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The Energy and Seidel Energy of the Prime Order Cayley Graph of a Dihedral Group

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Abstract
A graph consists of finite set whose elements are called vertices and a collection of unordered pairs of vertices called edges that often shown diagrammatically as a collection of dots and lines respectively. Meanwhile, Cayley graphs are graphs associated to a group and a set of generators for that group. Various type of Cayley graphs has been introduced due to the significance of this graph in combinatorics and finite group representation. The prime order Cayley graphs of a group is one of the new types of Cayley graphs with elements of the groups as vertices and two vertices u and v are adjacent if uv^{-1} is in the subset of prime order elements of the group. In this paper, the energy and Seidel energy of the prime order Cayley graph of the dihedral group of order six is determined. The energy of a graph is defined as the sum of the absolute values of its adjacency matrix's eigenvalues. Meanwhile, the Seidel energy is defined as the summation of the absolute values of the eigenvalues of the Seidel matrix of the graph. The prime order Cayley graph of the dihedral group of order six is constructed by using the definition of the prime order Cayley graph and the generator of the group. Then the adjacency of the vertices in the graph constructed is analyzed in order to determine the adjacency matrix and Seidel matrix of the graph. Finally, the energy and Seidel energy of the graph is computed based on the adjacency matrix and Seidel matrix. The energy and Seidel energy of the prime order Cayley graph of order six are found to be similar.
Keywords: graph; Cayley graphs; group; energy; Seidel energy; dihedral groups.

1. Introduction

A graph consists of a finite set whose elements are called the vertices and a collection of unordered pairs of vertices called edges [1]. For a graph Γ, its vertex set and edge set are denoted as $V(\Gamma)$ and $E(\Gamma)$, respectively. A graph is often shown diagrammatically as a collection of dots or circles representing the vertices and lines or curves representing the edges.

A group G is a finite or infinite set of elements together with a binary operation (called the group operation) that satisfy the four fundamental properties which are closure, associativity, the identity property, and the inverse property.

Various graphs of groups have been introduced including prime order Cayley graphs of group. In this paper, the prime order Cayley graphs of the dihedral group of order six is constructed. Let G be a group and S be the subset of prime order elements of G. The prime order Cayley graph of G with respect to S, denoted as $Cay_p(G,S)$, has the vertex set $V(Cay_p(G,S))$ equal to G and two vertices g and h are adjacent by an edge if and only if there exist $s \in S$ such that $g = sh$.

In addition, based on the structure of the prime order Cayley graph of the dihedral group of order six, the energy and Seidel energy of the graph are computed. In order to compute the energy and Seidel energy, the adjacency matrix and Seidel matrix are determined. The adjacency matrix of graph Γ, denoted by $A(\Gamma)$ is a square matrix of order n that can be obtained from the graph. The adjacency matrix, $A(\Gamma)$ is a matrix $A = [a_{ij}]$ consisting of $0's$ and $1's$ in which the entry of a_{ij} is 1 if there is an edge connecting with the vertices and 0 for otherwise [2]. Meanwhile the Seidel matrix, is a real square symmetric matrix consisting of 0 and ± 1 in which the entry of a_{ii} is 0, and the entry of a_{ij} is -1 if there is an edge connecting the vertices and 1 for otherwise [3].

The energy of a graph Γ, denoted as $\varepsilon(\Gamma)$, is defined as the sum of the absolute values of its adjacency matrix's eigenvalues [4]. This graph invariant is strongly related to the total π-electron energy of conjugated hydrocarbon molecules, which is a chemical quantity. Graph energy has attracted the interest of mathematicians and numerous variants have been proposed including the Seidel energy of graph. The Seidel energy is defined as the summation of the absolute values of the eigenvalues of the Seidel matrix of the graph [3].

In this paper, the energy and Seidel energy of the prime order of Cayley graph of the dihedral group of order six is computed.

2. Literature Review

2.1 Graph of Groups

Graph theory is a developing field with applications in mathematics, science, and technology. It is actively used in the fields of biochemistry, chemistry, communication networks, computer science (algorithms and computation), and operations research (scheduling), as well as in a variety of applications such as coding theory, x-ray crystallography, radar, astronomy, circuit design, communication network addressing, and data base management.

The definition of a graph is given as follows:

Definition 2.1 [6] Graph
A graph Γ is an ordered triple (V, E, f) consists of a nonempty set $V(\Gamma)$ of vertices, an edge set $E(\Gamma)$, and a function f associated with an edge of Γ.

