Proceedings of Science and Mathematics
Faculty of Science, Universiti Teknologi Malaysia

Vol. 16, 2023, page 76 – 85

Application of Queuing Theory and Discrete Event Simulation in Queuing System of Pusat Kesihatan Universiti

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Abstract

In recent days, the time spending in public hospital is increasing crucially. This is due to inefficient patient flow management, resource allocation, and service. Thus, this study is conducted and we focus on the queuing problem in Pusat Kesihatan Universiti (PKU). In this paper, queuing theory is suggested to determine the queuing structure of PKU and simulation is conducted based on the real life queuing system. By obtaining and analyse the data from PKU, the key performance indicators such as waiting times, service rates, and resource utilization are measured by using queuing theory. The models are compared and efficiency of resources are calculated to find out the ideal model to optimise the waiting time of patients in queuing system. The simulation models are constructed using Simul8 to test the performance and efficiency of the queuing system. In this study, there is no congestion in all the queue, however, the queuing system can be improved by rearrange the resources to achieve optimal utilization.

Keywords: Queuing problem; Queuing System; Discrete Event Simulation; Pusat Kesihatan Universiti

1. Introduction

The long queue in public health centres is always an issue for patients and hospital in most of the countries over the world. Moreover, some of the resources are not wisely allocated which leads to delays in the queue. These factors can be eliminated with mathematical approach. In this paper, queuing theory and discrete event simulation are used to study the data and simulate modification for the situation in the public health centre to obtain an optimised queue. The healthcare system in Malaysia is funded by tax revenues and the cost of care is heavily subsidised by the government. However, people still prefer private hospitals over public hospitals, despite having to pay more, due to the quality of health facilities, shorter waiting time and less crowded. In a public hospital, patients always have to wait for a long time from registration until the post-consultation stage and end up spending around 2 - 5 hours just to have a general outpatient visit. One of the patients from the public hospital, Hanisah Mashkuri shared her experience as she mentioned that she had to bear the long waiting times to get her treatment even though she had an appointment. But when she arrives at 8am, she will still need to wait until 11am to reach her turn (Hazim & Amin, 2022).

The World Health Organization (WHO) emphasized the importance of quality in the delivery of healthcare, as defined by the criteria of effectiveness, cost and social acceptability. Hence, it is necessary to overcome this problem by improving its queuing system to increase effectiveness and satisfaction. In 2022, the Budget 2023 was released and the healthcare sector was allocated RM36.14 billion by Malaysia's government. It is the highest among the Budget 2023, and higher compared to RM32.41 billion in Budget 2022 and looks promising in the government's efforts to strengthen the country's health services. In fact, Malaysia is in need of better accessibility and affordability for its healthcare system in general. (Hazim & Amin, 2022) Application of queuing theory for analysis and modelling of processes that involves waiting lines is used widely in industries for optimizing supply of fixed resources at variable demand conditions, however the healthcare industry views itself differently from other industries (Kritchanchai, D., 2012). The hospital operations managers should be aware of the status of business processes to improve operational performances and reduce waiting time. There are several analytical tools for understanding the complexity of a system's performance, among which the queuing theory is a tool to analyse systems which include queues and consist of clients, servers and queue. Queuing theory is a scientific approach to minimise system inefficiencies and increase the patients' satisfaction. The hospital operations managers can increase patients' satisfaction by making right decisions through a proper understanding of the queuing theory and variables related to the patients' waiting time. (Yaduvanshi et al., 2019).

Based on the experience of visiting PKU, most of the time the waiting area is crowded with people. By observations, we realise that the during peak hour, the maximum waiting time of a patient is up to 2 hours. It is normal to find patients waiting more than an hour for visiting doctor when delay happened as the demand and supply for service to the patients is uneven. Consequently, the queuing time to receive service increased. However, the situation of mismatch is temporary and due to the nature of variability in the timing of demands and the duration of service time. The queuing system is plagued by long waiting times and inefficiencies which causes the negatively impact to the patient experience, delayed access to care, and reduce overall operational efficiency. Therefore, PKU needs to create an effective patient flow to reduce queuing time of patients and improve the queuing system.

