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Portfolio Optimization of Exchange-Traded Funds Listed on the New York Stock Exchange Using Particle Swarm Optimization

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Abstract
Portfolio optimization employs mathematical models and algorithms to aid investors in making informed investment decisions and achieving the ideal asset allocation within their portfolio. Its main objective is to either maximize returns or minimize risks by assigning appropriate weights to different assets based on specific risk appetite and expected returns. This particular study endeavors to utilize Particle Swarm Optimization (PSO) to determine the Sharpe ratio and the optimal weights for a given portfolio. The Sharpe ratio serves as a measure of risk-adjusted return for the portfolio. The portfolio under consideration consists of 20 exchange-traded funds (ETFs) from various sectors listed on the New York Stock Exchange (NYSE). Historical prices of these 20 ETFs spanning from January 2018 to December 2021 will be gathered. The implementation of the particle swarm optimization algorithm will be conducted using Python version 3.11.3. The obtained results provide evidence of the effectiveness of PSO in optimizing portfolios.

Keywords: Particle swarm optimization (PSO); Portfolio optimization (PO); Exchange-traded funds (ETFs); New York Stock Exchange (NYSE)

1. Introduction

Portfolio optimization is a complex subject in the financial world and has troubled many investors and even industry professionals. As the topic of portfolios has become more and more popular, various portfolio methods and model calculations have emerged, while how to obtain the optimal solution for a portfolio has become a subject of interest to many. In the selection process, investors will invest according to their own risk tolerance and preferences. There are also investors who value funds based on their past performance and assess future performance.

In portfolio optimization, the proportion of each stock in the portfolio needed to determine based on capital to minimise risk and maximise return. Harry Markowitz, a Nobel Prize-winning American economist, is renowned for creating Modern Portfolio Theory, which enables investors to select their preferred level of risk-taking to maximize their returns. Markowitz's theory emphasizes the importance of purchasing low-correlated stocks for diversification of risks to form an optimal portfolio.

In 1952, Harry Markowitz used mathematical programming and variance to evaluate portfolio, mean and return, portfolio selection by optimizing two conflicting criteria of risk and return. His mathematical modelling was a long way from the real world, but it had a profound effect on improving the portfolio selection procedure, with many researchers thereafter refining his theory, but so far, a comprehensive model that investors can choose for the optimal portfolio of investments, to use it has not been introduced. The Markowitz model had two important drawbacks. First, its risk assessment criterion was not a suitable criterion for portfolio risk assessment, and second, the model was not appropriate for the long-term horizon. On the other hand, the issue of portfolio selection in the real world involves transaction costs or at least the number of transactions that make it a complex mathematical problem.

After averaging the variance models, the researchers focused on other models such as discrete-time models, continuous-time models, and random programming models. Each of these methods has its disadvantages and advantages that can be selected according to the investor decision making conditions. Continuous and discrete multi-period models are solved by dynamic scheduling and optimal control methods. Due to the large dimensions of the optimal portfolio selection problem, the men-Overview of Portfolio Optimization Advances in Mathematical Finance and Applications mentioned models face serious challenges. This report introduces the use of the Particle Swarm Optimization (PSO) approach to obtain portfolio optimization.

2. Scope of the Research

According to the Statista Research Department's May 2022 survey, there will be 8,522 ETFs worldwide in 2021. Besides that, ETFs worldwide managed assets up to more than 10 trillion U.S. dollars in 2021. Although there are over 8,000 ETFs worldwide, only 20 of them were selected for study in this research. In this research, the time of study is 5 year that is 1008 days, usually there are only 252 days of available data in a year due to there are 113 days when the stock market is closed because of the weekends and public holidays but this is not a certainty.

2.1 New York Stock Exchange (NYSE)

NYSE is one of the largest stock exchanges in the world and the first officially established stock exchange in the United States. The Market Identification Code (MIC) of this exchange is XNYS. It is a globally recognized stock exchange that provides a platform for investors to buy and sell stocks and other financial products. It is headquartered on Wall Street in New York City, USA, and is a free market institution that provides a venue for open market trading and related services. NYSE was founded in 1792 when 24 stockbrokers held a meeting under a buttonwood tree held a meeting where they entered into the Buttonwood Agreement and established a set of rules for organized securities trading in New York. Prior to this agreement, securities were sold through auctions rather than on an organized market.