In the construction of a graph, the structures of the graph such as connectivity and completeness of the graph can be observed.

Definition 2.2 [6] Connected Graph
A graph Γ is connected if every pair of vertices a and b, there exists a path from a and b. If no such path exists, then a graph is said to be disconnected.

Definition 2.3 [6] Complete Graph
For each integer $n \geq 1$, let K_n denotes the graph with vertices $\{v_1, v_2, \ldots, v_n\}$ and with an edge $\{v_i, v_j\}$ for every i and j. Every vertex in K_n is connected to every other vertex. The graph K_n is called the complete graph on n vertices.

Definition 2.4 [7] Graph of Group
A graph of a group *G*, is an object consisting of two vertices and an edge, and it is labelled as $\Gamma G = (V, E)$. Edge, *E*(*G*) are the lines that connect the two vertices, *V*(*G*).

Definition 2.5 [5] Cayley Graph
Let G be a finite group and S be a subset of G. The corresponding Cayley graph, $Cay(G, S)$ has the vertex set equal to G. A directed edge connects two vertices g and h if and only if there exist $s \in S$ such that $g = sh$.

The prime order Cayley graph has been introduced by Tolue [5] in 2019.

Definition 2.6 [5] Prime Order Cayley Graph
Let G be a group and S be the set of prime order elements of G. The prime order Cayley graphs denoted as $Cay_p(G, S)$ is a graph where the set of vertices of the graph is the elements of G and two distinct vertex g and h are adjacent with an edge whenever $gh^{-1} \in S$, that is $g = sh$ for some $s \in S$.

In [5], Tolue has constructed new type of Cayley graph namely prime order Cayley graphs of finite group.

Extending the result in [5], in this paper, the prime order Cayley graph of the dihedral group of order six is reconstructed in detail. Then, based on the visualization of the graph, the energy and Seidel energy of the graph are computed.

2.2 Graphs of Dihedral Groups

In this section, some basic concepts related to the dihedral groups are presented. A dihedral group is the group of symmetries of a regular polygon which includes rotations and reflections.

Definition 2.7 [8] Dihedral Group
For any integer $n \geq 3$, the dihedral group of order $2n$, D_{2n} is presented by:

$$D_{2n} = \langle a, b : a^n = b^2 = e, b^{-1}ab = a^{-1} \rangle.$$

Example 2.1:
From Definition 2.7, the dihedral group of order six, D_6 has the group presentation:

$$D_6 = \langle a, b : a^3 = b^2 = e, b^{-1}ab = a^{-1} \rangle$$

Based on the group presentation, the elements of D_6 are:

$$D_6 = \{e, a, a^2, b, ab, a^2b\}.$$

The Cayley table of D_6 is presented as follows;

$*$	$\boldsymbol{e}$	$\boldsymbol{a}$	$\boldsymbol{a^2}$	$\boldsymbol{b}$	$\boldsymbol{ab}$	$\boldsymbol{a^2b}$
$\boldsymbol{e}$	e	a	a^2	b	ab	a^2b
$\boldsymbol{a}$	a	a^2	e	ab	a^2b	b
$\boldsymbol{a^2}$	a^2	e	a	a^2b	b	ab
$\boldsymbol{b}$	b	a^2b	ab	e	a^2	a
$\boldsymbol{ab}$	ab	b	$a2b$	a	e	a^2
$\boldsymbol{a^2b}$	a^2b	ab	b	a^2	a	e

Table 2.1 The Cayley Table of D_6

In [5], Tolue has proof that $Cay_p(D_{2n}, S)$ is a connected graph and $Cay(G, S)$ is connected when a simple group contains an element of order p. Besides, Ali, Salman and Huang [9] has study on dimension of commuting graph on D_{2n} and compute its resolving polynomial.

2.3 Energy and Seidel Energy of a Graph

In order to compute the energy and Seidel energy of a graph, the adjacency matrix and Seidel matrix of the graph need to be determined.

Definition 2.8 [10] Adjacency Matrix

The adjacency matrix of a graph Γ denoted by $A(\Gamma)$ is known as the connection matrix of a graph Γ with n vertices and no parallel edges. It is defined as in the following:

$$A(\Gamma) = \begin{cases} x_{ij} = 1 & \text{if } v_i \sim v_j \\ x_{ij} = 0 & \text{if otherwise} \end{cases}$$

where $v_i \sim v_j$ represents the adjacency of vertex i to vertex j.