The objectives of the research are:

1. To apply queuing theory in solving the patient flow problem in Pusat Kesihatan Universiti (PKU) UTM, Johor, Malaysia.
2. To construct a simulation model of the queuing problem in Pusat Kesihatan Universiti (PKU) UTM, Johor, Malaysia.
3. To investigate the number of resources that can lead to optimal waiting time.

2. Queuing Theory

Queuing theory is a mathematical discipline that studies the behaviour and characteristics of waiting lines, or queues, and the systems that generate them. It provides a framework for analysing and optimizing queuing systems, which are prevalent in various sectors, including healthcare. It is emerged from the concept of Markov probability chains, then improved by Kendall to an outlining of three major characteristic which are the arrival and service patterns, queue discipline, system capacity, number of service channels and service phase (Kendall, 1953). The notation for the major component of Kendall can be written as *A/S/c.* The notation was subsequently extended to the form *A/S/c/K/N/D*, by the inclusion of three additional parameters which are the number of places in system, size of population of customers and queuing discipline. For healthcare queuing problems, usually the notation will be *M/M/s as* we will have a single queue, unlimited waiting room served by *s* identical servers. Folake et al. (2020) and Venkateswarlu & Murty (2016) use the same queuing system in their paper. Besides, the patients arrive according to Poisson distribution and the service rate is exponentially distributed which are represented by *M* (Markovian).

Many queueing models in research deal with the study of reducing waiting time in the outpatient clinic. Afrane (2014) uses a queuing model to understand the patient flow through the outpatient department (OPD) in the hospital. The author highlights the relationship between wait times and allocated resources such as doctors and supporting staff. Yaduvanshi et al. (2019) pointed out that the waiting time can be reduced by having a proper system for registration and improving the quality of serving such as allocation of registration counters, printer problem, and serving time of receptionists. Aziati and Hamdan (2018) performed a hospital-wide study of patient flow using queuing theories and assessed its impact on the outpatient department.

2.1 Types of Simulation

Simulation is widely used in various fields to model and analyse complex systems. There are four types of simulation method can be categorised which are Monte Carlo simulation (MC) , discrete event simulation (DES), system dynamics (SD) and agent-based simulation (ABS). Before building a simulation model, it is crucial to identify the types of simulation that is suitable for the system to be modelled.

In DES, the system is represented as a sequence of discrete events that occur at specific points in time. The simulation tracks the changes in the system state based on events, such as arrivals, departures, or changes in process states. DES is often used to model systems with dynamic behaviour and interactions between entities, such as queuing systems, transportation networks, and manufacturing processes (Ullrich & Lückerath, 2017). MC simulation uses random sampling to model uncertainty and variability in a system. It requires to generate a large number of random samples from probability distributions and simulate the system's behaviour based on these samples. MC simulation is commonly used for risk analysis, optimization, and decision-making in various domains, including finance, engineering, and project management (Raychaudhuri, 2008). ABS models complex systems by representing individual entities, called agents, and their interactions within the system. Each agent has its own set of rules, behaviours, and decision-making processes. The simulation tracks the actions and interactions of these agents over time to understand the emergent behaviour of the system as a whole. ABS is particularly useful for studying social systems, ecological systems, and complex organizational dynamics (Crooks & Heppenstall, 2012). SD simulation focuses on understanding the behaviour of systems over time by capturing the relationships and feedback loops among different variables. System dynamics models represent the system as a set of interconnected stocks, flows, and feedback loops. The simulation allows for studying the long-term behaviour, policy impacts, and dynamic behaviour of complex systems, such as population dynamics, economic models, and environmental systems (Coyle, 1997).

However, these simulation methods have their limitations that should be considered. DES can become challenging when modelling systems with high complexity and a large number of entities and interactions. The model's complexity may lead to increased simulation time and computational requirements. DES is also not well-suited for systems with continuous processes that change continuously over time, such as fluid flow or chemical reactions. Modelling such systems accurately may require alternative simulation methods as it may not capture real-time constraints and dynamic changes in systems where timing and synchronization are critical factors, such as real-time control systems. For MC simulation, since it relies on random sampling, which introduces randomness and imprecision into the results. Some applications may require higher precision than what can be achieved through random sampling. Besides, the accuracy of MC simulation depending on the sample size used may lead to inaccurate results or unreliable statistical estimates when there is insufficient sample size. To construct an ABS model, it can be difficult when comes to validate and calibrate due to the heterogeneity of agent behaviours and the lack of precise real-world data for all agents. Moreover, ABS can be computationally intensive, especially when modelling large-scale systems with numerous agents and complex interactions. Running simulations with a high number of agents may require significant computational resources. In addition, representing the behaviours and decision-making processes of individual agents accurately can be a challenging task. Simplifying or abstracting agent behaviour may lead to limitations in the fidelity of the simulation results. Another similar limitation from all simulation, SD simulation also faces challenges at building a system dynamics model as it requires a good understanding of the system being modelled and the relationships among variables. Developing an accurate model can be time-consuming and resource intensive. On other hand, SD simulation typically operates at an aggregate level, focusing on the system's overall behaviour rather than individual entities. This may limit the ability to capture fine-grained details and individual-level interactions.