On 16 November 2005, the Intercontinental Exchange (ICE) was listed on the NYSE. 2006 saw the merger of the NYSE, Archipelago (Arca), and Pacific Exchange (PCX) to form the publicly traded NYSE Group. 2008 saw the NYSE acquired the American Stock Exchange and became the third largest options market in the United States. By 2013, the Intercontinental Exchange (ICE) acquired the NYSE which remains the parent company of the exchange today.

2.2 Exchange-Traded Funds (ETFs)

ETFs have emerged from their fledgling beginnings in 1993 to a full-blown revolution in the mutual fund industry. The number of ETFs offerings increase by the hundreds each year. ETF is an investment tool designed to track the performance of a specific index, sector, commodity, bond or other portfolio of assets. an ETF is a fund that holds a variety of underlying assets, rather than just one, as is the case with stocks. ETFs consist of a basket of stocks, bonds, futures or commodities based on an index, providing immediate broad diversification and avoiding the risks involved in holding a single company's stock. the liquidity of an ETF reflects the liquidity of the underlying basket of stocks. There are several types of ETFs in general, including equity ETFs, leveraged and inverse ETFs, stock ETFs, sector ETFs, bond ETFs, and commodity ETFs.

ETFs typically have lower management fees compared to traditional mutual funds. In addition, because they can be traded through an exchange, investors can take advantage of market prices to buy or sell without having to pay front-end or back-end sales charges. ETF trading is more transparent, and investors can know exactly what stocks or underlying assets are held in an ETF by visiting the ETF's website or the ETF's announcement on the exchange's website.

ETFs offer a convenient and flexible way to invest, providing broad asset diversification while offering good liquidity and low transaction costs. Investors can participate in a variety of market opportunities by selecting the right ETF based on their investment objectives and preferences.

2.3 Portfolio Optimization (PO)

Portfolio optimization is a mathematical approach involving mathematical models, and algorithms for optimization to help investors make investment decisions in a range of financial instruments and to find the optimal allocation of assets in a portfolio. The objective is to maximize the return or minimize the risk of a portfolio by rationally allocating the weights of different assets given a certain risk preference and expected return.

Portfolio optimization begins with determining which investment assets to invest in, such as stocks, bonds, exchange-traded funds, mutual funds, or other financial instruments. Each asset is associated with a certain level of expected return and risk. Expected return represents the average return an investor can expect from holding the asset, while risk measures the uncertainty associated or volatility with the asset's return.

Portfolio optimization requires considering the risks and expected returns of assets and their relative potential correlation. Portfolio optimization can be achieved by combining assets with diverse risk and return characteristics to create a portfolio that provides the best trade-off between risk and return which is optimal portfolio. An optimal portfolio is considered to have the highest Sharpe ratio which measures the expected return generated per unit of risk.

2.4 Modern Portfolio Theory (MPT)

Portfolio optimization is based on Modern Portfolio Theory (MPT). The foundation for MPT was established in 1952 by Harry Markowitz with the writing of his doctoral dissertation in statistics. It is a theoretical framework for portfolio construction and asset allocation, by combining a variety of assets with different risk and return profiles, the overall portfolio risk can be effectively reduced while increasing the expected return. Here it will introduce the efficient frontier, which is a curve consisting of various combinations of assets with different risk and return profiles, which represents the optimal portfolio that can be achieved for a given level of risk. Investors can choose a suitable portfolio on the efficient frontier according to their risk tolerance and investment objectives.

In a nutshell, the goal of MPT is to help investors find their own optimal portfolio based on their risk tolerance level. However, there are some limitations of MPT exist, such as the level of rationality of investors towards the market. So investors should combine other methods to make investment decisions.