Definition 2.9[11] Seidel Matrix

Seidel matrix, $S(r)$ of graph Γ is a symmetric matrix has 0 on the diagonal and ± 1 off diagonal, where -1 indicates adjacency and $+1$ indicates non adjacency.

Definition 2.10 [10] Characteristic Polynomial
The characteristic polynomial of a matrix, f is given by $det\ [A - \lambda I] = 0.$,

Then, the energy and Seidel energy of a graph can be computed from the sum of absolute values of eigenvalues of the adjacency matrix and Seidel matrix.

Definition 2.11 [11] Energy of Graph
The energy of a simple graph Γ, $\varepsilon(\Gamma)$ is defined as the sum of the absolute values of eigenvalues of the adjacency matrix of Γ.

In 2019, Fadzil et al. [4] has found the Cayley graphs and calculated the eigenvalues of adjacent matrix and compute the generalized energy of the specific subset of dihedral groups.

Definition 2.12 [12] Seidel Energy of a Graph
The Seidel energy of a simple graph Γ is the sum of the absolute values of the eigenvalues of the Seidel matrix of Γ.

In [13] , Zhou found the relation between eigenvalues and eigenvectors of an adjacency matrix with Seidel matrix of graph that will be used to find the Seidel energy.

3. Materials and Methods

In this paper, the prime order Cayley graph of the dihedral group of order six is constructed using the definition of the graph and the group presentation. The adjacency matrix and Seidel matrix are determined based on the adjacency of the vertices in the graph. Then, energy and Seidel energy of the prime order Cayley graph of the dihedral group of order six is calculated based on the adjacency matrix and Seidel matrix.

4. Results and Discussion

4.1 The Prime Order Cayley Graph the Dihedral Groups of Order Six

In this section, the prime order Cayley graph of the dihedral groups of order six, D_6 is constructed. Based on Example 2.1, the group presentation of D_6 is given as:

$$\begin{aligned} D_6 &= \langle a, b : a^3 = b^2 = e, b^{-1}ab = a^{-1} \rangle \\ &= \langle a, b : a^3 = b^2 = e, bab = a^3 \rangle \\ &= \langle a, b : a^3 = b^2 = e, ab = a^2 b \rangle. \end{aligned}$$

Since the element of D_6 are $e, a, a^2, b, ab, a^2b,$ then by Definition 2.5, the set of vertices of the prime order Cayley graph of D_6 is as follows:

$$V(Cay_p(D_6, S)) = \{e, a, a^2, b, ab, a^2b\}.$$

To proceed with the construction of prime order Cayley graph of D_6, the order of each element of D_6 is calculated to determine which element has prime order. The order of the elements of D_6 are:

$g \in D_6$	e	a	a^2	b	ab	a^2b
$\lvert g \rvert$	1	3	3	2	2	2

Hence, the elements of D_6 with prime order are $\{a, a^2, b, ab, a^2b\}$.

The order of b, ab and a^2b are 2 and the order a and a^2 are 3. Thus, by Definition 2.6, the subsets of S of elements of D_6 with prime order are:

$$S = \{a, a^2, b, ab, a^2b\}.$$

Let $g, h \in D_6$. Vertex g is adjacent to vertex h, denoted as $g \sim h$ if $gh^{-1} \in S$ which implies that exists $s \in S$ such that $gh^{-1} = s$ or $g = sh$. Thus, g is adjacent to h in $Cay_p(D_6, S)$. Therefore, $\{g, h \in$ E$(Cay_p(D_6, S))$.