2.2 Simulation in Healthcare

In a healthcare system, DES and ABS are more preferred methods used to simulate the model (Siebers et al., 2010). To have a clearer comparison, the simulation method used in modelling healthcare environments is listed in Table 1 based on the past research. The table included the simulation type, simulation model and simulation platform used in the publications.

Table 1: Simulation Systems in Healthcare.

Reference	Simulation Type	Simulation Model
Zeinal et al., 2015	DES	Resource planning in the emergency departments
Bartz-Beielstein et al, 2020	DES	Hospital capacity planning during Covid-19
Cubukcuoglu et al., 2020	DES	Space planning in hospital
Yousefi. M & Yousefi. M, 2020	ABS	Human resource allocation in emergency department
Castanheira-Pinto et al., 2021	DES	Modelling and analysis of the emergency department
Mohamed, 2021	DES	Modelling and analysis of the emergency department

Based on the papers, we can see that most of the time DES is used to model and simulate the healthcare queuing problem. By considering the methods and limitations of these methods, DES is chosen for this study. As it can effectively describe the events occur in PKU such as patient arrivals, service times and departures and is not restricted with its limitation.

3. Data Collection

The data are collected from the patients from the beginning of registration at the counter until the patients complete their consultation. The data are extracted from the database of PKU UTM from 1 Jan 2023 until 31 April 2023. Descriptive analysis and observations study was used to determine the time taken of patients from the registration until pharmacy. The collected data were the arrival time (λ) which was the number of patients entered to the outpatient counter during standard study time (30-minutes intervals) and the service time (μ) which was the period of giving services to each patient per 30 minutes. The data required to develop the patient flow as follows:
a) Patients' arrival times
b) Registration time at the counter.
c) Service time at each station.
d) The number of patients (at each phase).
e) The number of doctors, staffs involved at each phase.

Lengths of the intervals between arrivals are independently and identically distributed and described by a continuous density function. It is assumed that interarrival times and service times follow the exponential distribution or equivalently that the arrival rate and service rate follow a Poisson distribution. Description of different variables and characteristics used in these cases are as follows:
λ = the arrival rate (outpatient arrival rate)
μ = the service rate
S = the number of serves (doctor)
n = the number of patients in system awaiting service or being served

3.1 Queuing Model

There are 14106 patients visiting PKU UTM between 8:00 am and 5:00 pm from 1 Jan 2023 until 31 March 2023 excluding Friday and Saturday. In this study, we are required to decide between the register time and patient's scan time to calculate the arrival rate of patients. A one-test is used to test whether

the mean time difference between the patients' register time and scan time exceeds two minutes. The test statistic $z_{test} = 0.9247$ is less than the critical value, $z_{0.05} = 1.6449$. Hence, there is no statistical evidence to reject H_0 at $\alpha = 0.05$. Thus, we can conclude that the mean time difference between register time and scan time do not exceed two minutes.
The interarrival time of patients are calculated by using the register time of patients.

$$\text{Mean Interarrival Time} = \frac{\text{Total interarrival time (hour)}}{\text{Total number of patients}}$$

Thus, the interarrival time of patients in the morning is 1.9777 minutes per patient while in the afternoon is 3.07046 minutes per patient.

Since there is no completion time of patients from each station, the service time of a patient is obtained by calculating the time difference between the calling time of the first and second patient. The assumption of time difference between calling time should be less than 40 mins is made to ensure continuous patient flow. For OPD consultation rooms and dental rooms, one-way Analysis of Variance (ANOVA) is conducted to determine whether there is a significant difference between the mean service time of the same type of consultation rooms.