2.5 Particle Swarm Optimization (PSO)

Particle Swarm Optimization (PSO) is a meta-heuristic algorithm and was first proposed by Kennedy and Eberhart. Later developed by an electronic engineer and a social psychologist. The idea was inspired by the habits of group animals, such as the flight of birds in groups and the swimming of fish in groups. Like any other meta-simulation method, this algorithm begins with the creation of an initial random population, here called a group of particles. The properties of each particle in the group are determined based on a set of parameters that must determine their optimal values. In this way, each particle would represent a point in the solution space of the problem. Each particle has a memory, meaning it remembers the best position that it can find in the search space. Each particle then goes in two directions, first to its best possible location and secondly to the best possible place for all particles combined. As a result, under this strategy, each particle's change in location in the search area will be impacted by both its neighbours and its own experience and knowledge.

PSO does not require derivative information and is relatively easy to implement. It has been widely applied in various fields, including engineering, economics, finance, and data analysis, to solve optimization problems such as parameter tuning, feature selection, and portfolio optimization. Overall, Particle Swarm Optimization is a heuristic optimization algorithm that mimics the social behaviour of particles to iteratively search for the optimal solution within a given search space. Its simplicity and effectiveness make it a popular choice for solving a wide range of optimization problems.

3. Data

In this research, the portfolio is composed of different ETFs listed on the New York Stock Exchange. There are various ETFs in the market, and the ETFs used in this research are from the Sectors ETFs. This research will introduce the Global Industry Classification Standard (GICS), which was developed by S&P Dow Jones Indices in conjunction with Morgan Stanley Capital International (MSCI) in 1999. GICS is a framework for industry analysis. It provides a standard and consistent industry definition for global finance. The GICS standard classifies companies in the global marketplace into multiple levels of industries and sub-industries to help investors better understand the business characteristics, competitive market environment, and risk characteristics of companies.

The GICS industry classification structure is made up of 11 sectors, 25 industry groups, 74 industries, and 163 sub-industries. These 11 sectors are energy, materials, industrials, consumer discretionary, consumer staples, health care, financial, information technology, communication services, utilities, and real estate. The GICS methodology has been recognized as the industry analysis framework for investment research, portfolio management, and asset allocation. Its globally accepted industry approach enhances the transparency and efficiency of the investment process.

This portfolio has total 20 ETFs all from the GICS industry classification structure which are Vanguard Information Technology Index Fund ETF Shares (VGT), iShares Expanded Tech-Software Sector ETF (IGV), Financial Select Sector SPDR Fund (XLF), SPDR S&P Insurance ETF (KIE), SPDR S&P Biotech ETF (XBI), iShares U.S. Medical Devices ETF (IHI), Consumer Discretionary Select Sector SPDR Fund (XLY), iShares U.S. Home Construction ETF (ITB), iShares Transportation Average ETF (IYT), Industrial Select Sector SPDR Fund (XLI), Vanguard Communication Services Index Fund ETF Shares (VOX), Consumer Staples Select Sector SPDR Fund (XLP), Energy Select Sector SPDR Fund (XLE), United States Oil Fund (USO), VanEck Oil Services ETF (OIH), Vanguard Real Estate Index Fund ETF Shares (VNQ), Materials Select Sector SPDR Fund (XLB), VanEck Gold Miners ETF (GDX), Invesco MSCI Global Timber ETF (CUT), and Utilities Select Sector SPDR Fund (XLU).

Those data are come from Yahoo Finance and www.investing.com website, adjusted price per Exchange Traded fund at close of market on the day. This research will use the adjusted prices of these ETFs from 2 January 2018 to 31 December 2021, for a total of 1008 days.

4. Results and Discussion

4.1 Adjusted Price

The closing price is the final price at which a stock or any other specific type of security traded during the market session on that trading day. The adjusted price of a stock refers to the price of the stock after it has been adjusted for certain events or factors, such as stock splits, dividends, or stock distributions. These adjustments are made to reflect the true value of the stock and to ensure that historical price comparisons are accurate. The adjusted closing price makes it extremely convenient to make a full evaluation of a stock's price. Investors can quickly estimate the value they will obtain from a specific stock.