Now consider $s = a \in S$.

h	$g = sh$	$g \sim h$
e	$a \cdot e = a$	$a \sim e$
a	$a \cdot a = a^2$	$a^2 \sim a$
a^2	$a \cdot a^2 = a^3 = e$	$e \sim a^2$
b	$a \cdot b = ab$	$ab \sim b$
ab	$a \cdot ab = a^2b$	$a^2b \sim ab$
a^2b	$a \cdot a^2b = a^3 \cdot b = e \cdot b = b$	$b \sim a^2b$

Let $s = a^2 \in S$.

h	$g = sh$	$g \sim h$
e	$a^2 \cdot e = a^2$	$a^2 \sim e$
a	$a^2 \cdot a = a^3 = e$	$e \sim a$
a^2	$a^2 \cdot a^2 = a^3 \cdot a = e \cdot a = a$	$a \sim a^2$
b	$a^2 \cdot b = a^2b$	$a^2b \sim b$
ab	$a^2 \cdot ab = a^3 \cdot b = e \cdot b = b$	$b \sim ab$
a^2b	$a^2 \cdot a^2b = a^3 \cdot ab = e \cdot ab = ab$	$ab \sim a^2b$

Let $s = b \in S$.

h	$g = sh$	$g \sim h$
e	$b \cdot e = b$	$b \sim e$
a	$b \cdot a = a^2b$	$a^2b \sim a$
a^2	$b \cdot a^2 = a^2b \cdot a = a^2 \cdot a^2\mathrm{b} = \mathrm{a} \cdot a^3 \cdot \mathrm{b} = \mathrm{a} \cdot \mathrm{e} \cdot \mathrm{b} = \mathrm{ab}$	$ab \sim a^2$
b	$b \cdot b = b^2 = e$	$e \sim b$
ab	$b \cdot ab = ba \cdot b = a^2b \cdot b = a^2 \cdot b^2 = a^2e = a^2$	$a^2 \sim ab$
a^2b	$b \cdot a^2b = ba \cdot ab = a^2b \cdot ab = a^2 \cdot a^2b \cdot b$ $= a \cdot a^3b^2 = a \cdot e \cdot e = a$	$a \sim a^2b$

Let $s = ab \in S$.

h	$g = sh$	$g \sim h$
e	$ab \cdot e = ab$	$ab \sim e$
a	$ab \cdot a = a \cdot ba = a \cdot a^2b = a^3 \cdot b = e \cdot b = b$	$b \sim a$
a^2	$ab \cdot a^2 = a \cdot ba \cdot a = a \cdot a^2b \cdot a = a^3 \cdot ba = a^3 \cdot a^2b$ $= e \cdot a^2b = a^2b$	$a^2b \sim a^2$
b	$ab \cdot b = a \cdot b^2 = a \cdot e = a$	$a \sim b$
ab	$ab \cdot ab = a \cdot ba \cdot b = a \cdot a^2b \cdot b = a^2 \cdot b^2 = e$	$e \sim ab$
a^2b	$ab \cdot a^2b = a \cdot ba \cdot ab = a \cdot a^2b.ab$	$a^2 \sim a^2b$

	$= a^3 \cdot a^2b \cdot b = a^3 \cdot a^2 \cdot b^2 = e \cdot a^2 \cdot e$	

Let $s = a^2b \in S$.

$\boldsymbol{h}$	$\boldsymbol{g = sh}$	$\boldsymbol{g \sim h}$
e	$a^2b \cdot e = a^2b$	$a^2b \sim e$
a	$a^2b \cdot a = a^2 ba = a^2 \cdot a^2b = aa^3b = a \cdot e \cdot b = ab$	$ab \sim a$
a^2	$a^2b \cdot a^2 = a^2 \cdot ba \cdot a = a^2 \cdot a^2b \cdot a = a^4 \cdot a^2b$ $= a^3 \cdot a^3 \cdot b = e \cdot e \cdot b = b$	$b \sim a^2$
b	$a^2b \cdot b = a^2 \cdot b^2 = a^2 \cdot e = a^2$	$a^2 \sim b$
ab	$a^2b \cdot ab = a^2 \cdot ba \cdot b = a^2 \cdot a^2b \cdot b = a \cdot a^3 \cdot b^2 = a \cdot e \cdot e = a$	$a \sim ab$
a^2b	$a^2b \cdot a^2b = a^2 \cdot ba \cdot ab = a^2 \cdot a^2b \cdot ab = a^2 \cdot a^2 \cdot ba \cdot b$ $= a \cdot a^3 \cdot a^2b \cdot b = a^3 \cdot a^3 \cdot b^2 = e$	$e \sim a^2b$

Hence, the set of edges of $Cay_p(D_6, S)$ is given as follows:

