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Determination Of Hamiltonian Polygonal Paths of Assembly Graph for $FTTM_6$ Using $\Omega -$ Algebra

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Abstract
This research study focuses on investigating and visualizing Hamiltonian polygonal paths in the assembly graph for $FTTM_6$, a mathematical model used in solving the neuromagnetic inverse problem in epilepsy patients. The research objectives include determining the existence of Hamiltonian paths, incorporating $\Omega - Algebra$ in path determination, constructing algorithms, and developing programming for path analysis and visualization. The study explores the significance of visualizing Hamiltonian polygonal paths using $\Omega - Algebra$. The research framework integrates $\Omega - Algebra$, C++ programming, and Python coding techniques to explore and analyze the connectivity and structure of the graph. The findings contribute to the understanding of graph theory and have implications in network routing, optimization algorithms, and medical treatments for epilepsy patients. In conclusion, this research investigates and visualizes Hamiltonian polygonal paths in the assembly graph for $FTTM_6$, offering insights into the connectivity and structure of the generated paths. By incorporating $\Omega - Algebra$, C++ programming, and Python coding techniques, the study advances the understanding of graph theory and provides practical approaches for path analysis and visualization. The findings have implications in various domains, including network routing, optimization algorithms, and mathematical model to solve the neuroinverse problems..

Keywords Hamiltonian polygonal paths; assembly graph; $FTTM_6$; $\Omega - Algebra$; path determination; graph visualization.

Introduction

The human brain is our body's most significant structure. It is also the most complicated ordered structure yet discovered. Epilepsy is one of the most dangerous neurological disorders that happen anywhere, it is characterised by unprovoked electrical activity in the brain known as seizures. An estimated 5 million people worldwide are diagnosed with epilepsy each year [1]. Abnormal pattern such as spikes, sharp wave and wave complexes can be observed by using EEG [2]. Epileptic patients undergo EEG recordings to capture the electrical activity in their brains. However, analyzing the patterns and outcomes of these recordings lacks mathematical approach.

To address this, previous researchers have introduced the concept of Fuzzy Topographic Topological Mapping (FTTM). The Fuzzy Topographic Topological Mapping (FTTM) model that is composed of four different topological spaces, namely the magnetic contour plane (MC), base magnetic plane (BM), fuzzy magnetic field (FM), and topographic magnetic field (TM). A sequence of $FTTM$, $\boldsymbol{FTTM_n}$, is an expansion of $FTTM$ that is organized symmetrically. The special characteristic of $FTTM$, namely the homeomorphisms between its components, makes it possible to create new $FTTM$. Several previous researchers have attempted to create, extend, propose, and implement the Fuzzy Topological Topographic Mapping ($FTTM$) for problem resolution in the neuro magnetic inverse problem for epileptic patients [1][2][3][5][6].The concept of $\boldsymbol{\Omega - algebra}$ is a set of catalytic relations which can be modelled as a algebraic operation [4]. This concept is suitable to compute the algorithm for the determination of the Hamiltonian Polygonal Paths of Assembly Graph for $\boldsymbol{FTTM_6}$.

In this project, we investigate and visualize all the Hamiltonian Polygonal Paths of Assembly Graph for $FTTM_6$. There are a total of 8,032 Hamiltonian polygonal paths in assembly graph of $FTTM_6$.

Definition 1 (Shukor, Ahmad, Idri, Awang, & Fuad, 2021)
Let $FTTM_i = (M_i, B_i, F_i, T_i)$ such that $\{M_i, B_i, F_i, T_i\}$ are topological spaces with $M_i \cong B_i \cong F_i \cong T_i$ for $i = 1,2,3,\cdots,n$.