Table 2: ANOVA for OPD consultation rooms and Dental rooms.

Hypothesis Testing
H_0: The mean service time of (rooms) are same.
H_1: The mean service time of (rooms) have at least one is different.
Reject H_0 if $F_{test} > F_{critical}$
OPD: $F_{test} = 1.4335 < F_{critical} = 2.2148$
Dental: $F_{test} = 2.7394 < F_{critical} = 3.8497$

From Table 2, we can see that both F_{test} are less than the $F_{critical}$. This indicates that there is insufficient evidence to reject H_0 at $\alpha = 0.05$ for both types of room. Thus, the mean service time for same type of rooms is same. The service rate, μ is calculated according to each station and is shown in Table 3.

Table 3: Service Time for each Station in PKU UTM.

Station	OPD	Dental	X-ray	Lab
Service time, (Minutes per Patient)	7.94975	31.74025	8.84798	5.97418

Hypothesis Testing

Table 4: Anderson-Darling Goodness-of-Fit Test for Interarrival Time of PKU.

Hypothesis Testing
H_0: The interarrival time for (period) of PKU follows exponential distribution.
H_1: The interarrival time for (period) of PKU does not follow exponential distribution.
Reject H_0 if $AD_{test} > AD_{0.05} = 1.321$
Morning: $AD = -0.16399 < AD_{0.05} = 1.321$
Afternoon: $AD = 0.73284 < AD_{0.05} = 1.321$

Since both test statistic are less than $AD_{0.05} = 1.321$, which indicates that there is no evidence to reject H_0. Hence, we may conclude that the interarrival time of PKU for morning and afternoon period follows exponential distribution.

Table 5: Anderson-Darling Goodness-of-Fit Test for Service time of each station.

Hypothesis Testing	
H_0: The interarrival time for (station) of PKU follows exponential distribution.	
H_1: The interarrival time for (station) of PKU does not follow exponential distribution.	
Reject H_0 if $AD_{test} > AD_{0.05} = 1.321$	
Lab Sample:	$AD = 0.27525 < AD_{0.05} = 1.321$
X-ray:	$AD = 0.70023 < AD_{0.05} = 1.3217$

Since all stations' test statistic are less than $AD_{0.05} = 1.321$, which indicates that there is no evidence to reject H_0. Hence, we may conclude that the service time of PKU data follows exponential distribution for all stations.

3.2 Simulation Model

Simulation modelling is the method by which a computer prototype of a physical object is generated and analysed to predict its real life results. In this study, Simul8 will be the simulation software to conduct the simulation for this research. Simul8 is a discrete event simulation software that enables the users to construct a simulation model considering the real life constraints, failure rates and other aspects that influence the overall output performance and quality.

In this study, the duration of the simulation is 5 days per week, starting from 8 a.m. and the duration is 9 hours per day. The simulation model starts with a starting point which is the arrival of patients in the healthcare centre. There is queue before every work centre. Patients are first directed to the registration counter which is Work Centre 1. By observations, the service time of the patients at registration counter is assumed with a mean of 2 minutes per patient. After the registration counter, the patients will either go to the consultation room or dental room as their first station. In this queuing system, there are six consultation rooms which will show as OPD in the model with 6 replicates. The queue follows the first come first serve (FCFS) queue discipline. A label of arriving at OPD and dental is made based on the arrival of patients in both stations. Then, all the patients will go to their next station such as lab sample collecting room, X-ray room, pharmacy or exit by following their requirement to visit the healthcare centre. Since the simulation model is constructed according to the actual scenario of the healthcare centre, hence the routing out percentage to each station is calculated. After that, all the patients are eventually going to the same endpoint in the healthcare centre. For the resources, doctor and dentist are considered as there are shifts for the staffs in both stations while for other stations there will be 1 staff on duty throughout the working hour. The details of setting for the simulation model are show as below.