4.2 Risk Free Rate

The risk-free rate refers to the hypothetical rate of return on an investment that carries no risk. The risk-free rate is only a theoretical concept, this is because all investments carry some degree of risk. For business valuation, many return models assume the presence of a "risk-free rate". The exact value of the risk-free rate can vary over time and from nation to nation or region to region. Usually, government bonds or treasury bills that have been issued by stable and creditworthy governments are risk-free investments. The interest rates offered by these government securities are often used as an estimate of the risk-free rate. The probability of failure on such bond issues is virtually zero due to the central government guarantee. As a result, government bonds are considered the safest investment asset class in which investors can invest their money.

$$Nominal\ Risk\ Free\ Rate\ = \frac{1+Real\ Risk\ Free\ Rate}{1+Inflation\ Rate} \tag{1}$$

$$Real\ Risk\ Free\ Rate\ = \frac{1+Nominal\ Risk\ Free\ Rate}{1+Inflation\ Rate} \tag{2}$$

In this research, 5-year Treasury Bill Rate of United State will be taken as free-risk rate which is 3.92%.

4.3 Sharpe Ratio Model

The Sharpe Ratio combines the information from mean and variance of an asset. It is quite simple, and it is a risk-adjusted measure of mean return, which is often used to evaluate the performance of a portfolio. It is described with the following equation:

$$Sharpe\ Ratio\ = \frac{R_p\ -\ \mathrm{R}_f}{\sigma_p} \tag{3}$$

where p is the portfolio, R_p is the mean return of the portfolio, R_f is the test available rate of return of a risk-free security. σ_p is the standard deviation of portfolio's excess return, in other words, it is a measure of risk of the portfolio. Adjusting the portfolio weights w_i, we can maximize the portfolio Sharpe Ratio in effect balancing the trade-off between maximizing the expected return and at the same time minimizing the risk. In this study, Sharpe Ratio is used in the PSO to find the most valuable portfolio with good ETFs combinations.

4.4 Formulation of Portfolio Optimization

Portfolio optimization was first developed by Markowitz (1952) in modern portfolio theory. The theory presents the efficient frontier, which illustrates various combinations of maximum portfolio return given each level of risk, or minimum portfolio risk for each return level. With variance (or standard deviation) as a risk measure, portfolio returns, and risk are calculated after considering the correlation between assets' returns. Based on the variance-return framework, the optimization process is obtained by changing combinations of assets with the objective function to maximize the portfolio returns or to minimize the portfolio risk.

The minimization objective is described as follows:

$$min \sum_{i=1}^{n}\sum_{k=1}^{n} x_i x_k \sigma_{ik} \quad (4)$$

subject to:

$$\sum_{i=1}^{n} x_i = 1, \quad (5)$$

$$\sum_{i=1}^{n} r_i x_i = r_p, \quad (6)$$

$$0 \le x_i.\ i = 1, 2, \ldots, n$$

where x_i are asset weights, r_i is the assets' rate of return, and σ_{ik} is the covariance between returns on assets i and k. Total weights of assets are 100 percent while the weight for any individual asset cannot be negative, and the weighted average expected return of the portfolio equals a predetermined level r_p.

4.5 Particle Swarm Optimization Algorithm (PSO)

In Particle Swarm Optimization Algorithm (PSO), a population of potential solutions, called particles, move through the search space to find the optimal solution. Each particle represents a potential solution to the optimization problem and has a position and velocity. The position of a particle corresponds to a potential solution, and the velocity determines the direction and speed of its movement through the search space. The behaviour of particles in PSO is influenced by their own best-known position (personal best) and the best-known position among all particles in the population (global best). The personal best represents the best solution found by an individual particle so far, while the global best represents the best solution found by any particle in the population.

During each iteration of the algorithm, particles adjust their velocities based on their personal and global best positions. By doing so, particles are attracted toward the global best position while exploring the search space. This balance between exploitation (moving towards the best-known solution) and exploration (searching for new solutions) helps the algorithm converge toward an optimal solution. Each particle i can be represented with three vectors v_{iz}. current position $X_i = (x_{i1}, x_{i2}, \ldots, x_{iD})$, particle's optimal position, i.e., previous best position $P_i = (p_{i1}, p_{i2}, \ldots, p_{iD})$, and velocity $V_i = (v_{i2}, v_{i2}, \ldots, v_{iD})$; the optimal swarm position, global best position $g_{best} = (g_1, g_2, \ldots, g_D)$ is known to all *m* particles. For iteration $t + 1$, velocity and position coordinates of each particle are updated as given in Eq. (1) and (2), in order to restrict the particle velocity within a defined boundary, V_{min} and V_{max} are defined as the minimum and maximum allowable velocities, respectively. An iteration of PSO-based particle movement has been demonstrated in Figure 1.