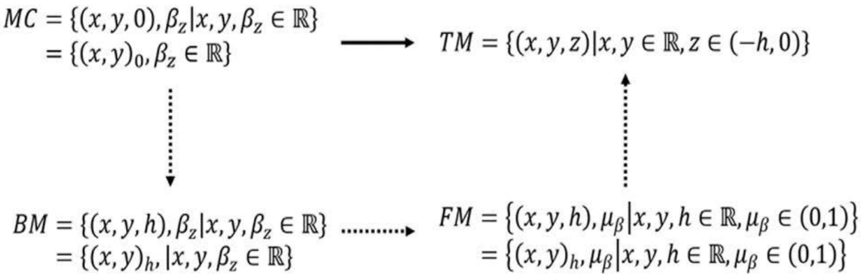

Figure 1 $FTTM$ model by algorithm.

Definition 2 (Ahmad, Shukor, Idris, & Mahamud, 2019)
The vertices of FTTM are the 4 components in FTTM which are M, B, F and T. The sequence of vertices of $FTTM_n$ can be defined as $vFTTM_1$, $vFTTM_2$, $vFTTM_3$, $\cdots$ and given recursively by equation

$$vFTTM_n = 4n \; for \; n \geq 1$$

Definition 3 (Ahmad, Shukor, Idris, & Mahamud, 2019)
The edges of FTTM are lines that connect the components in FTTM. The sequence of edges of $FTTM_n$ can be defined as $eFTTM_1$, $eFTTM_2$, $eFTTM_3$, $\cdots$ and given recursively by equation

$$eFTTM_n = 4 + (n-1)8 \; for \; n \geq 1$$

Definition 4 (Ahmad, Shukor, Idris, & Mahamud, 2019)

The faces of FTTM are squares where each face has 4 components in FTTM. The sequence of faces of $FTTM_n$ can be defined as $fFTTM_1$, $fFTTM_2$, $fFTTM_3$, $\cdots$ and given recursively by equation

$$fFTTM_n = 1 + (n-1)5 \; for \; n \geq 1$$

Definition 5 (Ahmad, Shukor, Idris, & Mahamud, 2019)
The maximal assembly graph of $FTTM_n$ is defined as

$$\Gamma_{FTTM_n} = FTTM_n - [E(FTTM_1) \cup E(FTTM_n)] \; for \; n \geq 3,$$

such that $|\Gamma_{FTTM_n}|$ denotes the number of 4-valent vertices. Noted that the only assembly graph considered in this study is the maximal assembly graph of $FTTM_n$ if there is not specific mentioned. Then, an assemble graph is exists in any sequence of $FTTM_n$ for $n \geq 3$

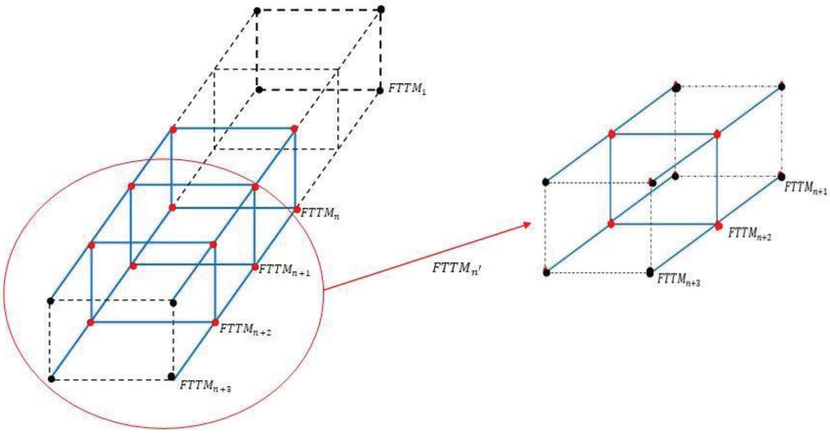

Figure 2 Assembly graphs of $FTTM_{n'}$.