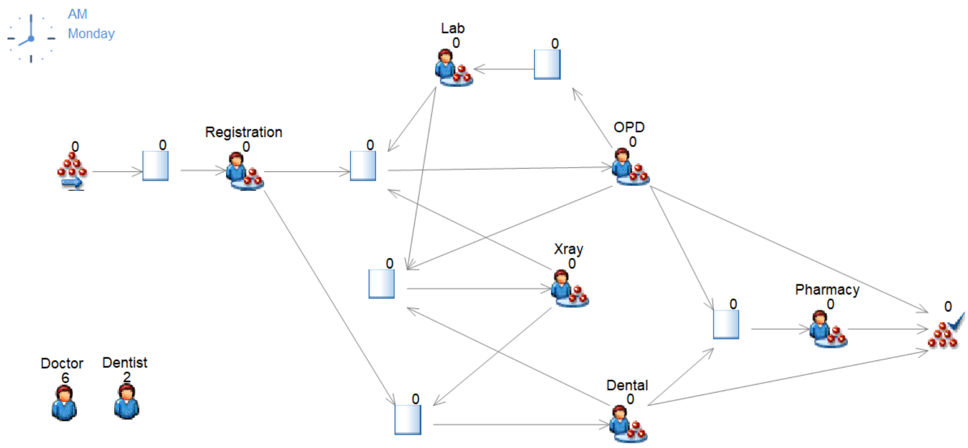

Figure 1 Simulation model of PKU UTM.

Table 6: Setting of Arrival and Service Time of Model.

Activity	Distribution	Setting
Starting Point	Exponential	Mean interarrival time = WEP (WEP: Morning: 8am - 1pm = 1.9777, Afternoon: 1pm - 5pm = 3.07046)
Registration (Replicate: 2)	Average	Mean service time = 2
OPD (Replicate: 6)	Exponential	Mean service time = 7.94975
Dental (Replicate: 2)	Exponential	Mean service time = 31.74025
Lab	Exponential	Mean service time = 5.9871
X-ray	Exponential	Mean service time = 8.87
Pharmacy	Average	Mean service time = 2

Table 7: Setting of Shift of Model

Resources	Day	Shift	No. of Resources
Doctor	Mon, Tue, Fri	8 am – 1 pm	6
(Available:6)	Wed, Thu	8 am – 1 pm	5
	Mon - Fri	2 pm – 5 pm	4
Dentist	Mon, Tue, Fri	8 am – 1 pm	2
(Available:2)	Wed, Thu	8 am – 1 pm	2
	Mon - Fri	2 pm – 5 pm	1

Table 8: Routing Out Percent Discipline

	OPD	Dental	Lab	X-ray	Pharmacy	End Point
Registration	96.0441	3.9559	-	-	-	-
OPD	-	-	9.785	3.0948	73.7603	13.3599
Dental	-	-	-	26.2164	47.9606	28.823

4. Results and Discussion

After setting up the system based on the setting mentioned earlier in simulation model, the simulation is run and the results are recorded in Table 9.

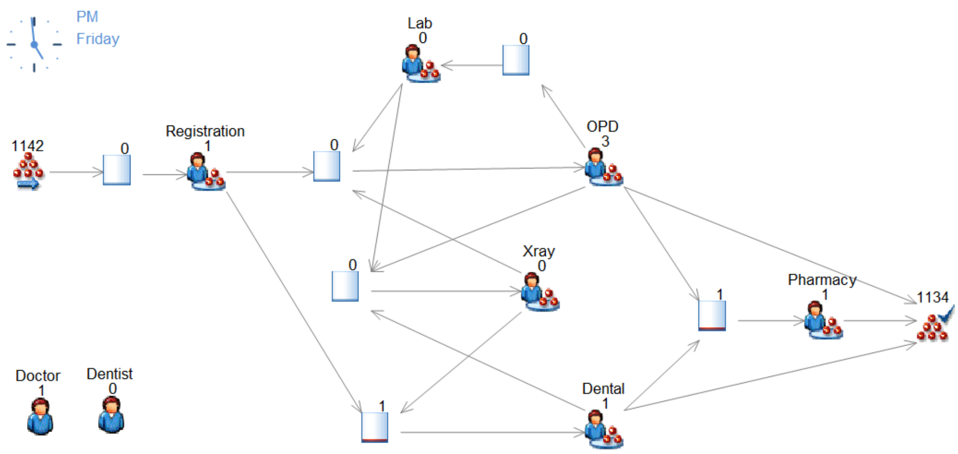

Figure 2 Simulation result of PKU UTM.

Table 9: Results of Queue for Simulation Model 1.