$$v_i = \omega * v_i + c_1 * rand\,() * (pbest_i - x_i) + c_2 * rand() * (gbest_i - x_i) \quad (7)$$

$$x_i = x_i + v_i \quad (8)$$

where, ω is the inertia weight; c_1 and c_2 present acceleration constants; $rand()$ generates a random value within the interval $[0, 1]$; the velocity ranges within $[\, v_{min} , v_{max}\,]$.

Flow of Standard PSO Algorithm

1. Initialize a group of particles (group size is N), including random position and velocity;
2. Evaluate the fitness of each particle;
3. For each particle, compare its fitness value with the best position $pbest$ it passes through, and if it is better, take it as the current best position $pbest$;

4. For each particle, compare its fitness value with the best position $gbest$ it passes through, and if it is better, take it as the current best position $gbest$;
5. Adjust particle velocity and position according to formulas (2) and (3);
6. If the end condition is not met, go to step (2).

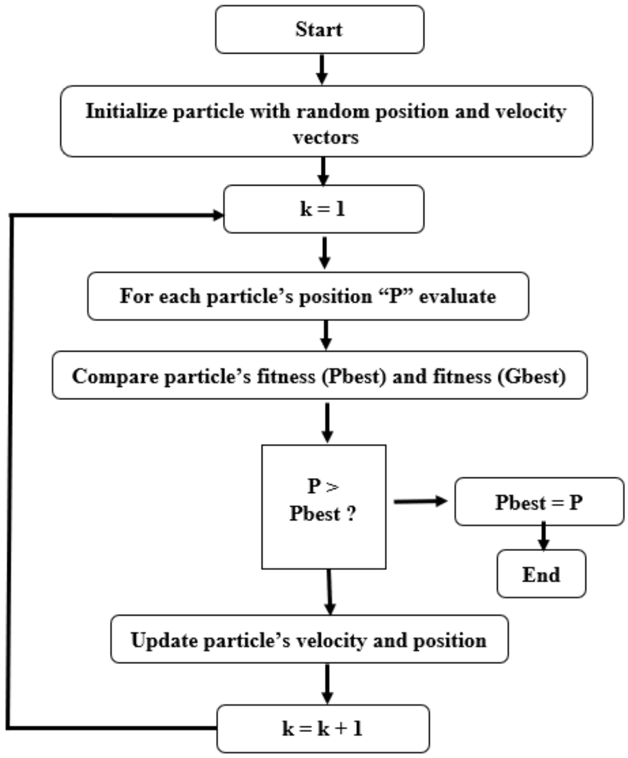


Figure 1 The flowchart depicting the general algorithm of PSO.

4.6 Optimize Portfolio using PSO Algorithm

The following is one of the formulas of the PSO algorithm:

$$v_i = \omega * v_i + c_1 * rand\,() * (pbest_i - x_i) + c_2 * rand() * (gbest_i - x_i) \tag{9}$$

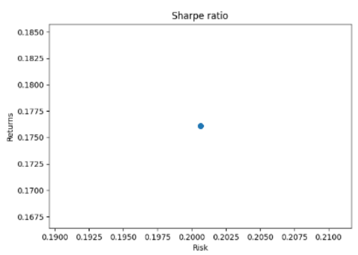
Since the parameters ω, c_1, and c_2 are widely discussed, it will be assumed in this research that $c_1 = c_2 = 1$. Inertia weight, ω is to manipulate the direction of the particles on the future displacement. Generally, the value of ω is between 0 and 1. To test what ω is, 0.1, 0.3, 0.5, 0.7, and 0.9 will be chosen to find the difference and select the most suitable ω for the following portfolio optimization. Both number of iteration and the number of particles generated in this portfolio is 100. The sum of weights is equal to 1.