Methodology

The concept of omega algebra is defined as a structure (M, Ω), where M is a set and Ω is a collection of operations and relations on M. The collection Ω consists of n-ary operations and relations for all natural numbers n. Each operation or relation in Ω takes elements from M as inputs and produces an element of M as output. The connection between n elements will produce one of these elements as a final product.

$$\omega_2 : M \times M \to M$$
$$\omega_3 : M \times M \times M \to M$$
$$\vdots$$
$$\omega_n : \underbrace{M \times M \times \cdots \times M}_{n \text{ times}} \to M$$

The notation $\omega_{n(v_i,v_j)}$ represent there exist v_i and v_j that have n-ary relation with v_i traverses to v_j through n-ary Cartesian product with $\Omega - Algebra$ operation. The $*$ operation of $\Omega - Algebra$ is used to represent the hexadecimal operation in $FTTM_6$.

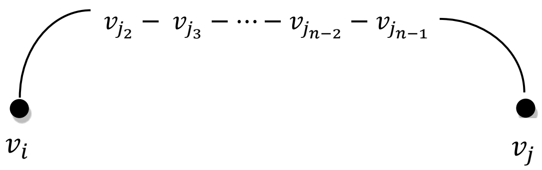

Figure 3 Example of $\omega_{n(v_i,v_j)}$ with $\Omega - Algebra$ operation.

Definition 6

A hexadecimal operation on $FTTM_6$ that applies the concept of $\Omega - Algebra$ is defined as follow

$$\omega_{16} : *_{16} : \underbrace{V \times V \times \cdots \times V}_{16 \text{ times}} \to V$$

such that $\exists v_i, v_j \in V$ and $v_i *_{16} v_j = v_j \in V$ through all other $v_j \in V$ (16 vertices with length 15).

An example of a path in hexadecimal operation that applies the concept of $\Omega - Algebra$ is as follows:

$$v_{j_1} \to v_{j_2} \to v_{j_3} \to v_{j_4} \to v_{j_5} \to v_{j_6} \to v_{j_7} \to v_{j_8} \to v_{j_9} \to v_{j_{10}} \to v_{j_{11}} \to v_{j_{12}} \to v_{j_{13}} \to v_{j_{14}} \to v_{j_{15}} \to v_{j_{16}}$$

