lowering the vibrations to the passengers.

Conclusion

To perform comparison between the passive and active for suspension control system, one of the first things that must be done during controller design is deciding upon a criterion for measuring how good a response is. However, in dynamic systems where the transient behaviour is also important, it becomes important to introduce several other criterions.

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