Table 1: The result of different inertia weight

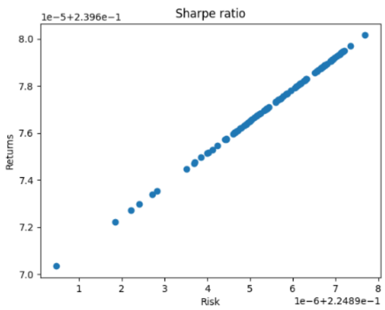
FUND	$\omega = 0.1$	$\omega = 0.3$	$\omega = 0.5$	$\omega = 0.7$	$\omega = 0.9$
VGT	0.0937	0.2120	0.2498	0.9021	0.9950
IGV	0.0998	0.1642	0.3666	0.0000	0.0000
XLF	0.0515	0.0000	0.0000	0.0000	0.0000
KIE	0.0990	0.0000	0.0000	0.0000	0.0000
XBI	0.1004	0.0000	0.0000	0.0000	0.0000

IHI	0.1011	0.0431	0.2340	0.0074	0.0050
XLY	0.0592	0.2038	0.1495	0.0000	0.0000
ITB	0.0315	0.0000	0.0000	0.0000	0.0000
IYT	0.0000	0.1400	0.0000	0.0000	0.0000
XLI	0.0937	0.0322	0.0000	0.0000	0.0000
VOX	0.0000	0.0009	0.0000	0.0000	0.0000
XLP	0.0000	0.0000	0.0000	0.0000	0.0000
XLE	0.0000	0.0000	0.0000	0.0000	0.0000
USO	0.0000	0.0000	0.0000	0.0000	0.0000
OIH	0.0000	0.0000	0.0000	0.0000	0.0000
VNQ	0.0078	0.0000	0.0000	0.0000	0.0000
XLB	0.0988	0.1850	0.0000	0.0000	0.0000
GDX	0.0509	0.0189	0.0000	0.0904	0.0000
CUT	0.0173	0.0000	0.0000	0.0000	0.0000
XLU	0.0951	0.0000	0.0000	0.0000	0.0000
Annualized Risk	0.2121	0.2249	0.2382	0.2475	0.2649
Annualized Expected Return	0.1977	0.2397	0.2976	0.3298	0.3473
Sharpe Ratio	0.7438	0.8879	1.0816	1.1707	1.1600

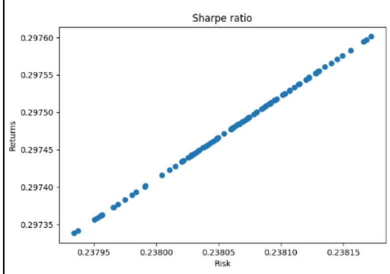
$\omega = 0.7$ has the highest Sharpe ratio, 1.1707 and $\omega = 0.1$ has the lowest Sharpe ratio, 0.7438. When ω is larger, the optimal local search ability is weak, and the global search ability is strong. A higher $\omega = 0.7$ and $\omega = 0.9$ is considered as best because its Sharpe ratio is more than 1 which is represents a higher return than volatility risk. However, the result shows that both have an ETF that is more than 90% weighted, resulting in a risky portfolio that cannot effectively rely on other ETFs to diversify risk. Therefore, $\omega = 0.5$ will be used for the following optimization.



Sharpe ratio ($\omega = 0.1$)



Sharpe ratio ($\omega = 0.3$)



Sharpe ratio ($\omega = 0.5$)