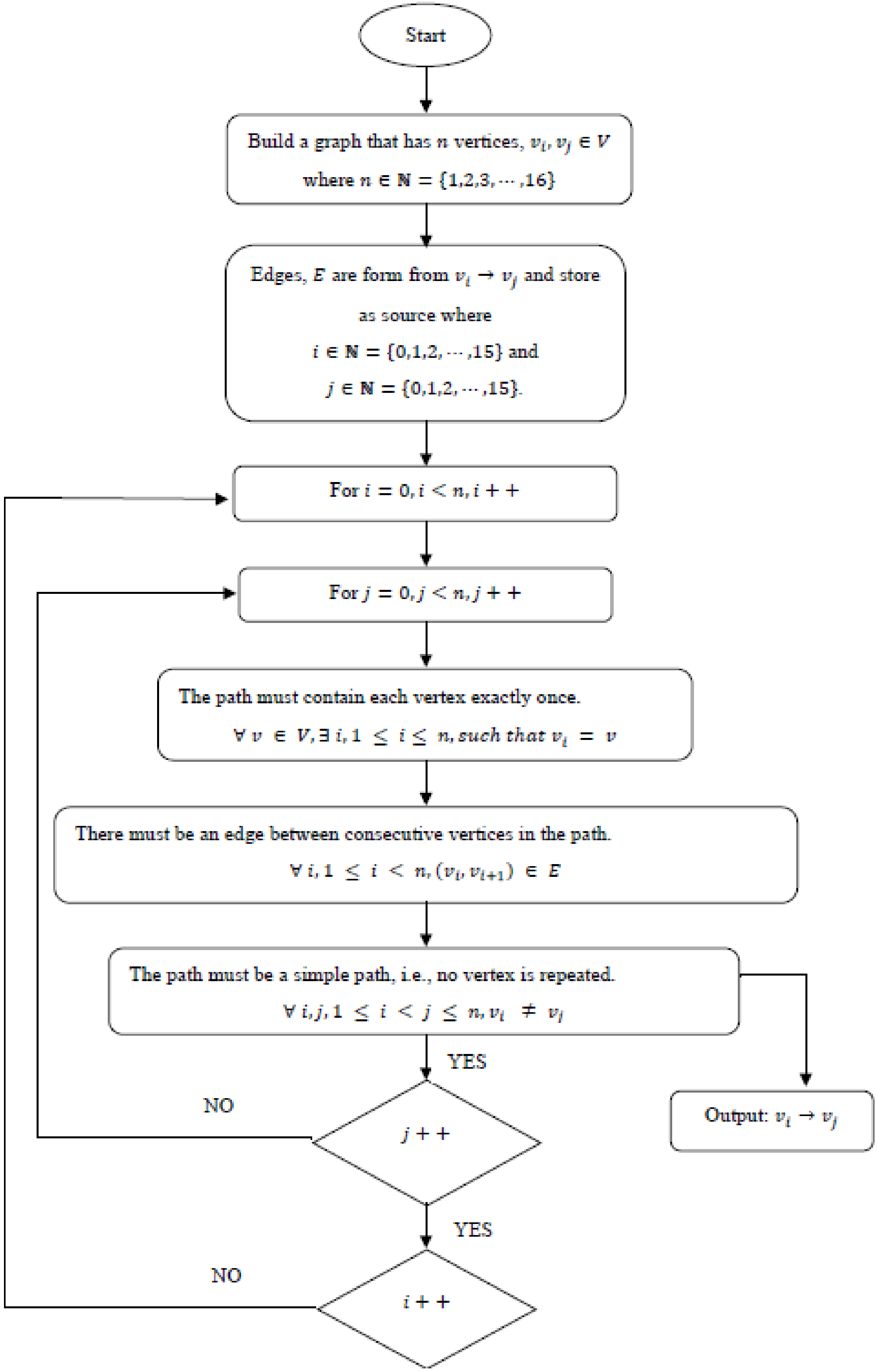

Figure 4 Algorithm flow chart of the possible paths.

Implementation and Result

The geometrical features of $FTTM_6$ based on Definition 2, Definition 3, and Defination4 are shown as follow:

$$vFTTM_6 = 4(6) = 24$$
$$eFTTM_6 = 4 + (6-1)8 = 44$$
$$fFTTM_6 = 1 + (6-1)5 = 26$$

Table 1 Geometrical Features of $FTTM_6$.

Geometrical Features of $FTTM_n$	$FTTM_6$
Vertices	24
Edges	44
Faces	26

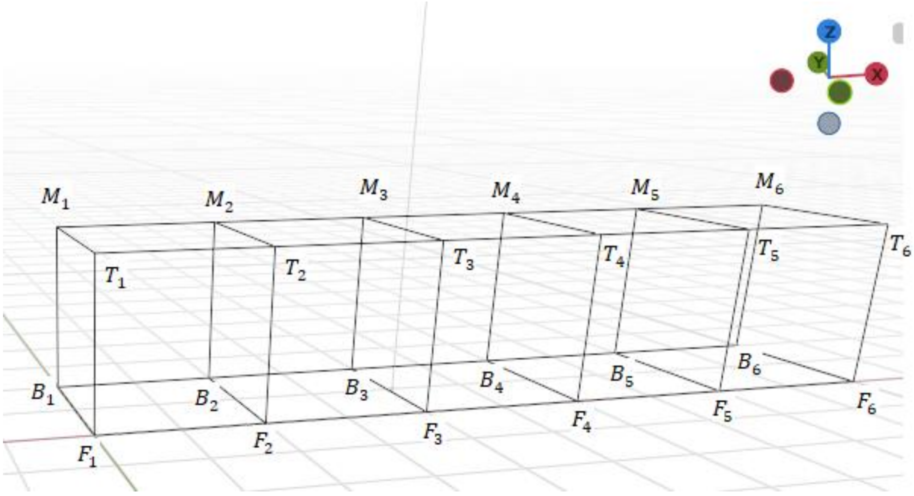

Figure 5 Graph of sequence of $FTTM_6$.