$E(Cay_p(D_6,S)) = \{\{a,e\},\{a^2,a\},\{e,a^2\},\{ab,b\},\{a^2b,ab\},\{b,a^2b\},\{a^2,e\},\{e,a\},\{a,a^2\},\{a^2b,b\},\{b,ab\},$
$\{ab,a^2b\},\{b,e\},\{a^2b,a\},\{ab,a^2\},\{e,b\},\{a^2,ab\},\{a,a^2b\},\{ab,e\},\{b,a\},\{a^2b,a^2\},\{ab,a^2\},\{e,b\},\{a^2b,e\},$
$\{ab,a\},\{b,a^2\},\{a^2,b\},\{a,ab\},\{e,a^2b\}\}$

Based on $V(Cay_p(D_6,S))$ and $E(Cay_p(D_6,S))$, the prime order Cayley graph of D_6, $Cay_p(D_6,S)$ is visualized as in Figure 3.1.

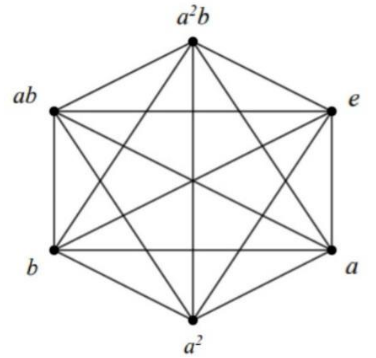

Figure 3.1 Prime order Cayley Graph of D_6

Based on Figure 3.1, it can be seen that the prime order Cayley graph of D_6 is a complete graph with six vertices, K_6. Note that the degree of each vertex is five since all the vertex is adjacent to each other. The only element of D_6 which is not in S is e. So, if $gh^{-1} = e$, then $g \nsim h$.

4.2 The Energy of the Prime Order Cayley Graph of the Dihedral Group of Order Six

In this section, the energy of the prime order Cayley graph of the dihedral group of order six $Cay_p(D_6,S)$ is computed. First, the adjacency matrix of $Cay_p(D_6,S)$ is determined based on the adjacency of $Cay_p(D_6,S)$ as given in Figure 3.1 and Definition 2.8. As example, the entry of a_{12} is equal to 1 since e and a are adjacent in $Cay_p(D_6,S)$. Meanwhile, the entry a_{22} is 0 since the vertex a is not adjacent to itself in $Cay_p(D_6,S)$. The adjacency matrix of the prime order Cayley graph of dihedral group of order six is obtained as follows:

$$e \quad a \quad a^2 \quad b \quad ab \quad a^2b$$

$$A(Cay_p(D_6,S)) = \begin{array}{c}\begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}\end{array} \begin{array}{c} e \\ a \\ a^2 \\ b \\ ab \\ a^2b \end{array}$$

Based on $A(Cay_p(D_6,S))$, the characteristics polynomial $f(\lambda)$ can be determined using $\det\left(A(Cay_p(D_6,S)) - \lambda I\right)$.

$$f(\lambda) = \left|\begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda \end{bmatrix}\right|$$

$$f(\lambda) = \left|\begin{bmatrix} 0-\lambda & 1 & 1 & 1 & 1 & 1 \\ 1 & 0-\lambda & 1 & 1 & 1 & 1 \\ 1 & 1 & 0-\lambda & 1 & 1 & 1 \\ 1 & 1 & 1 & 0-\lambda & 1 & 1 \\ 1 & 1 & 1 & 1 & 0-\lambda & 1 \\ 1 & 1 & 1 & 1 & 1 & 0-\lambda \end{bmatrix}\right|$$

$$f(\lambda) = \lambda^6 - 15\lambda^4 - 40\lambda^3 - 45\lambda^2 - 24\lambda - 5$$

To find the eigenvalue of the matrix, λ, the characteristic equation $f(\lambda) = 0$ is considered.

$$\lambda^6 - 15\lambda^4 - 40\lambda^3 - 45\lambda^2 - 24\lambda - 5 = 0$$
$$(\lambda + 1)^5(\lambda - 5) = 0$$

Hence, the eigenvalues of the adjacency matrix of the prime order Cayley graph of dihedral group of order six are $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 =- 1$ and $\lambda_6 = 5$.