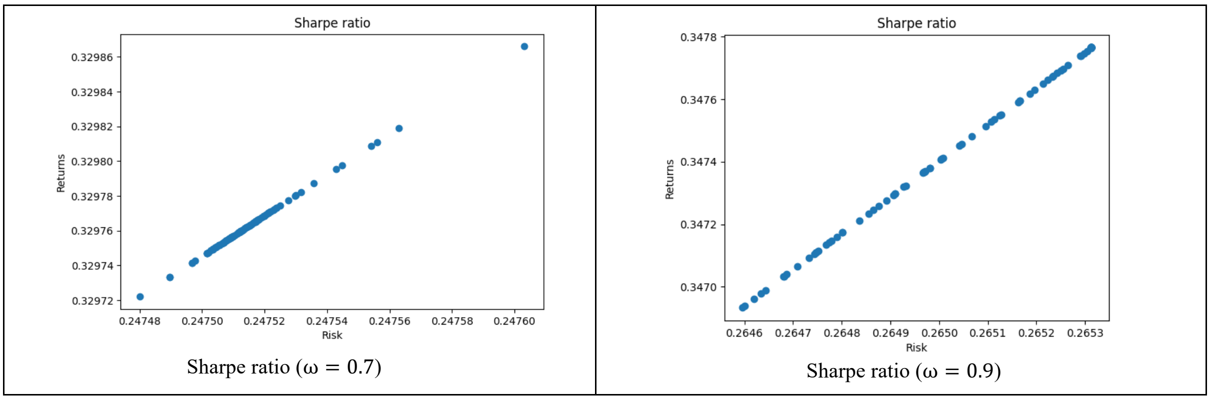


Figure 2 Sharpe ratio graph for different ω.

Figure 2 shows the graph of Sharpe ratio for different. Sharpe ratio graph of $\omega = 0.3, 0.5, 0.7, 0.9$ shows the linear pattern graph. The Sharpe ratio graph of $\omega = 0.1$ shows all the points concentrated in the middle. It is possible that this is due to its low inertia weight. When ω is smaller, the optimal local search ability is strong, and the global search ability is strong. This is the one leading to the absence of other particles in the periphery.

Table 2: The result of inertia weight, $\omega = 0.5$

FUND	Weight 1	Weight 2	Weight 3	Weight 4	Weight 5	Average Weight
VGT	0.0826	0.4511	0.372	0.6291	0.1203	0.3310
IGV	0.3401	0.0579	0.2596	0.3008	0.3284	0.2574
XLF	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
KIE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XBI	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
IHI	0.2032	0.4653	0.3684	0.0701	0.1713	0.2557
XLY	0.0789	0.0000	0.0000	0.0000	0.3119	0.0782
ITB	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
IYT	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XLI	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
VOX	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XLP	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XLE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
USO	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
OIH	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
VNQ	0.1394	0.0000	0.0000	0.0000	0.0000	0.0279
XLB	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
GDX	0.1558	0.0257	0.0000	0.0000	0.0682	0.0499
CUT	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XLU	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Annualized Risk	0.2097	0.2296	0.2390	0.2582	0.2213	0.2316

Annualized Expected Return	0.2543	0.2986	0.3054	0.3311	0.2744	0.2928
Sharpe Ratio	1.0219	1.1263	1.1102	1.127	1.0594	1.0890

The PSO algorithm of $\omega = 0.5$ was carried out 5 times to find the optimal weights and Sharpe ratio. Because of the presence of random numbers, the average Sharpe ratio and average optimal weights will be used. The results show that it will allocate 33.10% in VGT, 25.74% in IGV, 7.82% in XLY, 25.57% in IHI, 2.79% in VNQ, and 4.99% in GDX. The Sharpe ratio is 1.0890 and is considered good because its returns are greater than risks.

4.7 Portfolio Optimization using the PSO Algorithm after Adjusting

But at a realistic level, they are still too heavily weighted to help diversify the portfolio well. And most of the funds in the portfolio have a weight of 0. So, to better diversify the risk, the portfolio optimization will be adjusted so that each fund will have a weight of at least 0.5% and no more than 20%. This is because even if one fund performs poorly, it will not affect the return of the portfolio significantly.