The maximal assembly graph of $FTTM_6$ based on definition 5 is defined as

$$\Gamma_{FTTM_6} = FTTM_6 - [E(FTTM_1) \cup E(FTTM_6)]$$

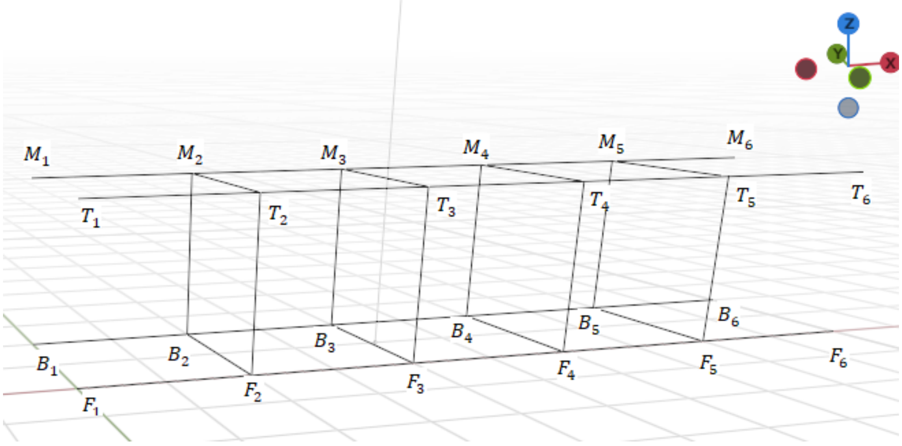

Figure 6 Assembly graph of $FTTM_6$.

Based on definition 5 of assembly graph, an assembly graph is a finite connected graph, where all rigid vertices are from valency 1 or 4. Let $\Gamma = (V, E)$ be a finite graph with a set of vertices V and a set of edges E. A vertex of valency 1 is an end point of the graph. The $|\Gamma|$ represents the number of 4-valent vertices in Γ.

$$4-\text{valent vertices in } FTTM_6 = \{M_2, B_2, T_2, F_2, M_3, B_3, T_3, F_3, M_4, B_4, T_4, F_4, M_5, B_5, T_5, F_5\}$$
$$|FTTM_6| = 16$$

$$1-\text{valent vertices in } FTTM_6 = \{M_1, B_1, T_1, F_1, M_6, B_6, T_6, F_6\}$$

Since all vertices in $FTTM_6$ are from valency 1 or 4, therefore $FTTM_6$ is an assembly graph.
The output result in C++ of Hamiltonian polygonal paths of assembly graph for $FTTM_6$ using $\Omega - Algebra$ is shown as figure below in vertices, v_n form where $0 \le n \le 15$.