Thus, by using Definition 2.12, the energy of a graph can be computed from the sum of absolute values of eigenvalues of adjacency matrix. Hence, the energy of prime order Cayley graph of dihedral group of order six is:

$$\varepsilon(Cay_p(D_6,S)) = 5|-1| + 5 = 10$$

4.3 The Seidel Energy of the Prime Order Cayley Graph of the Dihedral Group of Order Six

Next, the Seidel matrix is determined using the adjacency of the vertices in the prime order Cayley graph of D_6, $Cay_p(D_6,S)$ as given in Figure 3.1 and Definition 2.9. As example, the entry of a_{12} is equal to -1 since e and a are adjacent in $Cay_p(D_6,S)$. Meanwhile, the entry a_{22} is 0 since the vertex a is not adjacent to itself in $Cay_p(D_6,S)$. The Seidel matrix of the prime order Cayley graph of dihedral group of order six is obtained as follows:

$$S(Cay_p(D_6,S)) = \begin{array}{c} \begin{array}{cccccc} e & a & a^2 & b & ab & a^2b \end{array} \\ \begin{bmatrix} 0 & -1 & -1 & -1 & -1 & -1 \\ -1 & 0 & -1 & -1 & -1 & -1 \\ -1 & -1 & 0 & -1 & -1 & -1 \\ -1 & -1 & -1 & 0 & -1 & -1 \\ -1 & -1 & -1 & -1 & 0 & -1 \\ -1 & -1 & -1 & -1 & -1 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ e \\ a \\ a^2 \\ b \\ ab \\ a^2b \end{array}$$

Based on $S(Cay_p(D_6,S))$, the characteristics polynomial $f(\lambda)$ can be determined using $det\left(S(Cay_p(D_6,S)) - \lambda I\right)$.

$$f(\lambda) = \left| \begin{bmatrix} 0 & -1 & -1 & -1 & -1 & -1 \\ -1 & 0 & -1 & -1 & -1 & -1 \\ -1 & -1 & 0 & -1 & -1 & -1 \\ -1 & -1 & -1 & 0 & -1 & -1 \\ -1 & -1 & -1 & -1 & 0 & -1 \\ -1 & -1 & -1 & -1 & -1 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda \end{bmatrix} \right|$$

$$f(\lambda) = \left| \begin{bmatrix} 0-\lambda & -1 & -1 & -1 & -1 & -1 \\ -1 & 0-\lambda & -1 & -1 & -1 & -1 \\ -1 & -1 & 0-\lambda & -1 & -1 & -1 \\ -1 & -1 & -1 & 0-\lambda & -1 & -1 \\ -1 & -1 & -1 & -1 & 0-\lambda & -1 \\ -1 & -1 & -1 & -1 & -1 & 0-\lambda \end{bmatrix} \right|$$

$$f(\lambda) = \lambda^6 - 15\lambda^4 + 40\lambda^3 - 45\lambda^2 + 24\lambda - 5$$

To find the eigenvalue of the matrix, λ, the characteristic equation $f(\lambda) = 0$ is considered.

$$\lambda^6 - 15\lambda^4 + 40\lambda^3 - 45\lambda^2 + 24\lambda - 5 = 0$$
$$(\lambda - 1)^4(\lambda + 5)(\lambda - 1) = 0$$

Hence, the eigenvalues of the Seidel matrix of the prime order Cayley graph of the dihedral group of order six $S(Cay_p(D_6, S))$, are $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 1$, $\lambda_5 = -5$ and $\lambda_6 = 1$.

Thus, by using Definition 2.13, the Seidel's energy of a graph can be computed from the sum of absolute values of eigenvalues of Seidel's adjacency matrix. Hence, the Seidel's energy of prime order Cayley graph of dihedral group of order six is:

$$\varepsilon(S(G(Cay_p(D_6,S)))) = 4|1| + |-5| + 1 = 10$$

5. Conclusion

In this paper, the prime order Cayley graph are constructed for the dihedral group of order six. Then, the energy and Seidel energy of the prime order Cayley graph of dihedral group of order six, $Cay_p(D_6,S)$has been determined. It is found that for $Cay_p(D_6,S)$, the energy and Seidel energy are similar.

Below is the summarisation from this research.

$\boldsymbol{n}$	**Group** $\boldsymbol{D_{2n}}$	$\boldsymbol{Cay_p(D_{2n},S)}$	**Structure of** $\boldsymbol{Cay_p(D_{2n},S)}$	$\boldsymbol{\varepsilon(Cay_p(D_{10},S))}$	$\boldsymbol{\varepsilon(S(G(Cay_p(D_{2n},S)))}$
3	D_6	$Cay_p(D_6,S)$	Complete graph with six vertices, K_6.	10	10

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