Table 3: The results after adjusted

FUND	Weight 1	Weight 2	Weight 3	Weight 4	Weight 5	Average
VGT	0.0678	0.0437	0.0929	0.0702	0.0634	0.0676
IGV	0.0788	0.1093	0.0920	0.1137	0.1156	0.1019
XLF	0.1063	0.0220	0.0246	0.0166	0.0630	0.0465
KIE	0.0828	0.0953	0.0338	0.0463	0.0361	0.0589
XBI	0.0857	0.0447	0.0533	0.0068	0.0353	0.0452
IHI	0.0482	0.0693	0.0880	0.1081	0.0316	0.0690
XLY	0.0246	0.0281	0.0583	0.0766	0.0488	0.0473
ITB	0.0021	0.0886	0.0733	0.0873	0.0871	0.0677
IYT	0.0546	0.0881	0.0079	0.0184	0.0252	0.0388
XLI	0.0443	0.0682	0.0343	0.0046	0.0196	0.0342
VOX	0.0241	0.0163	0.0600	0.0043	0.1020	0.0413
XLP	0.0407	0.0697	0.0954	0.0594	0.0819	0.0694
XLE	0.0022	0.0101	0.0118	0.0787	0.0485	0.0303
USO	0.0194	0.0116	0.0155	0.0074	0.0086	0.0125
OIH	0.0031	0.0108	0.0006	0.0006	0.0112	0.0053
VNQ	0.0801	0.0202	0.0244	0.0122	0.0116	0.0297
XLB	0.0839	0.1272	0.1213	0.0964	0.0065	0.0871
GDX	0.0990	0.0379	0.0444	0.0369	0.1204	0.0677
CUT	0.0105	0.0228	0.0381	0.0801	0.0303	0.0364
XLU	0.0419	0.0162	0.0299	0.0755	0.0532	0.0433
Annualized Risk	0.2055	0.2157	0.2067	0.2117	0.2041	0.2087
Annualized Expected Return	0.1748	0.1817	0.1936	0.1915	0.1819	0.1847

Sharpe Ratio	0.6559	0.6572	0.7433	0.7157	0.6954	0.6935

The PSO algorithm of $\omega = 0.5$ was carried out 5 times to find the optimal weights and Sharpe ratio. Because of the presence of random numbers, the average Sharpe ratio and average optimal weights will be used. The results show that it will allocate 6.76% in VGT, 10.19% in IGV, 4.65% in XLF, 5.89% in KIE, 4.52% in XBI, 6.90% in IHI, 4.73% in XLY, 6.77% in ITB, 3.88% in IYT, 3.42% in XLI, 4.13% in VOX, 6.94% in XLP, 3.03% in XLE, 1.25% in USO, 0.53% in OIH, 2.97% in VNQ, 8.71% in XLB, 6.77% in GDX,, 3.64% in CUT, and 4.33% in XLU. The highest weighted fund is IGV, and the lowest weighted fund is OIH. The Sharpe ratio is 0.6935 and is considered to be not preferred because its returns are less than risks.

By comparing the Sharpe ratio between portfolio optimization not adjusted and adjusted, we can conclude that the Sharpe ratio of the unadjusted portfolio optimization is better and that the strategy yields higher returns while taking the same level of risk. A low Sharpe ratio does not mean that it has a low return, but that it takes on more volatility. But in investment, it is possible to choose different strategies in different situations, not necessarily based on the Sharpe ratio. It is possible to use a more aggressive risk-taking strategy to pursue high returns, or to use a safe and low-risk approach to pursue the corresponding returns.

Therefore, the objective of this research was achieved. The PSO is well-suited for optimizing portfolios, as it can effectively find the optimal weight allocation for the portfolio.

Conclusion

The Particle Swarm Optimization (PSO) is suitable to be applied in portfolio optimization. The advantages of PSO are the algorithm is simple, with few parameters, and fast convergence. It can be easily implemented in programming such as Python, C++, and MATLAB. Besides that, PSO does not require the optimized function to be differentiable and derivable, and is suitable for solving nonlinear, nonconvex, and high-dimensional function optimization problems. By using PSO, it is possible to use the Sharpe ratio as an indicator to allocate the weighting of assets in a portfolio according to risk-taking level.

However, the local search capability of PSO is poor, and when there are functions with multiple local optimal solution, it is easy to be trapped into the local optimal solution. Moreover, the search rate of PSO is less consistent, and the number of iterations is occasionally high. In each step of the iteration, only the global best and the personal best information are utilized, and the PSO algorithm often unable to obtain accurate outcomes.

The Sharpe ratio is a better indicator of risk and is a very convenient way to measure how well a strategy is doing. Therefore, it is often used in fund performance, asset allocation and other long-term investment performance measurement. But it is not sufficient to choose a fund product based on the Sharpe ratio as a whole; it needs to be evaluated in combination with long-term returns, investment win rate and other metrics to make a comprehensive assessment before investing.

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