Total number of Hamiltonian paths: 8032

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Path 7999: 15 12 13 14 10 11 7 3 2 6 5 1 0 4 8 9
Path 8000: 15 12 13 14 10 11 7 3 2 6 5 9 8 4 0 1
Path 8001: 15 12 13 14 10 11 7 3 0 1 2 6 5 4 8 9
Path 8002: 15 12 13 14 10 11 7 3 0 1 2 6 5 9 8 4
Path 8003: 15 12 13 14 10 11 7 3 0 4 8 9 5 6 2 1
Path 8004: 15 12 13 14 10 11 7 3 0 4 8 9 5 1 2 6
Path 8005: 15 12 13 14 10 6 5 4 0 1 2 3 7 11 8 9
Path 8006: 15 12 13 14 10 6 5 1 2 3 0 4 7 11 8 9
Path 8007: 15 12 13 14 10 6 5 9 8 11 7 4 0 1 2 3
Path 8008: 15 12 13 14 10 6 5 9 8 11 7 4 0 3 2 1
Path 8009: 15 12 13 14 10 6 5 9 8 11 7 3 2 1 0 4
Path 8010: 15 12 13 14 10 6 5 9 8 4 0 1 2 3 7 11
Path 8011: 15 12 13 14 10 6 7 4 0 3 2 1 5 9 8 11
Path 8012: 15 12 13 14 10 6 7 3 2 1 0 4 5 9 8 11
Path 8013: 15 12 13 14 10 6 7 11 8 9 5 4 0 1 2 3
Path 8014: 15 12 13 14 10 6 7 11 8 9 5 4 0 3 2 1
Path 8015: 15 12 13 14 10 6 7 11 8 9 5 1 2 3 0 4
Path 8016: 15 12 13 14 10 6 7 11 8 4 0 3 2 1 5 9
Path 8017: 15 12 13 14 10 6 2 1 0 3 7 4 5 9 8 11
Path 8018: 15 12 13 14 10 6 2 1 0 3 7 11 8 9 5 4
Path 8019: 15 12 13 14 10 6 2 1 0 3 7 11 8 4 5 9
Path 8020: 15 12 13 14 10 6 2 1 0 4 5 9 8 11 7 3
Path 8021: 15 12 13 14 10 6 2 1 5 4 0 3 7 11 8 9
Path 8022: 15 12 13 14 10 6 2 1 5 9 8 11 7 4 0 3
Path 8023: 15 12 13 14 10 6 2 1 5 9 8 11 7 3 0 4
Path 8024: 15 12 13 14 10 6 2 1 5 9 8 4 0 3 7 11
Path 8025: 15 12 13 14 10 6 2 3 0 1 5 4 7 11 8 9
Path 8026: 15 12 13 14 10 6 2 3 0 1 5 9 8 11 7 4
Path 8027: 15 12 13 14 10 6 2 3 0 1 5 9 8 4 7 11
Path 8028: 15 12 13 14 10 6 2 3 0 4 7 11 8 9 5 1
Path 8029: 15 12 13 14 10 6 2 3 7 4 0 1 5 9 8 11
Path 8030: 15 12 13 14 10 6 2 3 7 11 8 9 5 4 0 1
Path 8031: 15 12 13 14 10 6 2 3 7 11 8 9 5 1 0 4
Path 8032: 15 12 13 14 10 6 2 3 7 11 8 4 0 1 5 9
Total number of Hamiltonian paths: 8032

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Figure 7 Output result of Hamiltonian polygonal paths of assembly graph for $FTTM_6$ using $\Omega - Algebra$.

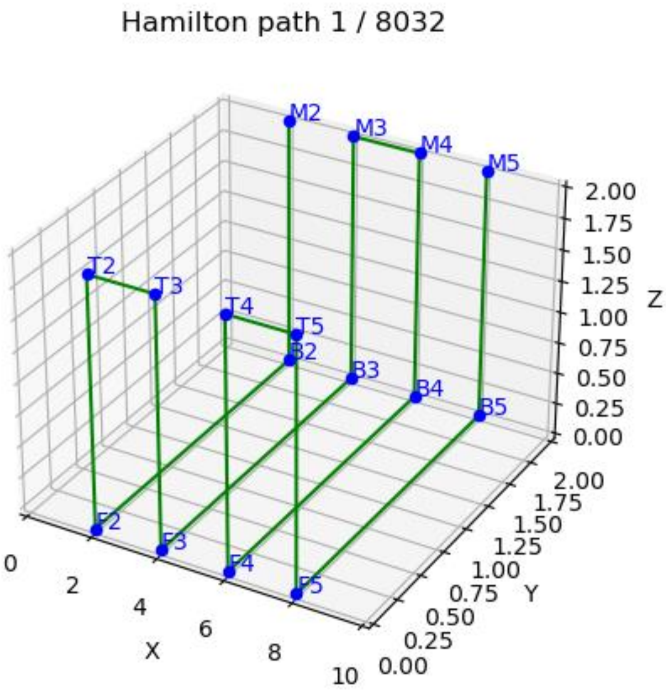

Figure 8 Hamiltonian path 1 in $FTTM_6$ model.