Queue	Registration	OPD	Dental	Lab	X-ray	Pharmacy
Average Queue Size	0.11	2.33	0.14	0.06	0.04	0.85
Maximum Queue Size	4	21	3	3	2	7
Average Queuing Time	0.26	5.03	6.76	1.62	1.55	2.36
Maximum Queueing Time	3.78	38.25	71.44	19.32	25.19	13.67

From the table above, we can see that there is no congestion in the queuing system. The maximum queue size for all stations is less than 30. Besides, the utilization of resources is 77.53% for doctors and 28.45% for dentist. However, the maximum queuing time for dental room is crucial which are more than 45 minutes, followed by OPD room which is 38.25 minutes. Next, for simulation model 2, we increase the number of doctors for night shift from 4 to 5 and also increase the number of dentists for night shift from 1 to 2, and other settings remained the same. And for simulation 3, we decrease the number of doctors for morning shift (1) from 6 to 5 and also decrease the number of dentists for morning shift (2) from 2 to 1, and other settings still remained the same.

Table 10: Results of Queue for Simulation Model 2 and 3.

			Simulation Model 2			
Queue	**Registration**	**OPD**	**Dental**	**Lab**	**X-ray**	**Pharmacy**
Average Queue Size	0.11	1.16	0.05	0.07	0.04	0.75
Maximum Queue Size	4	21	3	3	2	7
Average Queuing Time	0.26	2.49	2.65	1.87	1.71	2.11
Maximum Queueing Time	3.78	28.2	34.05	24.48	25.19	13.67
			Simulation Model 3			
Queue	**Registration**	**OPD**	**Dental**	**Lab**	**X-ray**	**Pharmacy**
Average Queue Size	0.11	2.66	0.19	0.06	0.04	0.83
Maximum Queue Size	4	21	3	3	2	7
Average Queuing Time	0.26	5.74	9.39	1.62	1.63	2.36
Maximum Queueing Time	3.78	38.25	71.44	19.32	25.19	13.02

Table 11: Improvement Percent of Adjustment Model for Adjusted Station.

	Station	Improvement of Queuing Time	Improvement Utilization of Resources
Simulation Model 2	OPD	50.5%	-8.33%
	Dental	60.8%	-20.67%
Simulation Model 3	OPD	0%	7.31%
	Dental	0%	16.66%

From Table 10 we can see that the increase of number of doctors will improve the queuing size and queuing time in terms of average and also maximum. While for dentist, we can see that when we increase the number of it, the queuing time and queuing size reduce significantly. However, from Table 11, we can see that by increasing the resources might not only bring the good effect but also bad effect. For those increased the number of resources, the utilization of the resources is reduced which indicates waste of cost. The ideal adjustment for the queuing system is to have a great improvement in queuing time and least decline in percentage for resources. However, based on the results in Table 11, none of them is best solution in terms of ideal adjustment mentioned. But depends on the main objective, to reduce the waiting time with optimised number of resources, simulation model 2 might be the ideal solution.

Conclusion

This research investigates the effectiveness of the queuing system of PKU UTM. Primary data were obtained from PKU UTM for 1 Jan 2023 to 31 March 2023. As mentioned earlier in the introduction, the purposes of this study were to apply queuing theory and construct simulation of the queuing system of PKU UTM and also investigate the number of resources to achieve optimal queuing time. By applying queuing theory to the queuing system in PKU, we realise that the queuing system in PKU UTM is improvable. First, the average time spent of a patient in the system is 27.1 minutes while at the peak hour, the maximum time spent of a patient is increased to 128.77 minutes. The time of patients is wasted just to wait in the queue for service. Consequently, patients that are frustrated for waiting too long and just leave without getting service. The results of this study highlighted that the allocation of resources is an important factor which will affect the time spent of patients in the PKU UTM system. The ideal model for PKU system is increase 1 unit of doctor and dentist during night shift will help to reduce the maximum waiting time in the specific station band for the overall, the average time spent of a patient in the system will be reduced to 24 minutes and the maximum time spent of a patient is reduced to 116 minutes. However, these findings cannot be generalized to other healthcare centre as the situation and condition such as arrival rate of patient, number of resources will be different. However, the same study can be conducted to investigate the problem and improve the queuing system in any other healthcare centre.

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