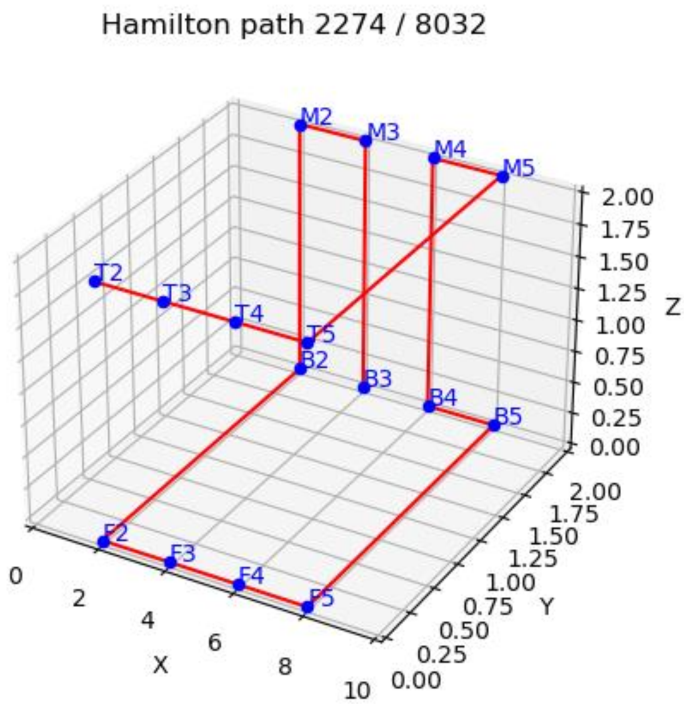

Figure 9 Hamiltonian path 2274 in $FTTM_6$ model

The implementations contribute to the objectives of the project, showcasing the effectiveness of the implemented algorithms in solving the Hamiltonian path problem and visualizing the paths. The findings can be used for further analysis, optimization, or application in various domains such as network routing, graph theory, or optimization algorithms.

Overall, the project demonstrates the application of programming techniques to solve and visualize the Hamiltonian path problem, providing valuable insights into graph connectivity, structure, and properties.

Conclusion

We know that all vertices in $FTTM_6$ are from valency 1 or 4, therefore $FTTM_6$ is an assembly graph based on the chapter 4.1. There exist Hamiltonian polygonal paths in assembly graph for $FTTM_6$. The total number of Hamiltonian polygonal paths for $FTTM_6$ using $\Omega - Algebra$ is 8032 paths. The algorithm and a workable coding in programming language for the Hamiltonian polygonal paths of assembly graph $FTTM_6$ using $\Omega - Algebra$ were constructed in this study.

In conclusion, this research study successfully investigated and visualized the Hamiltonian polygonal paths of the assembly graph for $FTTM_6$using $\Omega - Algebra$. The research outcomes confirmed the existence of these paths and demonstrated the applicability of $\Omega - Algebr$a in their determination. The development of coding implementations in C++ and Python enabled the analysis and visualization of the paths, providing valuable insights into the connectivity and structure of the graph. The contributions of this research include expanding the understanding of $FTTM_6$, advancing the knowledge and application of algebraic sets, and providing practical approaches for path analysis and visualization. These findings have implications for various fields, including network routing, graph theory, and optimization algorithms, and hold the potential to improve neuroinverse mathematical modelling.

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